

A Process-Ontological Model: A More Formal Approach

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Abstract. The term of a “process” is used in Software Engineering (SE) theories and practices in many different ways, which cause confusion. In this paper we give a more formal description a Process-Ontological Model which can be used to analyze some problematic nature of software engineering. Firstly we present a process ontology in which everything is in a process. There are two kinds of processes: “eternal” and actual, where actual processes are divided into physical and mental processes. Secondly, we propose a topological model T for actual processes. Thirdly we propose an algebraic model for eternal processes, i.e. concepts. Lastly, by using category theory we connect these two models of processes together in order to get a category theoretical description of the Process-Ontological Model. That model is a functor category $\mathcal{C}^{O(T)^{op}}$, i.e. the category of presheaves of concepts on T .

Keywords. Process, ontology, modelling, software engineering, concept

1 Introduction

The term of a “process” is used in many widely known theories and essential practices of Software Engineering (SE). “Process thinking” has become one of the major efforts to make software engineering to an engineering, which has a qualitative value. To serve this aim we are developing a Process-Ontological Model (POM), see [1], [2]. In this paper we will present some of the basic ideas of POM more formally. We think that a process-ontology will provide an appropriate philosophical and conceptual framework for the SE researches as well as for the SE practice. It gives a possibility to compare different SE models and concepts and to interpret the dependencies between them. For example, in [2] that model was using to positioning different standards.

This paper is composed as follows. Firstly we will shortly consider a process ontology in which everything is in a process. Our overall view will be that everything in the world is composed of processes. Secondly, we propose a topological model for actual processes. Thirdly we propose an algebraic model for eternal processes, i.e.

concepts. Lastly, by using a category theory we are connecting these two models together to get a category theoretical description of the Process-Ontological Model.

2 A Process-Ontology

In *Process and Reality* [3], Alfred North Whitehead presented a view that the world is a process which is the becoming of *actual entities* (or actual occasions). There are also *eternal objects* to be understood as conceptual objects. As potentialities they enter into the actual entity becoming concrete without being actualities themselves. Thus, everything consists of processes, and that these processes are divided into eternal processes interpreted as *concepts*, and actual processes, which we will interpret to be events occupying a finite amount of a four dimensional space-time. Thus, the world is constructed out of *events*.

Accordingly, from the ontological point of view, everything consists of processes. Among processes, firstly, there are eternal processes and actual processes. Eternal processes are interpreted as concepts, whereas actual processes are interpreted as space-time events. Eternal processes are instantiated in actual processes. Secondly, among actual processes there are mental events and physical events. Mental events consist of bundles of compresent qualities which can be known without inferences, whereas physical events, if known, are known only by inference as regards to their space-time structure.

3 A Topological Model for Actual Processes

We shall give a topological model for actual processes, in which events are interpreted as *open sets*, i.e., space-time events will have a one-one correspondence with four-dimensional open sets. To get an idea, a few topological concepts are defined as follows. Consider a set T . Let $\{O_{i \in I}\}$ to be a set of open subsets of T satisfying the following axioms:

- A1 The union of any number of open sets is an open set.
- A2 The intersection of two open sets is an open set.
- A3 T itself and the empty set \emptyset are open sets.

A *topology* on a set T is then the specification of open subsets of T which satisfy these axioms, and this set T is called a *topological space*.

A set of open subsets $\{O_{i \in I}\}$ of T is said to be an *open covering* of T , if the union of $O_{i \in I}$ contains T . An open covering $\{V_{j \in J}\}$ of a space T is said to be a *refinement* of an open covering $\{O_{i \in I}\}$, if for each element V_j of $\{V_{j \in J}\}$ there is an element O_i of $\{O_{i \in I}\}$ such that $V_j \subseteq O_i$. If $\{O_{i \in I}\}$ is any open covering of T , and there is some finite subset $\{O_{i_1}, O_{i_2}, \dots, O_{i_m}\}$ of $\{O_{i \in I}\}$, then a space T is called a *compact*.

A topological space T is *separated*, if it is the union of two disjoint, non-empty open sets. A space T is *connected*, if it is not separated. A space T is said to be *path-*

connected if for any two points x and y in T there exists a continuous function f from the unit interval $[0, 1]$ to T with $f(0) = x$ and $f(1) = y$. This function is called a *path* from x to y . A space T is *simply connected* if only if it is a path connected, and it has no “holes”.¹ A space T , which is *connected*, but not simply connected, is called *multiply connected*.

Given two points a and b of a space T , a set $\{O_1, O_2, \dots, O_n\}$ of open sets is a *simple chain from a to b* provided that O_1 (and only O_1) contains a , O_n (and only O_n) contains b , and $O_i \cap O_j$ is non-empty if and only if $|i - j| \leq 1$. That is, each link intersects just the one before it and the one after it, and, of course, itself. It can be proved that if a and b are two points of connected space T , and $\{O_{i \in I}\}$ is a set of open sets covering T , then there is a simple chain of elements of $\{O_{i \in I}\}$ from a to b , (for the proof, see the Theorem 3-4 in [4]). Moreover, let $C_1 = \{O_{11}, O_{12}, \dots, O_{1n}\}$ and $C_2 = \{O_{21}, O_{22}, \dots, O_{2m}\}$ be simple chains from a point a to a point b in a space T . The chain C_2 will be said to *go straight through* C_1 provided that i) every set O_{2i} is contained to some set O_{1j} and ii) if O_{2i} and O_{2k} , $i < k$, both lie in a set O_{1r} , then for every integer j , $i < j < k$, O_{2j} also lies in O_{1r} . Accordingly, the finer chain C_2 goes straight through the coarser chain C_1 .

Given two points a and b of a space T , we can define a *valuation* on T as a function $v: T \rightarrow \mathbf{R}$, where \mathbf{R} is the set of real numbers, having the following properties: i) $v(a) = r$, and ii) $v(b) = s$. Then there is a non-negative real number $|s - r|$ called the *distance* between a and b , which is denoted as $d_v(a, b)$. For each point x, y , and z in T , the distance from x to y satisfy the following four properties: 1) $d_v(x, y) \geq 0$, 2) $d_v(x, y) = 0 \Leftrightarrow x = y$, 3) $d_v(x, y) = d_v(y, x)$, and 4) $d_v(x, z) \leq d_v(x, y) + d_v(y, z)$. Thus the space T is a *metrizable*.

A topological model for actual processes is used as follows: a process as a whole is interpreted as a topological space T , which, at least for empirical reasons, is compact and, depending on the number of parallel processes, is either a simply- or a multiply connected. The space T contains a start point a and an end point b of the process. The start point a is an event, which is included in the open set O_1 , and, similarly, the endpoint b is an event, which is included in the open set O_n . The simple chain from a to b consists of sequences of events interpreted as a set $\{O_1, O_2, \dots, O_n\}$ of open sets. Moreover, it is possible to get as coarse or as fine a chain from a to b as necessary. In a case there are parallel processes, i.e., processes which we want to keep distinct in a certain moment, for example feedbacks, we just add “holes” to our space T . This prevents the parallel processes from deforming to each other. The space T will then be multiply connected. In addition, the space T can be made a metrizable space as well.

¹ More formally, a path-connected space T is *simply connected* if given two points a and b in T and two paths $p: [0, 1] \rightarrow T$ and $q: [0, 1] \rightarrow T$ joining a and b , i.e., $p(0) = q(0) = a$ and $p(1) = q(1) = b$, there exists a homotopy in T between p and q . Two maps $p, q: X \rightarrow Y$ are said to be *homotopic* if there is a map $H: [0, 1] \times X \rightarrow Y$ such that for each point x in X , $H(0, x) = p(x)$ and $H(1, x) = q(x)$. The map H is called a *homotopy* between p and q . Intuitively, maps p and q are homotopic, if p can be continuously deformed to get q while keeping the endpoints fixed, and a path-connected space T is simply connected, if every closed path in T can be continuously deformed into a point.

4 An Algebraic Model for Eternal Processes

We will interpret eternal processes as concepts, i.e. they are like “frozen” processes. The relations between concepts enable us to make *conceptual structures*, where the basic relation between concepts is an intensional containment relation [5], [6], [7], [8]. Based on the intensional containment relation, we have an axiomatic intensional concept theory, denoted by *KC*, in a first-order language *L* that contains individual variables *a*, *b*, *c*,..., which range over the *concepts*, and one non-logical 2-place *intensional containment relation*, denoted by “ \geq ”. We shall first present four basic relations between concepts defined by “ \geq ”, and then, briefly, the basic axioms of the theory. A more complete presentation of this theory, see [5], and Palomäki [7].

Two concepts *a* and *b* are said to be *comparable*, denoted by $a \text{ H } b$, if there exists a concept *x* which is intensionally contained in both.

$$\text{Df}_{\text{H}} \quad a \text{ H } b =_{\text{df}} (\exists x) (a \geq x \wedge b \geq x).$$

If two concepts *a* and *b* are not comparable, they are *incomparable*, which is denoted by $a \text{ I } b$.

$$\text{Df}_{\text{I}} \quad a \text{ I } b =_{\text{df}} \sim a \text{ H } b.$$

Dually, two concepts *a* and *b* are said to be *compatible*, denoted by $a \perp b$, if there exists a concept *x* which contains intensionally both.

$$\text{Df}_{\perp} \quad a \perp b =_{\text{df}} (\exists x) (x \geq a \wedge x \geq b).$$

If two concepts *a* and *b* are not compatible, they are *incompatible*, which is denoted by $a \text{ Y } b$.

$$\text{Df}_{\text{Y}} \quad a \text{ Y } b =_{\text{df}} \sim a \perp b.$$

The two first axioms of *KC* state that the intensional containment relation is a *reflexive* and *transitive* relation.

$$\begin{aligned} \text{AX}_{\text{Ref}} \quad & a \geq a. \\ \text{AX}_{\text{Trans}} \quad & a \geq b \wedge b \geq c \rightarrow a \geq c. \end{aligned}$$

Two concepts *a* and *b* are said to be *intensionally identical*, denoted by $a \approx b$, if the concept *a* intensionally contains the concept *b*, and the concept *b* intensionally contains the concept *a*.

$$\text{Df}_{\approx} \quad a \approx b =_{\text{df}} a \geq b \wedge b \geq a.$$

The intensional identity is clearly a reflexive, symmetric and transitive relation, hence an equivalence relation.

A concept *c* is called an *intensional product* of two concepts *a* and *b*, if any concept *x* is intensionally contained in *c* if and only if it is intensionally contained in both *a* and *b*. If two concepts *a* and *b* have an intensional product, it is unique up to the intensional identity and we denote it then by $a \otimes b$.

$$\text{Df}_{\otimes} \quad c \approx a \otimes b \stackrel{\text{df}}{=} (\forall x) (c \geq x \leftrightarrow a \geq x \wedge b \geq x).$$

The following axiom Ax_{\otimes} of KC states that if two concepts a and b are comparable, there exists a concept x which is their intensional product.

$$\text{Ax}_{\otimes} \quad a \text{ H } b \rightarrow (\exists x) (x \approx a \otimes b).$$

It is easy to show that the intensional product is idempotent, commutative, and associative.

A concept c is called an *intensional sum* of two concepts a and b , if the concept c is intensionally contained in any concept x if and only if it contains intensionally both a and b . If two concepts a and b have an intensional sum, it is unique up to the intensional identity and we denote it then by $a \oplus b$.

$$\text{Df}_{\oplus} \quad c \approx a \oplus b \stackrel{\text{df}}{=} (\forall x) (x \geq c \leftrightarrow x \geq a \wedge x \geq b).$$

The following axiom Ax_{\oplus} of KC states that if two concepts a and b are compatible, there exists a concept x which is their intensional sum.

$$\text{Ax}_{\oplus} \quad a \perp b \rightarrow (\exists x) (x \approx a \oplus b).$$

The intensional sum is idempotent, commutative, and associative.

The next axiom of KC concerns the distributivity of an intensional sum and a product whenever both sides are defined,

$$\text{Ax}_{\text{Distr}} \quad (a \otimes b) \oplus (a \otimes c) \geq a \otimes (b \oplus c).$$

A concept b is an *intensional negation* of a concept a , denoted by $\neg a$, if and only if it is intensionally contained in all those concepts x , which are intensionally incompatible with the concept a . When $\neg a$ exists, it is unique up to the intensional identity.

$$\text{Df}_{\neg} \quad b \approx \neg a \stackrel{\text{df}}{=} (\forall x) (x \geq b \leftrightarrow x \text{ Y } a).$$

The following axiom Ax_{\neg} of KC states that if there is a concept x which is incompatible with the concept a , there exists a concept y , which is the intensional negation of the concept a .

$$\text{Ax}_{\neg} \quad (\exists x) (x \text{ Y } a) \rightarrow (\exists y) (y \approx \neg a).$$

However, to prove that $a \approx \neg \neg a$ we need a further axiom,

$$\text{Ax}_{\neg\neg} \quad b \text{ Y } \neg a \rightarrow b \geq a.$$

If a concept a is intensionally contained in every concept x , the concept a is called a *general concept*, and it is denoted by G . The general concept is unique up to the intensional identity, and it is defined as follows:

$$\text{Df}_G \quad a \approx G \stackrel{\text{df}}{=} (\forall x) (x \geq a).$$

The next axiom of KC states that there is a concept, which is intensionally contained in every concept.

$$\text{Ax}_G \quad (\exists x)(\forall y) (y \geq x).$$

Adopting the axiom of the general concept it follows that all concepts are to be comparable.

A *special concept* is a concept a , which is not intensionally contained in any other concept except for concepts intensionally identical to itself. Thus, there can be many special concepts.

$$\text{Df}_S \quad S(a) =_{\text{df}} (\forall x) (x \geq a \rightarrow a \geq x).$$

The last axiom of *KC* states that there is for any concept y a special concept x in which it is intensionally contained.

$$\text{Ax}_S \quad (\forall y)(\exists x) (S(x) \wedge x \geq y).$$

By Completeness Theorem, every consistent first-order theory has a model. Accordingly, it is shown in [7] that a model of *KC* is a *complete semilattice*, where every concept $a \in C$ defines a *Boolean algebra* $B_a = \langle \downarrow a, \otimes, \oplus, \neg, G, a \rangle$, where $\downarrow a$ is an ideal, known as the *principal ideal generated by a*, i.e. $\downarrow a =_{\text{df}} \{x \in C \mid a \geq x\}$, and the intensional negation of a concept $b \in \downarrow a$ is interpreted as a *relative complement of a*.

5 Putting Things Together: A Process-Ontological Model

A basic idea behind a Process-Ontological Model is that everything consists of processes. There are two kinds of processes: eternal processes, which are interpreted as concepts, and actual processes, which are interpreted as space-time events. Moreover, actual processes are either mental or physical. Now, eternal processes are instantiated in actual processes. So, given the models for actual processes and eternal processes, i.e. the topological model for actual processes and the algebraic model for eternal processes, we should put these two models together. For this purpose we will use category theory, but owing to the limitation of space, only rudiment of it is presented just to get an idea.

Let \mathbf{X} be a set of object, x, y, z, \dots together with two functions as follows:

- i) A function assigning to each pair (x, y) of objects of \mathbf{X} a set $\text{hom}_{\mathbf{X}}(x, y)$. An element $f \in \text{hom}_{\mathbf{X}}(x, y)$ is called an *arrow* $f: x \rightarrow y$, with *domain* x and *codomain* y .
- ii) A function assigning to each triple (x, y, z) of objects of \mathbf{X} a function $\text{hom}_{\mathbf{X}}(x, z) \times \text{hom}_{\mathbf{X}}(x, y) \rightarrow \text{hom}_{\mathbf{X}}(x, z)$. For arrows $g: y \rightarrow z$ and $f: x \rightarrow y$, the function is written as $g \circ f: x \rightarrow z$, and it is called the *composite* of f and g .

The set \mathbf{X} with these two functions is called a *category*, if the following two axioms hold:

- C 1 *Associativity*: If $h: z \rightarrow w$, $g: y \rightarrow z$ and $f: x \rightarrow y$ are arrows of \mathbf{X} with indicated domains and codomains, then $h \circ (g \circ f) = (h \circ g) \circ f$.

- C 2 *Identity*: For each object y in \mathbf{X} there exists an *identity arrow* $1_y: y \rightarrow y$ such that if $f: x \rightarrow y$, then $1_y \circ f = f$, and if $g: y \rightarrow z$, then $g \circ 1_y = g$.

Given a category \mathbf{X} , we can form a new category \mathbf{X}^{op} , called the *opposite category* of \mathbf{X} , by taking the same objects but reversing the direction of all arrows and the order of compositions.

If \mathbf{X} and \mathbf{Y} are two categories, a *functor* $F: \mathbf{X} \rightarrow \mathbf{Y}$ is a pair of functions; an *object function*, which assigns to each object x of \mathbf{X} an object $F(x)$ of \mathbf{Y} , and a *mapping function*, which assigns to each arrow $f: x \rightarrow y$ of \mathbf{X} an arrow $F(f): F(x) \rightarrow F(y)$ of \mathbf{Y} . These functions are to satisfy two requirements:

- i) $F(1_x) = 1_{F(x)}$, for each identity 1_x of \mathbf{X} ; and
- ii) $F(g \circ f) = F(g) \circ F(f)$, for each composite $g \circ f$ defined in \mathbf{X} .

For categories \mathbf{X} and \mathbf{Y} , a functor $F: \mathbf{X}^{\text{op}} \rightarrow \mathbf{Y}$ is called a *contravariant functor* from \mathbf{X} to \mathbf{Y} . Ordinary functors from \mathbf{X} to \mathbf{Y} are sometimes called *covariant functors*.

If $F, G: \mathbf{X} \rightarrow \mathbf{Y}$ are two functors, a *natural transformation* $\tau: F \rightarrow G$ from F to G is a function, which assigns to each object x of \mathbf{X} an arrow $\tau_x: F(x) \rightarrow G(x)$ of \mathbf{Y} in such a way that every arrow $f: x \rightarrow y$ of \mathbf{X} it follows, $G(f) \circ \tau_x = \tau_y \circ F(f)$. In case each τ_x is invertible in \mathbf{Y} , we call $\tau: F \rightarrow G$ a *natural isomorphism*.

Two categories \mathbf{X} and \mathbf{Y} yields a new category $\mathbf{Y}^{\mathbf{X}}$, called a *functor category*. The objects of $\mathbf{Y}^{\mathbf{X}}$ are functors from \mathbf{X} to \mathbf{Y} , while the arrows of $\mathbf{Y}^{\mathbf{X}}$ are natural transformations between such functors. Accordingly, a functor is a morphism of categories, whereas a natural transformation is a morphism of functors.

Let us have two categories \mathbf{X} and \mathbf{Y} , and two functors $F: \mathbf{X} \rightarrow \mathbf{Y}$ and $G: \mathbf{Y} \rightarrow \mathbf{X}$ in opposite directions between them. For an object x in \mathbf{X} and an object y in \mathbf{Y} we may compare the set $\text{hom}_{\mathbf{Y}}(F(x), y)$ of all arrows in \mathbf{Y} from $F(x)$ to y with the set $\text{hom}_{\mathbf{X}}(x, G(y))$ of all arrows in \mathbf{X} from x to $G(y)$. Now, an *adjunction* of the functor F to the functor G is a natural isomorphism $\varphi: \text{hom}_{\mathbf{Y}}(F(x), y) \rightarrow \text{hom}_{\mathbf{X}}(x, G(y))$, defined for all objects x in \mathbf{X} and y in \mathbf{Y} , and moreover, this natural isomorphism φ is natural in these arguments x and y , which means that it preserves categorical structure as x and y vary. The functor F is called a *left adjoint* of G , and G is called a *right adjoint* of F . An important corollary for our purpose is the following one, (for the proofs, see [9, p. 83]):

Corollary If the functor $F: \mathbf{X} \rightarrow \mathbf{Y}$ has two right adjoints G and H , then G and H are naturally isomorphic. The same is true for left adjoints.

Now, since a category can be seen as a set of objects with a structure, we can think our topological model for actual processes as a category, where open subsets are objects and subset relations between open sets are arrows. Similarly, our algebraic model for eternal processes can be thought to be a category as well, where concepts are objects and intensional containment relations between concepts are arrows. Thus

we can denote the category of topological space as $\mathbf{O}(T)$ and the category of concepts as \mathbf{C} .²

Given the categories $\mathbf{O}(T)$ and \mathbf{C} , they can be connected by two functors $P: \mathbf{O}(T) \rightarrow \mathbf{C}$ and $S: \mathbf{C} \rightarrow \mathbf{O}(T)$, which are opposite between them. For a concept a in \mathbf{C} and an open set O in $\mathbf{O}(T)$ we may compare the set $\text{hom}_{\mathbf{C}}(P(O), a)$ of all intension containment-relations in \mathbf{C} from $P(O)$ to a with the set $\text{hom}_{\mathbf{O}(T)}(O, S(a))$ of all subset-relations in $\mathbf{O}(T)$ from O to $S(a)$. Hence, an adjunction of the functor P to the functor S is a natural isomorphism $\tau: \text{hom}_{\mathbf{C}}(P(O), a) \rightarrow \text{hom}_{\mathbf{O}(T)}(O, S(a))$, defined for all open sets O in $\mathbf{O}(T)$ and a in \mathbf{C} and is natural in these arguments O and a .

This adjunction τ can now be interpreted as what Whitehead calls “ingression”, that is, eternal processes participating actual processes. Also, since both mental processes and physical processes are space-time events, they are actual processes, and thus they are modeled by topological model, that is $\mathbf{O}(T)$. Now the connection of mental processes and physical processes can be modelled, based on the Corollary mentioned above, by natural transformations.

Moreover, a \mathbf{C} -valued *presheaf* on T is the same as a contravariant functor E from $\mathbf{O}(T)$ to \mathbf{C} , which is same as a covariant functor $F: \mathbf{O}(T)^{\text{op}} \rightarrow \mathbf{C}$, i.e. $E^{\text{op}} = F$. This means, that if U and O are open subsets of T , and $U \subseteq O$, then $F(U \subseteq O): F(O) \rightarrow F(U)$.³ Accordingly, we can describe the Process-Ontological Model as a functor category from the category of topological space $\mathbf{O}(T)$ to the category of concepts \mathbf{C} , where the objects are contravariant functors, i.e. presheaves, and the arrows are natural transformations between these functors. This functor category is the category of presheaves of concepts on T .

6. Conclusion

In this paper we have introduced a process-ontological model which can be used to analyze some problematic nature of software engineering. Firstly we considered a process ontology in which everything is in a process. Our overall view is that everything in the world is composed of processes. There are two kinds of processes, “eternal” and actual, where actual processes are divided into physical and mental processes. Secondly, we proposed a topological model for actual processes, which is a four-dimensional, simply or multiply connected, and metrizable topological space T . Thirdly we proposed an algebraic model for eternal processes, i.e. concepts, that is, a complete semi-lattice, where every concepts as a principal ideal determines a Boolean algebra. Lastly, by using category theory we connected these two models of processes

² Of course, more category theoretical notions should be given to have a more exact descriptions for the underlying topological and algebraic models. However, in this paper we give only those which are needed to understand the basic idea.

³ The transition from intensions to extensions reverses the containment relation, i.e., the intensional containment relation between concepts a and b is converse to the extensional set-theoretical subset relation between the sets of their extension, (see [5], [7], [8]).

in order to get the category theoretical description of the Process-Ontological Model. That model is a functor category $C^{O(T)^{op}}$, i.e. the category of presheaves of concepts on T .

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