Irregular Objects. Shape Detection and Characteristic Sizes

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Abstract. In this work, results of detecting the objects of irregular form with image analysis are described. The offered algorithm can reduce an irregular object to one of the standard shape, in which it looks like, and choose characteristic sizes of this standard shape, where the object area stays constant.

Keywords: Computer vision, image analysis, object search, contour search, shape detection

1 Introduction

Nizhny Tagil Technological Institute of Ural Federal University carries out investigations of shape detection of irregular plate objects with image analysis.

Problem of research includes three subproblems:

1. Find contour of object on a picture. It is considered that the source image is a binary-colored one. Binarization of multicolored images is the subject of another research. The object perimeter and area are formed by using contour.
2. Detect standard shape, in which it looks like. In this work, the following four shapes are examined: circle, rectangle, rhombus, and ellipse. Nevertheless, real objects can have very complex shape.
3. Find characteristic sizes of chosen standard shape, using equation (1). Characteristic sizes are shown in Table 1.

$$S_{\text{contour}} = S_{\text{shape}}. \tag{1}$$

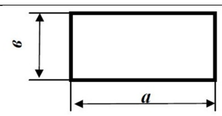
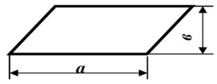
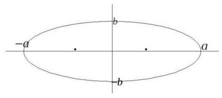
The `opencv` library is used to find contour of the object and its characteristics. The following three criteria of contour are used to detect the standard shape:

– *rectness* that characterizes closeness of contour to rectangle; it is calculated by Eq. 2

$$rec = \frac{S_{\text{contour}}}{S_{\text{rect}}}, \tag{2}$$

where rec is the rectness, S_{contour} is the area of a contour, S_{rect} is the area of minimum rectangle that contains this contour. It can be found by the method `GetMinAreaRect` [2] in `opencv`. $rec \leq 1$ ($rec = 1$ for the rectangle).

Table 1. Characteristic sizes of standard shapes.

Circle	d–diameter	
Rectangle	a–width b–height	
Rhombus	a–width h–height	
Ellipse	a–semimajor axis b–semiminor axis	

– *circless* that characterizes closeness of the contour to a circle; it is calculated by Eq. (3)

$$cir = \frac{S_{\text{contour}}}{S_{\text{cirle}}}, \tag{3}$$

where cir is the circless, S_{contour} is the area of a contour, S_{cirle} is the area of enclosing circle that contains this contour; it can be found by method `MinEnclosingCircle [3]` in `opencv`. $cir \leq 1$ ($cir = 1$ for the circle);

– *compactness* [1] is an universal shape criterion; if the contour is close to the standard shape, their compactnesses are approximataly equal; it is calculated by Eq. (4)

$$C = \frac{P^2}{S}, \tag{4}$$

where C is the compactness, P is the perimeter of the contour, S is the area of the contour.

2 Shape detection

All standard shapes are convex ones, but real irregular objects can have a lot of local concavities. So before calculations, a source contour must be smoothed and perimeter must be decreased to make it closer to perimeter of the standard form. Real and smoothed contours are shown in Fig. (1). It can be done by the `GetConvexHull` [4] method in `opencv`. It is interesting to note that perimeter of the source contour is 6804px, but perimeter of the smoothed contour is only 3702px.

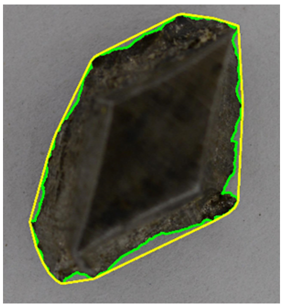


Fig. 1. Source and smoothed contour.

The coefficient $k = \dfrac{S_{\text{contour}}}{S_{\text{hull}}}$ is used to return to real contour in the next calculations. So, characteristic size a of a real contour can be calculated from size of the smoothed contour with this coefficient: $a_{\text{contour}} = \dfrac{a_{\text{hull}}}{\sqrt{k}}$.

Combination of three criteria (rectness, circless, and compactness) does not allow to detect one of the standard shape uniquely. The next strategy is used: if $rec > 0.8$ or $cir > 0.8$, than contour is counted as a rectangle or a circle respetively. In other cases, it is neccesary to choose a shape, which compactness is closest to the smoothed contour compactness.

3 Compactness of standard shapes

Compactness of standard shapes can be calculated by the following equations:

– circle (see Eq. 5)

$$C = \frac{(2\pi r)^2}{\pi r^2} = 4\pi; \tag{5}$$

the circle compactness is constant; it is minimum possible compactness; in all cases $C \geq 4\pi$. The characteristic parameter (diameter) can be calculated using Eq. (1) as $d = \sqrt{\dfrac{S_{\text{contour}}}{4\pi}}$;

– rhombus (see Eq. 6)

$$C = \frac{(4a)^2}{a^2 \sin\alpha} = \frac{16}{\sin(\alpha)}, \tag{6}$$

rhombus compactness $C \geq 16$ ($C = 16$ in the case of $\alpha = \pi/2$, *i.e.* quadrate);

– rectangle (see Eq. 7):

$$C = \frac{(2(a+b))^2}{ab} = \frac{4a^2 + 8ab + 4b^2}{ab}, \tag{7}$$

rectangle compactness $C \geq 16$ ($C = 16$ in the case of $a = b$, *i.e.* quadrate)

– ellipse perimeter is calculated with elliptic integral [5] (see Eq. 8)

$$P = \int_0^{\pi/2} \sqrt{1 - e^2 \cos^2 t} dt. \tag{8}$$

This intergal cannot be expressed in terms of elementary functions, so the following approximation (Eq. 9) of ellipse perimeter with the maximun error of 0.63% is used. The strcucture of this formula can help to simplify the following mathematical transformations,

$$P \approx 4\frac{\pi ab + (a-b)^2}{a+b}. \tag{9}$$

In this case, the ellipse compactness can be calculated by Eq. (10)

$$C = \frac{P^2}{S} = \frac{\left(\frac{4\pi ab + (a-b)^2}{a+b}\right)^2}{\pi ab} = 16\frac{\left(\pi ab + a^2 - 2ab + b^2\right)^2}{\pi a^3 b + 2\pi a^2 b^2 + \pi ab^3}. \tag{10}$$

The ellipse compactness $C \geq 4\pi$ ($C = 4\pi$ in the case of $a = b$, *i.e.* circle).

In the cases of rectangle and ellipse, compactness depends on two parameters, so, characteristic sizes cannot be determined uniquely. But if relation $e = b/a$ is used, compactness depends on one parameter, and all similar rectangles and ellipses have the same compactness. Compactness equations are shown on Table 2.

Table 2. Characteristic parameters of standard forms.

Shape	**Characteristic sizes**	**Compactness parameter**	**Compactness formula**
Circle	d–diameter	—	$C = 4\pi$
Rectangle	a–width b–height	$e = b/a$–ratio	$C(e) = \frac{4 + 8e + 4e^2}{e}$
Rhombus	a–base h–height	α–rhombus angle	$C(\alpha) = \frac{16}{\sin(\alpha)}$
Ellipse	a–semimajor axis b–semiminor axis	$e = b/a$–ratio	$C(e) = 16\frac{\left((\pi-2)e + e^2 + 1\right)^2}{\pi e^3 + 2\pi e^2 + \pi e}$

4 Characteristic parameters

In the case of circle, characteristic size (d) is calculated uniquely using Eq. (1) with Eq. (11)

$$d = \frac{1}{2}\sqrt{\frac{S_{\text{contour}}}{\pi}}. \tag{11}$$

In the case of rhombus, characteristic sizes (a, h) are calculated with Eq. (6) by Eq. (12 and Eq. 13)

$$a = \sqrt{\frac{S}{\sin\alpha}} = \sqrt{\frac{S}{16/C}} = \sqrt{\frac{S \cdot P^2}{S \cdot 16}} = \frac{P}{4}, \tag{12}$$

$$h = S/a. \tag{13}$$

In the cases of rectangle and ellipse, characteristic sizes can be calculated with well-known area equations $S = ab$ and $S = \pi ab$ respectively. Using the compactness parameter $e = b/a$, these equations are transformed to $S = a^2 e$ and $S = a^2 \pi e$, so $a = \sqrt{\frac{S}{e}}$, $b = \frac{S}{a}$ for rectangles, and $a = \sqrt{\frac{S}{\pi e}}$, $b = \frac{S}{\pi a}$ for ellipses. It means that parameters e and S are sufficiant to calculate the characteristic sizes. In the case of rectangle, e can be expressed with equation in Table 2 by Eq. (14)

$$e = \frac{C - 8 \pm \sqrt{C^2 - 16C}}{8}. \tag{14}$$

In the case of rhombus, dependence $C(e)$ is very complex, but this equation can be solved numerically.

5 Results of detection

This algorithm was applied to some real irregular objects. In this section some examples are introduced. All results have gathered in tables. The first column contains the original image, image with contour, and detected shape. The second column represents the calculated data. Bold font marks the base for shape detection.

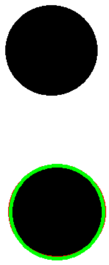
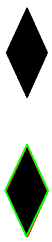
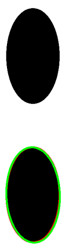
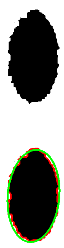
Notations in tables are

- S is the area of contour (smoothed)
- P is the perimeter of contour (smoothed)
- C is the compacntess of contour (smoothed)
- rec is the rectness
- cir is the circless
- C_r is the compacntess of rectangle
- C_c is the compacntess of circle
- C_{rh} is the compacntess of rhombus
- C_e is the compacntess of ellipse
- a, b, h, α, d are the characteristic parameters of detected figure (depending on shape).

5.1 Generated figures

This subsection contains image analysis of figures drawn upon the graphic primitives (Table 3). Columns 1,2 present source figures, columns 3,4 present artificially eroded figures.

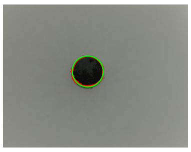
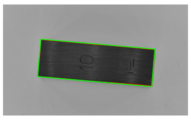
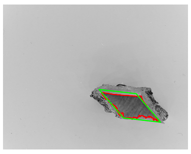
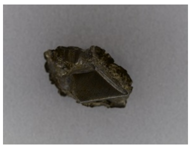
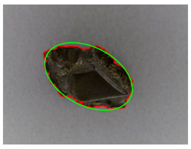
Table 3. Generated figures

generated figures		**ragged figures**	
	Circle S = 13074 P = 405 C = 12.59 rec = 0.79 **cir = 0.91** C_c = 12.56 C_e = 12.56 C_r = 16.00 C_{rh} = 16.00 d = 64.51		Circle S = 11088 P = 470 C = 19.98 rec = 0.78 **cir = 0.85** C_c = 12.56 C_e = 12.58 C_r = 16.00 C_{rh} = 16.00 d = 64.51
	Rectangle S = 42182 P = 846 C = 16.96 **rec = 1** cir = 0.53 C_r = 16.96 C_{rh} = 17.93 C_e = 12.56 C_c = 12.56 a = 161, b = 262		Rectangle S = 36323 P = 895 C = 22.05 **rec = 0.97** cir = 0.59 C_r = 16.94 C_e = 13.37 C_{rh} = 17.88 C_c = 12.56 a = 154.20, b = 249.34
	Rhombus S = 13690 P = 524 **C = 20.10** rec = 0.62 cir = 0.29 $\mathbf{C_{rh}}$ **= 20.66** $C_e = C$ = 15.60 C_r = 18.33 C_c = 12.56 a = 132.95, h = 102.96		Rhombus S = 11208 P = 576 **C = 29.67** rec = 0.64 cir = 0.31 $\mathbf{C_{rh}}$ **= 20.59** $C_e = C$ = 15.30 C_r = 18.29 C_c = 12.56 a = 128.25, h = 99.63
	Ellipse S = 27833 P = 662 **C = 14.18** rec = 0.79 cir = 0.53 $\mathbf{C_e = C}$ **= 14.21** C_r = 17.36 C_{rh} = 18.73 C_c = 12.56 a = 132.95, h = 102.96		Ellipse S = 24534 P = 711 **C = 14.47** rec = 0.77 cir = 0.50 $\mathbf{C_e = C}$ **= 14.31** C_r = 17.50 C_{rh} = 19.00 C_c = 12.56 a = 128.25, h = 99.63

5.2 Real figures similar to standard

This subsection contains image analysis of real figures similar to standard ones (Table 4).

Table 4. Real figures similar to standard

 	Circle $S = 274433$ $P = 3887$ $C = 12.81$ $rec = 0.78$ $\mathbf{cir = 0.83}$ $C_c = 12.56$ $C_e = 12.60$ $C_r = 16.03$ $C_{rh} = 16.07$ $d = 303$	 	Rectangle $S = 1472082$ $P = 5904$ $C = 21.32$ $\mathbf{rec = 0.99}$ $cir = 0.35$ $C_r = 21.49$ $C_e = 21.91$ $C_{rh} = 26.99$ $C_c = 12.56$ $a = 699.14$, $b = 2129.82$
 	Rhombus $S = 306123$ $P = 4640$ $\mathbf{C = 21.23}$ $rec = 0.65$ $cir = 0.26$ $\mathbf{C_{rh} = 23.56}$ $C_e = 16.83$ $C_r = 19.78$ $C_c = 12.56$ $a = 748.29$, $h = 508.11$	 	Ellipse $S = 1075559$ $P = 3915$ $\mathbf{C = 14.25}$ $rec = 0.77$ $cir = 0.54$ $\mathbf{C_e = 13.69}$ $C_r = 16.51$ $C_{rh} = 17.02$ $C_c = 12.56$ $a = 699.14$, $b = 2129.82$

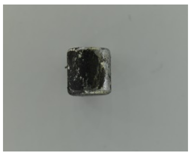
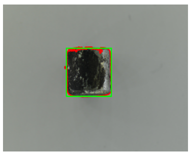
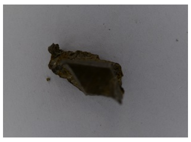
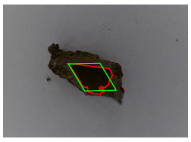
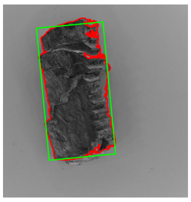
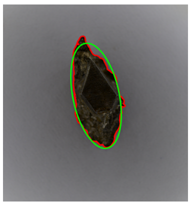
5.3 Irregular figures

This subsection contains image analysis of real irregular figures (Table 5).

6 Conclusion

The given algorithm provides to detecting the standard shape of irregular object, in which it looks like. Combination of three criteria (rectness, circless, and compactness) gives good results. Algorithm detects standard shape and characteristic sizes of it using area equivalence and compactness criterion.

Table 5. Irregular figures

 	Rectangle $S = 432967$ $P = 2455$ $C = 13.92$ $\mathbf{rec = 0.92}$ $cir = 0.72$ $C_r = 16.01$ $C_e = 12.66$ $C_c = 12.56$ $C_{rh} = 16.03$ $a = 637.61,\ b = 679.03$	 	Rhombus $S = 235564$ $P = 2012$ $\mathbf{C = 17.18}$ $rec = 0.68$ $cir = 0.45$ $\mathbf{C_{rh} = 18.43}$ $C_e = 13.59$ $C_r = 17.21$ $C_c = 12.56$ $a = 520.92,\ h = 452.20$
 	Rectangle $S = 2298000$ $P = 6027$ $C = 15.81$ $\mathbf{rec = 0.89}$ $cir = 0.51$ $C_{rh} = 23.56$ $C_e = 16.83$ $C_r = 19.78$ $C_c = 12.56$ $a = 748.29,\ h = 508.11$	 	Ellipse $S = 852466$ $P = 3798$ $\mathbf{C = 16.92}$ $rec = 0.76$ $cir = 0.39$ $\mathbf{C_e = 15.71}$ $C_r = 18.99$ $C_{rh} = 21.99$ $C_c = 12.56$ $a = 699.14,\ b = 2129.82$

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