Identity over Time

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Abstract. Young Socrates is the same person as old Socrates, but, differing in age, they cannot be absolutely identical. Leibnizian substitutivity of identity—one of the two great principles of absolute identity—fails for the terms "young Socrates" and "old Socrates". For substituting the former term for the latter one in the sentence "Old Socrates was sentenced to death" yields a falsehood; young Socrates has not been sentenced to death. Thus identity over time cannot be a species of absolute or Leibnizian identity. It's got to be a species of relative identity, then. The purpose of this paper is to offer an event-ontological clarification of this notion.

Key words: biography, character, event, identity over time, person, personal identity, relative identity

1 Introduction

Event-ontology treats natural individuals as events in space and time. Since the spatial dimension of event-ontology is still under construction I have to confine my treatment to their temporal dimension. All I need for doing so is the notion of a temporal ordering, that is an ordered pair $\langle T, < \rangle$, consisting of a non-empty set T and a two-place relation $<$ on T. The non-empty set is comprised of moments or instants of time; and the two-place relation is the earlier/later relation. Globally we shall not require more than that it be a subset of $T \times T$; but locally it may be irreflexive, transitive and branching towards the future. Because we permit branching some people might prefer to characterize the elements of T as tempo-modal rather than as temporal. In any case, my main concern is with their point-likeness. My notion of a point is not Euclidean; for Euclidean points do not have parts. Points in my sense do because in my ontology something is a point of a given type iff it can be bisected so that none of the two parts is of that type again. So the elements of the set T are required to be points of type T. Hence they may be years or days or milliseconds because none of the parts of these stretches of time are years or days or milliseconds in turn. The point requirement prohibits that material from T may be used to construct temporal entities which can be found among this very material. The upshot of our requirement is a sharp distinction between elements of construction and constructed items.

2 Biographies

Starting from our rather thin notion of a temporal ordering, we shall first introduce the concept of biography. This notion is needed in order to say what it means to age. Aging is kindred to living. For he who is living has lived and therefore must have grown older. As it appears aging or, for that matter, living is an event that combines occurring with having occurred. This combination lies at the heart of an individual's biography. Moreover, natural individuals have options for their future being. This presupposes the availability of alternative future courses of events. But having such options would be futile if we could not live to see these events occur. Hence biographies seem to branch towards the future and to exhibit a kind of hand-likeness.

Roughly speaking, biographies are events which contain temporal stretches of growing duration, each stretch corresponding to a particular age. This intuitive idea can be spelled out by a couple of definitions starting out from a rigorous definition of "stretch of time" or, as I prefer to say, "occurrence". For events are sets of occurrences. The event called "sunset", for example, is the totality of the particular episodes which consist in the sun's setting.

Definition 1. "occurrence"
v is an occurrence on $\langle T, < \rangle$ iff
(a) $v \subseteq T$ and $v \neq \emptyset$;
(b) for all $t, t' \in v$: $t < t'$ or $t = t'$ or $t' < t$;
(c) for all $t, t' \in v$: if $t < t'$ then not $t' < t$;
(d) for all $t, t', t'' \in v$: if $t < t'$ and $t' < t''$ then $t < t''$;
(e) for all $t, t'' \in v$ and $t' \in T$: if $t < t'$ and $t' < t''$ then $t' \in v$.

A set of occurrences on $\langle T, < \rangle$ will be called "a (generic) event on $\langle T, < \rangle$".

Consider an event that resembles the event of aging in that every occurrence harbours each of its initial segments. Living is such an event; opening a door is another one. And if Aristotle is correct even seeing belongs to this kind of events. He called events exhibiting this temporal structure "*energeiai*" [1, Θ 6]. I propose to use the adjective "entelic"[1] for them which may be defined as follows:

Definition 2. "entelicity"
ϵ is (an) entelic (event) on $\langle T, < \rangle$ iff (a) ϵ is a (generic) event on $\langle T, < \rangle$; (b) for all $v \in \epsilon$ and for all occurrences $v' \subseteq v$: if there is no $t \in v$ such that $t < v'$ then $v' \in \epsilon$.[2]

Although each biography has a unique past it is open to several future developments. Hence all occurrences belonging to one and the same biography must have the same past. Moreover, belonging to the same biography, they must have the same origin in time, that is, they must share an initial segment. But their

[1] In our days some people classify them as telic [2] and some people as atelic [3]. Since I do not want to stir up the wrong associations in my readers I suggest to use a completely new term which still has an Aristotelian ring about it.

[2] The formula "$t \in v'$" is short for "for all t': if $t' \in v'$ then $t < t'$".

behaviour vis-à-vis the future can be left undetermined. Therefore biographies present themselves in the form of trees or hands.

Definition 3. "hand-likeness"
ϵ is (a) hand-like (event) on $\langle T, <\rangle$ iff (a) ϵ is a (generic) event on $\langle T, <\rangle$; (b) for all $v, v' \in \epsilon$ and $t \in T$: $t < v$ iff $t < v'$; (c) for all $v, v' \in \epsilon$: $(v \cap v') \neq \emptyset$.

A (generic) event on $\langle T, <\rangle$ which is non-empty, entelic, and hand-like will be called "biography on $\langle T, <\rangle$".

If a temporal ordering $\langle T, <\rangle$ is to contain biographies it must possess a subset S such that $<$ is asymmetric, transitive, fusionless[3] and pentachotomous on S, as is clear from the following theorem:

Theorem 1. *If κ is a biography on $\langle T, <\rangle$ then $<$ has the following properties on $\bigcup \kappa$:*
(a) asymmetry: $\forall t, t' \in \bigcup \kappa \, (t < t' \rightarrow \neg t' < t)$;
(b) transitivity: $\forall t, t', t'' \in \bigcup \kappa \, ((t < t' \wedge t' < t'') \rightarrow t < t'')$;
(c) fusionlessness: $\forall t, t', t'' \in \bigcup \kappa \, ((t < t'' \wedge t' < t'') \rightarrow (t = t' \vee t < t' \vee t' < t))$;
(d) pentachotomy: $\forall t, t' \in \bigcup \kappa \, (t < t' \;\vee\; t = t' \;\vee\; t' < t \;\vee$
$\exists t'' \in \bigcup \kappa \, (t < t'' \vee t' < t'') \;\vee\; \exists t'' \in \bigcup \kappa \, (t'' < t \vee t'' < t'))$.[4]

There are different kinds of biographies. Some of them do not only grow in time but also spread over alternative histories. Some of them are flat like a line and might therefore be called "linear". Here is a definition of this notion:

Definition 4. "linear biography"
κ is a linear biography on $\langle T, <\rangle$ iff (a) κ is a biography on $\langle T, <\rangle$; (b) $\bigcup \kappa \in \kappa$.

If two linear biographies on $\langle T, <\rangle$ have a non-empty intersection this intersection is a linear biography on $\langle T, <\rangle$.[5]

3 Natural Individuals

Natural individuals are entities with both a temporal and a spatial dimension. Their temporal dimension is a biography. Their spatial dimension consists in a trajectory through space. To simplify matters I'll reduce the trajectory of a given individual to its birthplace. My reason for this reduction is that the exploration of the spatial dimension of natural individuals is still work in progress.

Since our experience presents time and temporal episodes to us in a linear order we may forget about individuals whose biographies are voluminous like trees or hands and concentrate on individuals with a linear biography. They might aptly be called "l-individuals". Here is a definition of them:

Definition 5. "l-individual"
$\langle \kappa, \gamma \rangle$ is a l-individual on $\langle T, <\rangle$ iff (a) κ is a linear biography on $\langle T, <\rangle$; (b) γ is the birthplace of $\langle \kappa, \gamma \rangle$.

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[3] Sometimes called "left linear" or "linear in the past".
[4] For a proof see [6, theorem 5.1].
[5] This holds even for biographies in general. For a proof of the general claim see [6, lemma 5.2 (2)].

4 L-individuals and their Identity

Although a l-individual is set-theoretically included in a single linear history its intuitive counterpart may be found in different alternative histories. This brings me to the gist of my argument. Since l-individuals are set-theoretical constructs they possess a set-theoretical identity. It is well known that this identity consists in identity of extension. But this is only one way of construing relative identity; identity of origin is another one. There are spatiotemporal origins and there are material origins. Since I do not consider the material dimension of natural individuals in this paper I want to focus on identity of spatiotemporal origin.

It is characteristic of l-individuals that they bear being divided. Take Mary's little lamb. If we cut off a piece of its skin we'll have two parts, namely one part being a lamb and the other one not. This procedure can be applied to l-individuals in particular as well as to individuals in general. We may even define an individual of type $\mathcal{T}$ to be something that can be cut into two pieces such that just one of them is of type $\mathcal{T}$, again. If we bisect a biography which is long enough to be divided at all we shall find out that it is an individual of type *biography*.[6]

Take two occurrences of one and the same biography. They must contain at least one initial occurrence in common, since sharing an initial occurrence guarantees a biography's identity over time. For the same reason it guarantees its identity across (possible) worlds or, if you prefer, world histories. Therefore we may consider two set-theoretically different biographies to be one and the same biography iff their set-theoretical intersection is not empty. The relation of being the same biography is an equivalence relation on the set of biographies.[7]

Since a l-individual is composed of a biography and a birthplace we may define its identity as follows:

Definition 6. "l-individual identity"
$\langle\kappa, \gamma\rangle$ is the same l-individual on $\langle T, <\rangle$ as $\langle\kappa', \gamma'\rangle$ iff (a) $\kappa \cap \kappa'$ is a linear biography on $\langle T, <\rangle$; (b) $\gamma = \gamma'$.

According to clause (a) κ and κ' must share at least one occurrence. Of course, this occurrence must include the origins both of κ and of κ'. Therefore our definition of l-individual identity realizes the idea of identity as identity of origin.

Let's condense these remarks into another theorem:

Theorem 2. *L-individual identity on $\langle T, <\rangle$ is an equivalence relation on the set of all l-individuals on $\langle T, <\rangle$.*[8]

5 Characters, Persons and their Identity

Persons are l-individuals who are endowed with a capability to choose. Typical specimens of persons are adult human beings. They are able to compare the

[6] For a proof see [6, lemma 5.3].
[7] The proof of this claim is an immediate corollary to theorem 5.3 in [6].
[8] For a proof see [6, theorem 5.6].

value of objects, properties, relations, states of affairs, events, actions, reasons, feelings, and so on, and so forth. They are able to evaluate both themselves and one another. They are able to estimate the quality of their lives and to choose among shorter or longer periods thereof. They are able to judge episodes in the past and to distinguish between good and bad actions to be done in the future. I propose to call their capability to compare, to evaluate, to estimate, to choose, to judge, to distinguish, *etc.* "character". It would be a gross mistake to conclude from this sketchy explanation that my understanding of the word "character" deviates considerably from its usual meaning. Usually, character is taken to be "the particular nature which distinguishes human beings from one another" [5, *s. v.* "*Charakter*"]. That is exactly what I have in mind.

People say things like "She is a good mother", "Your reply was good", "The new knife cuts well", "It's not good that you did not show up yesterday", thereby informing us about the results of their comparisons, evaluations, estimations, choices, judgments, and distinctions. As these examples suggest, and are meant to suggest, people can be characterized by what they choose to be good. This fact motivates my conception of character as something to do with choosing what is taken to be good. Let me emphasize that I am far from restricting a person's choices to what is socially, let alone morally, good; I want to include everything that she considers to be a good alternative in one respect or another. As I understand the adjective "good", its uses by default express the user's individual choices. So they unveil her particular nature and make it publicly accessible to everybody.

These considerations suggest that the study of the adjective "good" plays an important methodical role in our enquiry into the nature of character. Let's have a closer look at it, then, and consider the statement "Good apples are apples". Of course, that's true. Good apples do form a part of the class of apples. Extrapolating from this fact, we may describe what the adjective "good" stands for as the device which assigns to a given totality a uniquely determined particular choice thereof. Nobody would want to deny that good apples are a choice of apples, to be sure; but why should this choice be unique? Why identify the classes of apples which are good with respect to taste and which are good with respect to look? Obviously, the results of our choices depend on the respect of comparison. But that is not enough. We have to take into account the moment of choice as well. Thus we arrive at the conclusion that "good" stands for a three-place choice function $\mathfrak{g}$ which maps a moment t, a respect r and a comparison class[9] X onto a subset of X.

The fact that $\mathfrak{g}$ is a choice function can be rendered by the following postulate:

$$\mathfrak{g}(t, r, X) \subseteq X \,. \tag{1}$$

It is pretty clear that there must be some further postulates which regiment the behaviour of our choice function; and it is equally clear that they will restrict the chooser's predilections and aversions. But which restrictions are tolerable? Since there is a semantical connection between the positive "good" on the one

[9] This technical term for the class to choose from was coined by R. M. Hare in [4].

hand and the comparatives "better" and "equally good" on the other we should accept only postulates that contribute to establishing this connection. Thus we eventually arrive at the following postulates:

$$\text{If } X \subseteq Y \text{ and } \mathfrak{g}(t,r,X) \neq \emptyset \text{ then } \mathfrak{g}(t,r,Y) \neq \emptyset\,. \tag{2}$$

$$\text{If } x,y \in X \text{ then } (\mathfrak{g}(t,r,X) \cap \{x,y\}) \subseteq \mathfrak{g}(t,r,\{x,y\})\,. \tag{3}$$

$$\text{If } x \in \mathfrak{g}(t,r,\{x,y,z\}) \text{ then } \mathfrak{g}(t,r,\{x,y\}) \subseteq \mathfrak{g}(t,r,\{x,y,z\})\,. \tag{4}$$

Let's define the comparatives "better" and "equally good" as follows:

At t and with respect to r, x is better than $y :=$
$$x \in \mathfrak{g}(t,r,\{x,y\}) \text{ and } y \notin \mathfrak{g}(t,r,\{x,y\})\,.$$
At t and with respect to r, x and y are equally good $:=$
$$x \in \mathfrak{g}(t,r,\{x,y\}) \text{ iff } y \in \mathfrak{g}(t,r,\{x,y\})\,.$$

Our next theorem shows that our four choice postulates are sufficient for establishing the semantic connection between "good" and its comparatives:

Theorem 3. *Every function which satisfies postulates (1)–(4) induces on any given set S an ordering ⟨ better, equally good ⟩ with the following properties:*
(a) better *is transitive on S;*
(b) equally good *is an equivalence relation on S;*
(c) any two elements of S stand in just one of these two relations.[10]

Obviously, sometimes our choices are identical, sometimes they are different, and sometimes there is no choice at all. The last possibility is due to the fact that we do not always choose. Sometimes we are asleep, and sometimes we are not choosing because we are busy with something else. In order to account for the possibility of not choosing we are well advised to model the notion of character with partial choice functions. Thus we arrive at the following definition:

Definition 7. "character"
$\mathfrak{g}$ is the character of $\langle \kappa, \gamma \rangle$ on M iff $\langle \kappa, \gamma \rangle$ is a l-individual on $\langle T, < \rangle$; (b) $M \subseteq \bigcup \kappa$; (c) $\mathfrak{g}$ is a (possibly partial) choice function whose temporal arguments are in M and which satisfies postulates (1)–(4).

The character of $\langle \kappa, \gamma \rangle$ on $\bigcup \kappa$ will be called "the character of $\langle \kappa, \gamma \rangle$".

Unfortunately we cannot define persons as l-individuals endowed with a character because this definition does not make personal identity into an equivalence relation. For consider the following line diagram and suppose that each of the three persons π_1, π_2 and π_3 reaches from its beginning to the very end of the line that is tagged with her name and that all of them have the same birthplace. Since π_1, π_2 and π_3 share the occurrence that reaches from the beginning of the figure to the first branching point the intersections of their biographies are not empty. Therefore and because π_1, π_2 and π_3 have the same birthplace they are one and the same l-individual. In particular, π_3 is the same l-individual as π_1

[10] For a proof see [6, theorem 5.16].

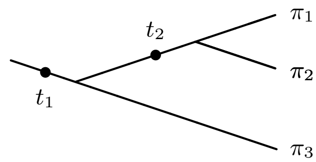

Fig. 1. Identity and possible futures

and π_2 notwithstanding the fact that π_3 does not occur at t_2. Otherwise it were not one and the same person who, at t_1, has three options.

But being the same l-individual is not sufficient for being the same person. Since the character of a person is defined on her biography, π_1, π_2 and π_3 may choose different things from the same class and in the same respect at t_1; and π_1 and π_2 may choose different things from the same class and in the same respect at t_2. If so our three l-individuals have different characters and therefore different personal identities. But this would permit a person's possible future past a given moment to influence her choice at this very moment. If we want to preclude this pathological kind of backward influence we should see to it that a person's choice at a given moment does not depend on her possible futures. Let's call persons whose characters have this property "common persons". This notion may be defined as follows:

Definition 8. "common person"
$\langle\kappa,\gamma\rangle$ is a common person on $\langle T,<\rangle$ iff (a) $\langle\kappa,\gamma\rangle$ is a l-individual on $\langle T,<\rangle$; (b) there is a $\mathfrak{g}$ such that $\mathfrak{g}$ is the character of $\langle\kappa,\gamma\rangle$ on $\bigcup\kappa$ and for all l-individuals $\langle\kappa',\gamma'\rangle$, characters $\mathfrak{h}$, moments t, respects r and classes X of comparison holds: if (i) $\langle\kappa',\gamma'\rangle$ is the same l-individual on $\langle T,<\rangle$ as $\langle\kappa,\gamma\rangle$, (ii) $\mathfrak{h}$ is the character of $\langle\kappa',\gamma'\rangle$ on $\bigcup\kappa'$, (iii) $t\in(\bigcup\kappa\cap\bigcup\kappa')$, and (iv) $\langle t,r,X\rangle\in\mathsf{dom}(\mathfrak{g})$ then $\mathfrak{g}(t,r,X)=\mathfrak{h}(t,r,X)$.

Given this notion of a common person we define the pertaining identity relation as follows:

Definition 9. "personal identity"
$\langle\kappa,\gamma\rangle$ is the same (common) person on $\langle T,<\rangle$ as $\langle\kappa',\gamma'\rangle$ iff (a) $\langle\kappa,\gamma\rangle$ is the same l-individual on $\langle T,<\rangle$ as $\langle\kappa',\gamma'\rangle$; (b) there are $\mathfrak{g}$ and $\mathfrak{g}'$ such that $\mathfrak{g}$ is the character of $\langle\kappa,\gamma\rangle$ on $\bigcup\kappa$, $\mathfrak{g}'$ is the character of $\langle\kappa',\gamma'\rangle$ on $\bigcup\kappa'$, and $\mathfrak{g}\cap\mathfrak{g}'$ is the character of $\langle\kappa\cap\kappa',\gamma\rangle$ on $\bigcup(\kappa\cap\kappa')$.[11]

That this definition induces an equivalence relation is the message of our final theorem:

Theorem 4. *Personal identity on $\langle T,<\rangle$ is an equivalence relation on the set of all common persons on $\langle T,<\rangle$.*[12]

[11] Clause (a) implies that $\gamma=\gamma'$ so that it doesn't matter whether we take $\langle\kappa\cap\kappa',\gamma\rangle$ or $\langle\kappa\cap\kappa',\gamma'\rangle$ as the bearer of the character $\mathfrak{g}\cap\mathfrak{g}'$.

[12] For a proof see [6, theorem 5.17].

As may be gathered from the very last conjunct of definition 9, identity of character is a matter of origin, too. Therefore personal identity itself, being composed of l-individual identity and identity of character, is a kind of identity of origin.

6 Concluding Remarks

The goal of my paper was to clarify the notion of identity over time. Because of time restrictions, I confined myself to elucidating the notion of personal identity which is the most interesting kind of identity over time in philosophy. If we treat young and old Socrates as common persons we can claim both that young Socrates $\neq$ old Socrates and that nevertheless young Socrates is the same person as old Socrates without contradicting ourselves.

Where else can we apply our results? Being myself a philosopher, I must forewarn you that my answer might need some professional rephrasing before it may appear plausible to you. People in the software business tell me that simulation may be an appropriate field of application. The objects of a simulation fare like persons in my sense. After instantiation, which is comparable to a person's birth, the objects live on without a corporeal component. Only some medium or other is needed in order to store their properties. Now, a person possesses a history, called "biography", and the capability to choose from alternatives, called "character". Since her character may change over time she is able to learn and so are the objects of a simulated social system.

Another field of application is software engineering. Any use of software leaves some traces in the form of adjustments, installations of extensions, service packs, patches *etc.* Normally, people tend to forget that every software has its own history. Taking its history to be a biography, the software could be treated as a kind of l-individual. This treatment might contribute to solve problems that arise in software identification.

Acknowledgments. I want to thank Reinhold Kienzle, Herzogenaurach, as well as the anonymous referees for their comments. The presentation of this paper was made possible through a travel allowance by *Exzellenzförderinitiave Mecklenburg-Vorpommern.*

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