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Trace shifts - minimal case for independence relations given by five node co-graphs

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Abstract: We study interrelations between symbolic descriptions of concurrently evolving systems and underlying sequential dynamics. We focus our interests on minimal shifts and t-shifts generated by them, assuming that an independence relation is given by a five vertex co-graph.

1 Introduction

In [11] we formulated a framework which allows to study parallel dynamics by tools used in a research of sequential symbolic dynamics. To be more precise, in place of words we consider traces introduced in the seminal papers [4, 16] and define a t-shift as a parallel counterpart of a shift. Then we raised a problem of interrelations between the sequential dynamics and their parallel counterparts.

The problem is well known in the theory of computing. Namely, a one-tape Turing machine is equivalent, in the sense of a computational power, to a Turing machine with multiple tapes. However, if we analyze moves of the tape then a "simple" computation of a multi-tape machine may force quite "complex" behavior of a one-tape machine during a simulation process. Some attempts have been made for the better understanding of relations between dynamics of computation and computation process itself - compare [3, 9, 20, 21, 22, 19]. There are also other models of computation considered in the literature [8]. For example, CRCW P-RAM - Concurrent Read Concurrent Write Parallel RAM that realizes the situation in which more than one processor can concurrently read from or write into the same memory location. These models could be described as dynamical systems and then one could analyze some of their dynamical properties. Basing our research on trace theory we try to combine, in some sense, these two approaches - parallel or concurrent execution of computational processes and its dynamics description using symbolic dynamics tools. This paper fits into this direction of a research.

Our recent results are published in [11, 12, 13, 14]. In this paper we focus our interests on minimal shifts and t-shifts generated by them, assuming that an independence relation, which determines infinite traces, is given by a five vertex co-graph. In fact there are 24 such co-graphs and we try to analyze all of them. The reason for choosing co-graphs as an independence relations follows from the general fact proved by B.Courcelle, M.Mosbah in [5] that all problems that can be formulated in monadic second-order logic (without quantification of edge sets) can be solved in a linear time on co-graphs. To make it more clear - for any co-graph its recognition, along with the construction of the corresponding co-tree, can be done in linear time. Co-trees form the basis for polynomial algorithms for problems such as isomorphism, coloring, clique detection, Hamiltonicity, tree-width and path-width, and dominating sets on co-graphs. In general case these problems are NP-hard. This is also a common reason for choosing a family of co-graphs as a subject of various theoretical researches with possible applications, for example in examination scheduling and automatic clustering of index terms [7].

2 Definitions and notations

Now we recall only basic concepts of graph theory, symbolic dynamics and theory of traces. All the missing notions may be found in [6, 15].

We consider only simple and finite graphs. By *a co-graph* we understand either a single vertex graph, or the disjoint union of two co-graphs, or the edge complement of a co-graph.

Let Σ be any finite set (alphabet) and denote by Σ^* and Σ^ω the set of all finite and infinite words over Σ, respectively. The set Σ^* with concatenation of words and the empty word, denoted 1 is a free monoid. The set of nonempty words is denoted by Σ^+ and $\Sigma^\infty = \Sigma^* \cup \Sigma^\omega$.

Let $I \subset \Sigma \times \Sigma$ be a symmetric and irreflexive relation. In the sequel I is called *an independence (commutation) relation*; its complement is denoted by D and called *a dependence relation*. For every letter $a \in \Sigma$ we denote by $D(a) = \{b \in \Sigma \,:\, (a,b) \notin I\}$, a set of all letters from Σ which depend on a. The relation I may be extended to a congruence $\sim_I$ on Σ^*. We have $u \sim_I v$ if and only if it is possible to transform u to v by a finite number of swaps $ab \to ba$ of independent letters. *A trace* is an element of the quotient space $\mathbb{M}(\Sigma, I) = \Sigma^* / \sim_I$. For

$w \in \Sigma^*$, $t \in \mathbb{M}(\Sigma, I)$ we denote by $|\, w\,|_a$ and $|\, t\,|_a$ the number of occurrences of the letter $a \in \Sigma$ in w and t respectively. $alph(w)$ and $alph(t)$ denote the set of all letters which occur in w, t. Two traces t_1 and t_2 are *independent,* denoted $t_1 I t_2$, if and only if $alph(t_1) \times alph(t_2) \subset I$. If $x \in \Sigma^\omega$ and $i \leq j$ are nonnegative integers then we denote $x_{[i,j]} = x_i x_{i+1} \ldots x_j$ and $x_{[i,j)} = x_{[i,j-1]}$.

We recall that a word $w \in \Sigma^*$ is in the *Foata normal form*, if it is the empty word or if there exist an integer $n > 0$ and nonempty words $v_1, ..., v_n \in \Sigma^+$ (called *Foata steps*) such that:

1. $w = v_1 .... v_n$,
2. for any $i = 1, ..., n$ the word v_i is a concatenation of pairwise independent letters and is minimal with respect to the lexicographic ordering,
3. for any $i = 1, ..., n-1$ and for an arbitrary letter $a \in alph(v_{i+1})$ there exists a letter $b \in alph(v_i)$ such that $(a,b) \in D$.

It is well known that for any $x \in \Sigma^*$ there exists the unique $w \in [x]_{\sim_I}$ in the Foata normal form.

In the theory of dynamical systems continuous maps acting on metric spaces are considered. Hence we endow Σ^ω with the following metric d. If $x = y$ then $d(x,y) = 0$ and otherwise $d(x,y) = 2^{-j}$ where j is the number of letters in the longest common prefix of x and y. Now, define a shift map $\sigma : \Sigma^\omega \to \Sigma^\omega$ by

$$(\sigma(x))_i = x_{i+1}$$

where $(\cdot)_i$ denotes the i-th letter of a sequence. It is easy to observe that σ is continuous. Σ^ω together with the map σ is referred to as the *full shift* over Σ. Any closed and σ-invariant (i.e. $\sigma(X) \subset X$) set $X \subset \Sigma$ is called a *shift* or a *subshift*.

For any word $w = (w_i)_{i \in \mathbb{N}} \in \Sigma^\omega$ the *dependence graph* $\varphi_{\mathbb{G}}(w) = [V, E, \lambda]$ is defined as follows. We put $V = \mathbb{N}$ and $\lambda(i) = w_i$ for any $i \in \mathbb{N}$. The function λ successively labels nodes of $\varphi_{\mathbb{G}}(w)$ by letters of w. There exists an arrow $(i,j) \in E$, if and only if $i < j$ and $(w_i, w_j) \in D$. Let us denote the set of all possible dependence graphs (up to an isomorphism of graphs) by $\mathbb{R}^\omega(\Sigma, I)$ and let $\varphi_{\mathbb{G}} : \Sigma^\omega \to \mathbb{R}^\omega(\Sigma, I)$ be a natural projection. We call elements of $\mathbb{R}^\omega(\Sigma, I)$ *infinite (real) traces*. Each dependence graph is acyclic and it induces a well-founded ordering on $\mathbb{N}$. Then for any $v \in V$ the function $h : V \to \mathbb{N}$ given by $h(v) = \max \mathscr{P}(v)$ where
$\mathscr{P}(v) = \{n \in \mathbb{N} : \exists v_1, .., v_n \in V, v_n = v, (v_i, v_{i+1}) \in E \text{ for } i = 1, ..., n-1\}$ is well defined.

By $F_n(t)$ we denote a word $w \in \Sigma^*$ consisting of all the letters from the n-th level of infinite trace $t \in \mathbb{R}^\omega(\Sigma, I)$, that is from the set $\{\lambda(v) : v \in V, h(v) = n\}$. It follows from the definition of a dependence relation that for any infinite trace $t \in \mathbb{R}^\omega(\Sigma, I)$ the word $w = F_1(t) \ldots F_n(t)$ is in the Foata normal form with Foata steps given by $F_i(t)$ and $t = \varphi_{\mathbb{G}}(F_1(t) F_2(t) \ldots)$. Then in the same way as it was done for Σ^ω we may endow $\mathbb{R}^\omega(\Sigma, I)$ with a metric $d_{\mathbb{R}}(s,t)$ putting $d_{\mathbb{R}}(s,t) = 0$ if $s = t$ and $d_{\mathbb{R}}(s,t) = 2^{-j+1}$ if $s \neq t$ where j is the maximal integer such that $F_i(t) = F_i(s)$ for $1 \leq i \leq j$. By a *full t-shift* we mean the metric space $(\mathbb{R}^\omega(\Sigma, I), d_{\mathbb{R}})$ together with a continuous map $\Phi : \mathbb{R}^\omega(\Sigma, I) \to \mathbb{R}^\omega(\Sigma, I)$ defined by the formula $\Phi(t) = \varphi_{\mathbb{G}}(F_2(t) F_3(t) \ldots)$ for any $t \in \mathbb{R}^\omega(\Sigma, I)$. Analogically as a shift is defined, by a *t-shift* we mean any closed and Φ-invariant subset of $\mathbb{R}^\omega(\Sigma, I)$. It was proved in [11] that from a dynamical system point of view $(\mathbb{R}^\omega(\Sigma, I), \Phi)$ is equivalent to a shift of finite type (which means that dynamics of $(\mathbb{R}^\omega(\Sigma, I), \Phi)$ and (Σ^ω, σ) is to some extent similar). However, it frequently happens that the $\varphi_{\mathbb{G}}$ image of a shift is not a t-shift and there are also t-shifts which cannot be obtained as images of any (sequential) shift.

Let (X, d) be a compact metric space and let $f : X \to X$ be continuous. A point $y \in X$ is said to be an *ω-limit point* of x if it is an accumulation point of the sequence $x, f(x), f^2(x), \ldots$. The set containing all elements of the sequence $x, f(x), f^2(x), \ldots$ is called an orbit of x and denoted by $\mathrm{Orb}^+(x)$. The set of all ω-limit points of x is referred to as *ω-limit set* of x and denoted by $\omega(x, f)$. A point x is said to be *periodic (fixed)* if $f^n(x) = x$ for some $n \geq 1$ $(n = 1)$ and is said to be *recurrent* if $x \in \omega(x, f)$. If x is not periodic (fixed) point but $f^m(x)$ is periodic (fixed) for some positive integer m then we say that x is *eventually periodic (fixed).* A subset M of X is minimal if it is closed, nonempty, invariant (i.e. $f(M) \subset M$) and contains no proper subset with these three properties. It is well known that if a nonempty closed set $M \subset X$ is minimal then the orbit of every point of M is dense in M. We recall that a point x is referred to as *minimal* (or *almost periodic*) if it belongs to a minimal set.

Let X and Y be compact metric spaces and let $f : X \to X$ and $g : Y \to Y$ be continuous maps. If there is a homeomorphism $\phi : X \to Y$ with $\phi \circ f = g \circ \phi$, we will say that f and g are *(topologically) conjugate* and π is called *a (topological) conjugacy*. If there is a conjugacy from X to Y then Y is sharing all properties of X. Not formally we can think about these two shifts as they are the same.

Let X be a closed and σ-invariant subset of Σ^ω. We define the set $B_n(X)$ of n *admissible words for* X by:

$$B_n(X) = \left\{ x_{[i,i+n)} \in \Sigma^* \,:\, x \in X, i \in \mathbb{N} \right\}.$$

Let $w \in \Sigma^*$. The set of all subwords of w with the length equal to n is denoted by

$$S_n(w) = \{ u \in B_n(\Sigma^*) \,:\, \exists\, v_1, v_2 \in \Sigma^*\,,\, w = v_1 u v_2 \}.$$

We may extend canonically the definition of S_n to the case of $x \in \Sigma^\omega$, i.e. given $x \in \Sigma^\omega$ we define:

$$S_n(x) = \left\{ u \in B_n(\Sigma^*) \,:\, \exists\, i\,,\, u = x_{[i,i+n)} \right\}.$$

The least integer m (if it exists) such that for any $w \in B_m(X)$ it holds that $S_n(w) = S_n(x)$ is called the *n-th recurrency index of x in X* and is denoted by $R(n, x, X)$. If such m does not exist we put $R(n,x,X) = +\infty$. In the case of $X = \mathrm{cl}(\mathrm{Orb}^+(x))$ we will simplify the notation writing $R(n,x)$ instead of $R(n,x,\mathrm{cl}(\mathrm{Orb}^+(x)))$ where $\mathrm{cl}(A)$ denotes the closure of a set A. Let us recall some facts on recurrency indices.

Theorem 1 ([17, Thm. 7.2])**.** *Let $x \in \Sigma^\omega$. The following conditions are equivalent:*

1. *x is a minimal point,*
2. *$R(n,x) < +\infty$ for all positive integers n.*

Theorem 2 ([17, Thm. 7.1])**.** *If $M \subset \Sigma^\omega$ is a minimal set, then $R(n,x) = R(n,y)$ for any $x,y \in M$ and $n \in \mathbb{N}$.*

Let us consider an independence alphabet (Σ, I) and let $\mathscr{A}$ be the set of all nonempty words over Σ which are products of pairwise independent letters and are minimal with respect to the lexicographical ordering of Σ. Hence an element of $\mathscr{A}$ is a Foata step which may occur in some Foata normal form of a word (finite or not) over (Σ, I). We define a graph $\mathscr{G}(\Sigma, I)$ putting $V = \mathscr{A}$ as the vertex set and defining the set of edges $E \subset V \times V$ as follows. A pair $((a_1 \dots a_k),(b_1 \dots b_l))$ is in E if for any $0 \le j \le k$ there exists $1 \le i \le l$ such that $(a_j, b_i) \in D$. It is well known that the set

$$X_{\mathscr{G}(\Sigma,I)} = \{a_0a_1 \ldots \in \mathscr{A}^\omega : (a_i, a_{i+1}) \in E \text{ for } i = 0,1,\ldots\}$$

is a shift space over $\mathscr{A}$ and it was proved in [11] that there is a conjugacy $\pi : \mathbb{R}^\omega(\Sigma, I) \to X_{\mathscr{G}(\Sigma,I)}$.

Given a trace $t \in \mathbb{R}^\omega(\Sigma, I)$ we define *an n-th recurrency index of t* by $R(n,t) = R(n, \pi(t))$.

3 Main result

In our paper [14] we studied minimal shifts, their images by $\varphi_{\mathbb{G}}$ and interrelations between these objets assuming that an independency relation I is given by relatively small graphs (up to four vertices). We obtain a clear description of these cases except for I generated by C_4 - the cycle on four vertices. Now we present the main result of this paper, that is a description of trace counterparts of minimal shifts, assuming that an independency relation I is given by a cograph defined on five vertices.

We start our presentation with two useful theorems.

Theorem 3 ([14] Thm. 7)**.** *Let Σ, Θ be alphabets, $\Theta \subsetneq \Sigma$ and $X \subset \Sigma^\omega$ be a minimal shift with $alph(X) = \Sigma$. Let an independence relation I be given as follows:*

$$I = ((\Sigma \times \Theta) \cup (\Theta \times \Sigma)) \setminus \Delta_\Sigma,$$

where $\Delta_\Sigma = \{(a,a) : a \in \Sigma\}$.
Let $\pi : \Sigma^\omega \to (\Sigma \setminus \Theta)^\infty$ be a projection

$$\pi(a) = \begin{cases} 1 & \text{if } a \in \Theta \\ a & \text{if } a \notin \Theta \end{cases}$$

Then $\varphi_{\mathbb{G}}(X)$ and $\pi(X)$ are t-shift and shift respectively, they are conjugated and $(\pi(X), \sigma)$ is minimal.

The proof of a subsequent assertion is a modified version of the original one in [14].

Theorem 4. *Let X be a minimal shift, $alph(X) = \Sigma$. Let Σ_1, Σ_2 be a partition of Σ and assume that $\Sigma_1 \times \Sigma_2 \subset D$. Then there exists an integer M such that the sets*

$$Y = \bigcup_{i=0}^{M} \Phi^i(\varphi_{\mathbb{G}}(X)), \quad Z = \Phi^M(Y).$$

are t-shifts. Furthermore t-shift (Z, Φ) is minimal.

Proof. We will show that $\varphi_{\mathbb{G}}(x)$ is a minimal point for some $x \in X$. Let us fix an infinite word $x = x_0x_1 \ldots \in X$ such that $x_0 \in \Sigma_1$ and $x_1 \in \Sigma_2$ what is possible according to the minimality of X. It follows from the assumption that for every $i = 0,1,\ldots$ any Foata step $F_i(x)$ is a word in Σ_1^* or Σ_2^* exclusively, that is if $alph(F_i(x)) \cap \Sigma_j \neq \emptyset$ then $F_i(x) \in \Sigma_j^*$ where $j = 1,2$.

Now, let us fix a point $x = x_0x_1 \ldots \in X$ such that $x_0 \in \Sigma_1$ and $x_1 \in \Sigma_2$. We will show that $\varphi_{\mathbb{G}}(x)$ is a minimal point. In the word x there exist letters $x_{j_1}, x_{j_2} \in \Sigma_1$ and $x_{j_1+1}, x_{j_2-1} \in \Sigma_2$ for some $j_1 < j_2$. Then x_{j_1} and x_{j_2} determine some Foata steps, in the sense that $x_{j_1} \in F_{i_1}(x)$ and $x_{j_2} \in F_{i_2}(x)$ respectively for some $i_1 < i_2$. All the intermediate steps are uniquely determined by the intermediate letters, that is $F_i(x) = F_{i-i_1}(x_{(j_1,j_2)})$ for every $i_1 < i < i_2$ - by the definition of I and according to the fact that $x_{j_1}, x_{j_2} \in \Sigma_1$ and $x_{j_1+1}, x_{j_2-1} \in \Sigma_2$ it is not allowed to move any intermediate letter outside two-sided boundary given by the letters x_{j_1}, x_{j_2}. Now, let us introduce the following notation

$$M = \sup\left\{n \ : \ \exists\, i,\ x_i, x_{i+n} \in \Sigma_1,\ \Sigma_1 \cap alph(x_{(i,i+n)}) = \emptyset\right\}.$$

M is finite since X is minimal, in particular $M \le 2R(x,1)$. For any positive integers i,s there exist indices $i_1 \le i$, $i \le i_2 - s$ such that $i_1 - i \le M$, $i_2 - i - s \le M$ and $F_{i_1}(x), F_{i_2}(x) \in \Sigma_2^*$ and $F_{i_1-1}(x), F_{i_2+1}(x) \in \Sigma_1^*$. In particular, this implies that all the steps $F_{i_1}(x), \ldots, F_{i_2}(x)$ are uniquely determined by some subword u of x with the length $|u| < (2M+s+2)|\Sigma|$. But from the minimality of x each subword of x with the length $R((2M+s)|\Sigma|, x)$ contains u as a subword. Additionally $F_{i+1}(x) = F_i(\sigma(x))$ and $F_0(x) = x_0 \in \Sigma_1$. Finally

$$R(s, \varphi_{\mathbb{G}}(\sigma(x))) \le R((2M+s)|\Sigma|, x)$$

and then $\varphi_{\mathbb{G}}(\sigma(x))$ is a minimal point. If we fix $y \in X$ with $y_0 \in \Sigma_1$ then there exists an increasing sequence $\{i_k\}_{k=1}^\infty$ such that $\lim_{k\to\infty} \sigma^{i_k}(x) = y$. In particular, we may assume that $x_{[i_k, i_k+1]} = y_{[0,1]}$. There exists also an increasing sequence $\{j_k\}_{k=1}^\infty$ such that $\Phi^{j_k}(\varphi_{\mathbb{G}}(x)) = \varphi_{\mathbb{G}}(\sigma^{i_k+1}(x))$ and so $\sigma(y) \in \mathrm{cl}(\mathrm{Orb}^+(\varphi_{\mathbb{G}}(\sigma(x))))$. Thus we have just proved that all $x \in X$ with $x_0 \in \Sigma_1, x_1 \in \Sigma_2$ define exactly the same unique minimal t-shift. But it is also clear that for any $y \in X$ there is $j \le M$ such that $y_j \in \Sigma_1$ and $y_{j+1} \in \Sigma_2$ which immediately implies that $\Phi^i(\varphi_{\mathbb{G}}(y))$ is contained in a minimal set, for some $i \le j \le M$. $\square$

Now let us consider five letter alphabet and an independence relation given by a co-graph of order 5. We devide the set of 24 mentioned co-graphs into three subclasses according to their properties in order to facilitate analysis of the situation.

Theorem 5. *Let the independence relation I be represented by a co-graph G.*

1. *If G belongs to **subclass I** (Fig.1), then there exists a positive integer M and* $Y = \bigcup_{i=0}^{M} \Phi^i(\varphi_{\mathbb{G}}(X))$ *is a t-shift and* $Z = \Phi^M(Y)$ *is a minimal subshift.*
2. *If G belongs to **subclass II** (Fig.2), then two cases can occur. The first one is described in statement 1. and in the second one* $\varphi_{\mathbb{G}}(X')$ *is a minimal subshift.*

Proof.

- **subclass I**
  If the relation I is represented by any graph belonging to subclass I (see Fig. 1),

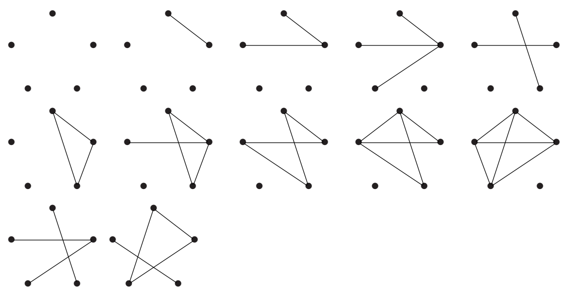

Figure 1: Subclass I of co-graphs of order 5

then there exists at least one letter (denote it by a) which is in relation D with any other letter. Then we can apply Theorem 4 because $\Sigma_1 = \{a\}$, $\Sigma_2 = \Sigma \setminus \{a\}$ fulfill the assumptions of the mentioned theorem. In particular, $Y = \bigcup_{i=0}^{M} \Phi^i(\varphi_{\mathbb{G}}(X))$ is a t-shift and after a finite number of iterations it becomes a minimal subshift $Z = \Phi^M(Y)$, where M is a positive integer which depends on the structure of X. In fact, exactly the same situation holds for the last two graphs from this subclass, but now Σ is decomposed into two sets containing two and three letters respectively.

- **subclass II**
  If the relation I is represented by any graph depicted at Fig 2, then there is at least one letter which is independent with any other letter.

  Then by Theorem 3 $\varphi_{\mathbb{G}}(X)$ is homeomorphic to $\varphi_{\mathbb{G}}(X')$ where X' is obtained from X by removing this particular letter. But this transformation allows us to modify the independence relation by removing exactly the same letter from I. Finally we obtain two cases. The first that allows to apply Theorem 4, and

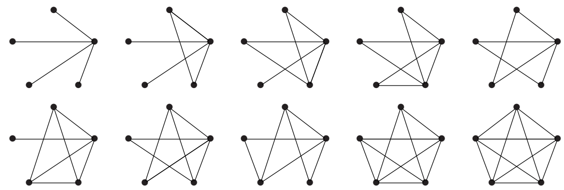

Figure 2: Subclass II of co-graphs of order 5

the second in which a form of the relation I obviously implies that $\varphi_{\mathbb{G}}(X')$ is minimal (for example I consisted of five isolated vertices).

□

One can see that the previous Theorem does not consider two remaining co-graphs on five vertices depicted in Fig. 3.

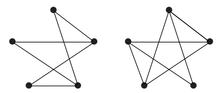

Figure 3: Subclass III of co-graphs of order 5

Unfortunately in this case we are unable to give any general description. So this case remains open for a further research.

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