Axioms for parthood and containment relations in bio-ontologies

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Abstract

To fix the semantics of different kinds of parthood relations we require axioms which go beyond those characterizing partial orderings. I formulate such axioms and show their implications for bio-ontologies. Specifically, I discuss parthood relations among masses, for example among body substances such as blood and portions thereof, and among components of complexes, for example between your stomach and your gastro-intestinal system. I contrast these with the relation of being contained in (as your lungs are contained in your thorax).

The axioms considered are rooted in mereology, the formal theory of parts and wholes. By making explicit the differences between the different kinds of relations they support different kinds of data integration in bioinformatics.

Introduction

The growth of bioinformatics has led to an increasing number of evolving ontologies which must be correlated with the existing terminology systems developed for clinical medicine. A critical requirement for such correlations is the alignment of the fundamental ontological relations used in such systems, and especially of the relation of part-of [16, 26].

However, there is one problem that stands in the way of achieving such integration: existing terminology systems and ontologies are marked by an inadequate degree of semantic consistency at their foundations [27]. The ambiguities and inconsistencies which result from the lack of a standard unified framework for understanding the basic ontological relationships that structure these domains are an obstacle to ontology alignment and data integration, and thus also to the sort of automatic processing of biomedical data which is the presupposition of advances in this field.

Part-whole relations play a critical role in medical concept representation. As Rogers and Rector [20] point out, this is most obvious in the modeling of anatomy; but it also true of the representation of surgical procedures, as well as of many physiological and disease processes, as also of the chemical pathways which lie beneath all of these.

Part-whole relations have long been the subject of extensive study in philosophy [2, 24], linguistics [31], knowledge representation [10, 9], and more recently in bio-informatics [11, 22, 20, 17]. In particular, it has long been recognized that several different subtypes of the part-of relation may be identified [19, 31, 9, 13]. This recognition underlies the modeling of the part-of relation in GALEN [20] and in the Foundational Model of Anatomy (FMA) [21, 16]. All such relations are, when taken singly, treated formally as partial orderings. However there does not exist a formal treatment of what *distinguishes* such relations one from another.

In this paper I give axiomatic theories for three sorts of partial ordering relations: (i) the component-of relation between components and the complexes they form (my mouth, my oropharynx, and my gastro-intestinal system are components of my alimentary system); (ii) part-of relations among masses such as body-substances in the sense of FMA (the blood in your left ventricle is part of the blood in your body); and (iii) containment relations (my brain is contained in my skull, my lungs are contained in my thorax).

The formal characterization will be purely mereological and will exploit the classification of formal theories given for example by Simons [23] or Varzi [29]. Thus no resources from topology or geometry are required. Moreover, in all that follows I consider entities at a single moment in time. The full formal characterization of all the part-whole relations contained in a system like the FMA or GALEN will need to go further than what is presented here. Distinctions of the type here discussed will however be indispensable to further progress in this field.

Partial ordering structures

In this paper formal theories of different kinds of partial order relations are discussed. Each of the theories is presented in a single-sorted first-order predicate logic with identity. I use the letters x, y, and z for variables. Predicates always begin with a capital letter. The logical connectors $\neg, =, \wedge, \vee, \rightarrow, \leftrightarrow$ have their usual meanings: not, identical-to, and, or, if ... then, if and only if (iff). I write (x) to symbolize universal quantification and $(\exists x)$ to symbolize existential quantification. Leading universal quantifiers are assumed to be understood and are omitted.

Properties of partial orderings

I introduce the binary primitive $x < y$ interpreted as the generic relation of proper partial ordering, i.e., x stands to y in the relation of proper partial ordering.
In terms of $<$, I define the relations of (improper) partial order and overlap: x and y are in the relation of improper partial order iff either $x < y$ or x and y are identical ($D_{\leq}$); x and y overlap iff they share a common entity in the partial ordering hierarchy (D_O):

$$D_{\leq} \quad x \leq y \equiv x < y \vee x = y$$
$$D_O \quad O\,xy \equiv (\exists z)(z \leq x \wedge z \leq y)$$

I now add axioms to the effect that the relation of proper partial ordering, $<$, is asymmetric and transitive (APO1-APO2).

$$APO1 \quad x < y \rightarrow \neg y < x$$
$$APO2 \quad (x < y \wedge y < z) \rightarrow x < z$$

It then follows that proper partial ordering is irreflexive (TPO1) and that (improper) partial ordering $\leq$ is reflexive, antisymmetric, and transitive (TPO2-4)[1]:

$$TPO1 \quad \neg x < x$$
$$TPO2 \quad x \leq x$$
$$TPO3 \quad (x \leq y \wedge y \leq x) \rightarrow x = y$$
$$TPO4 \quad (x \leq y \wedge y \leq z) \rightarrow x \leq z$$

Examples of partial ordering structures

I now discuss three examples of partial order relations: the component-of relation, the containment relation, and the part-of relation as it holds between masses.

The complement-of relation. Consider the component-of relation between components and complexes of my alimentary system. Figure 1 shows the component-of structure of my alimentary system according to the FMA [21]. My mouth, my oropharynx, and my gastrointestinal system are components of my alimentary system. In general, the nodes c and d in the graph structure are connected by an arrow iff entity c is a component of the complex d.

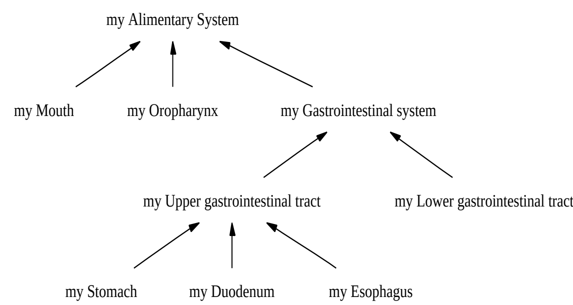


Figure 1: Component-of relations between the components of my Alimentary system

To see that the component-of relation satisfies the axioms of proper partial orderings (APO1-2) consider that components are distinct from the complexes they form. Since my stomach is a component of my alimentary system, the alimentary system is not a component of my stomach. Also the alimentary system is identical to itself but not a component of itself. Moreover, the component-of relation is transitive. My stomach is a component of my upper gastro-intestinal tract. My upper gastro-intestinal tract is a component of my gastro-intestinal system. And also my stomach is a component of my gastro-intestinal system.
As an example for overlap of complexes consider the alimentary system and the respiratory system according to the FMA. Both have the oropharynx as a component and hence overlap in the sense of definition D_O.

Containment is the second example of a proper partial ordering relation. For a non-medical example consider the relation between your backpack and the books therein, or the relation between your wallet and the coins therein, or the relation between the coins and the backpack in the case where the wallet with the coins is in the backpack.
For a medical example of containment consider the relation which holds between my pericardial sac and my thorax in the sense that my thorax forms a container for my pericardial sac, which in turn is contained in my thorax (Figure 2). The same relation of containment holds between my heart and my pericardial sac in the sense that my pericardial sac is a container for my heart. Clearly, containment understood in this sense is asymmetric and transitive. For example. The pericardial sac is a container for my heart, but the latter is not a container for the former. Since my heart is con-

[1] The formal proofs are omitted here but can be obtained from the author.

tained in my pericardial sac and my pericardial sac is contained in my thorax, and it also holds that my heart is contained in my thorax.

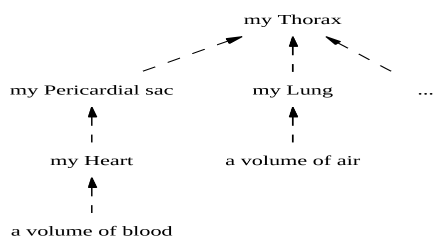


Figure 2: Containment relations

Notice that the interpretation of the containment relation employed here is different from those in the FMA [7] and GALEN [8]. Both interpret containment as a relation between an entity and (a part of) a space that is enclosed by a container. For example for GALEN [8] the heart is contained in the mediastinum, which is a part of the thoracic space.

Here, in contrast, the relation of containment always holds between entities – the contained entity (e.g., a volume of blood) and the container (e.g., my heart). Containers can themselves be contained in other containers (e.g., my heart is contained in my pericardial sac, which in turn is contained in my thorax).

Containers have properties, like having-a-cavity, which distinguish them from non-containers. The characterization of those properties, however, is beyond the realm of mereology. This requires at least the resources of topology and a theory of location [3, 6].

The advantage of the interpretation applied here is fourfold. Firstly, we focus on what *containment* means and not on what a container is. The former question can be answered within a mereological framework the latter cannot. Secondly we need only a single category in order to characterize containment – entities. In the interpretation of containment applied in the FMA and in GALEN one needs (at least) two categories: contained entities like the heart; and regions, like the thoracic space, which are enclosed by container-like entities. Thirdly, representing containment as relation of partial order between entities allows us to characterize the similarities and differences between parthood and containment in a very explicit manner.

Fourthly, representing containment as relation between entities allows us to distinguish it from the relation of *location*, which holds between entities and regions [4]. Often both relations are used in combination, for example, in order to say that the heart is *contained* in the thorax and within the thorax it is *located* in a region to which we refer to as the middle mediastinum, and which is a *part of* the region which is *enclosed by* the thorax. In general for specifying the semantics of relations in complex systems like the FMA or GALEN it is important to characterize relations in separation first by employing the simplest possible theory. Complex relations then can be described by combining the theories characterizing the components of the complex relation.

The parthood relation among masses is the third example of a partial ordering relation. Examples of masses are body-substances like saliva, semen, cerebrospinal fluid, inhaled air, urine, feces, blood, plasma, etc. The relation I have in mind here is the relation which holds between the blood in my body and the blood in my left ventricle. Notice that we do not have a relation of containment here. Rather names of containers like ‘my body’ or ‘my left ventricle’ are used here only in order to refer to certain quantities or portions of the blood in my body at a certain moment in time.

One can now verify that the parthood relation among masses is a proper partial ordering relation: the blood in my heart is a proper part of the blood in my body (but not *vice versa*), the blood in my right ventricle is a proper part of the blood in my heart, and the blood in my right ventricle is a proper part of the blood in my body.

From these examples we can see that all three relations share the property that they form partial ordering structures. Yet they are quite different in nature. It will our task in the remainder of this paper to characterize these distinctions formally.

Complexes

The characteristic property of complexes is that we can represent their partonomic structure using trees as indicated in Figure 1.

The formal theory of the relation component-of employs a binary primitive $x <_{cp} y$ which is interpreted as ‘the entity x is a component(-part) of the entity y’. We then add the axioms for asymmetry and transitivity for $<_{cp}$ (ACP1-2)

$$ACP1 \quad x <_{cp} y \rightarrow \neg y <_{cp} x$$
$$ACP2 \quad (x <_{cp} y \wedge y <_{cp} z) \rightarrow x <_{cp} z$$

together with definitions for the improper component-of relation (which includes identity) and for component-overlap (D $_{\leq_{cp}}$ and D $_{O_{cp}}$)

$$D_{\leq_{cp}} \quad x \leq_{cp} y \equiv x <_{cp} y \vee x = y$$
$$D_{O_{cp}} \quad O_{cp}\, xy \equiv (\exists z)(z \leq_{cp} x \wedge z \leq_{cp} y).$$

One can see that these axioms and definitions are exactly analogous to what was presented in the section on properties of partial orderings. As shown above it then follows that the component-of relation, $<_{cp}$, is irreflexive and that $\leq_{cp}$ is a partial ordering.

Axioms for the tree structure

We now characterize the specific character of the component-of relation beyond the fact that it has the structure of a (proper) partial ordering. We do so by adding axioms which constrain the partial order in such a way that the resulting component-of hierarchy is a finite tree structure.

For this purpose we introduce two additional predicates, one which holds for the root of the tree structure ($D_{root_{cp}}$) and another which holds for atomic components, i.e., entities without a component ($D_{At_{cp}}$).

$$
\begin{array}{ll}
D_{root_{cp}} & root_{cp}\ x \equiv (y)(y \leq_{cp} x) \\
D_{At_{cp}} & At_{cp}\ x \equiv \neg(\exists y)(y <_{cp} x)
\end{array}
$$

The component-of relation $\leq_{cp}$ is now governed by further axioms in addition to ACP1-2 (the $<_{cp}$ -counterparts of APO1-2). These additional axioms fall into two groups, axioms which enforce the tree structure and the finiteness of this structure respectively. We start by discussing the first group:

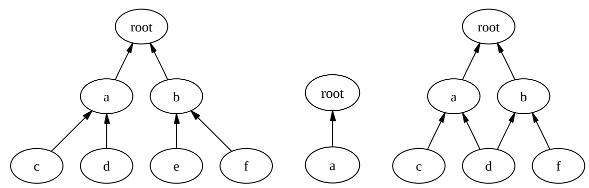
$$
\begin{array}{ll}
ACP3 & (\exists x) root_{cp}\ x \\
ACP4 & O_{cp}\ xy \rightarrow (x \leq_{cp} y \vee y <_{cp} x) \\
ACP5 & x <_{cp} y \rightarrow (\exists z)(z <_{cp} y \wedge \neg\, O_{cp}\ xz)
\end{array}
$$

ACP3 demands that every component-tree has a root. Using the antisymmetry of $\leq_{cp}$ we can then prove that there exists exactly one root. This rules out the structure in Figure 3(d) from being a component-of tree.

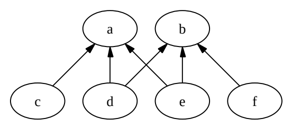
ACP4 is a version of what I shall call the *no-partial-overlap principle* (NPO). It rules out the possibility of partial overlap of components by demanding that if the complexes x and y share a common component then either x is a component of y, or x and y are identical, or y is a component of x. From this it follows that cycles like the one shown in Figure 3(c) cannot occur in component-of-trees.

Notice that the no-partial-overlap principle (NPO) also rules out the possibility that two different body systems which overlap (like the respiratory system and the alimentary system which share the component oropharynx) can exist within in the same component-of tree. This is because the two systems belong to distinct partitions of the human body (in the sense of the theory of granular partitions [1]), which is to say to different anatomical views or perspectives.

For example, the respiratory system has as components everything that is involved in the respiration process, and the alimentary system has as components everything that is is involved in the process of nutrition intake, digestion, and excretion. Clearly, there are parts of the body which have multiple functions, and therefore are components of different bodily systems. Each system has its own component-of tree with the particular system as a whole as the root. This corresponds to the view defended by Rector et al. [18] who argue that it is an important aspect of the design of ontologies to represent different views by means of separate tree structures.



(a) Component tree (b) Chain (c) Partial overlap (cycles)



(d) Multiple roots

Figure 3: Component trees and non-trees.

ACP5 demands that if x is a component of y then there exists a component z of y such that x and z do not overlap. This rules out cases where a complex has only a single proper component. In particular, it rules out graphs like the one shown in Figure 3(b) from being representations of component-of trees. ACP5 is a version of what, following Simons [23], I call the *weak supplementation principle* (WSP).

The second group of axioms that characterizes the component-of relation beyond the properties of being a partial ordering are axioms which enforce the finiteness of the component tree. ACP7 ensures that every complex has at least one atom as component. This ensures that no branch in the tree structure is infinitely long [30, 15]. Finally ACL8 is an axiom schema which enforces that every complex is either an atom or has only finitely many components. This ensures that component trees cannot be arbitrary broad.

$$
\begin{array}{ll}
ACP6 & (\exists y)(At_{cp}\, y \wedge y \leq_{cp} x) \\
ACP7 & \neg At_{cp}\, y \rightarrow (\exists x_1, \ldots, x_n)((\bigwedge_{1 \leq i \leq n} x_i <_{cp} y) \wedge \\
 & \quad (z)(z <_{cp} y \rightarrow \bigvee_{1 \leq i \leq n} z = x_i))
\end{array}
$$

Here $(\bigwedge_{1 \leq i \leq n} x_i <_{cp} y)$ is an abbreviation for $x_1 <_{cp} y \wedge \ldots \wedge x_n <_{cp} y$ and $\bigvee_{1 \leq i \leq n} z = x_i$ for $x_1 = z \vee \ldots \vee x_n = z$.

Extensionality

Extensionality is a property of the component-of relation which tells us that two complexes are identical if and only if they have the same components. For example if the complex c_1 has the components a and b and the complex c_2 has the components a and b then c_1 and c_2 are the same complex. This kind of reasoning might seem trivial from a human perspective, but it may be very useful to enable a computer to identify and to distinguish complexes by means of their components. Moreover, when specifying the semantics of the component-of relation it is important that the property of extensionality is covered by the formalism.

In this context it is important to stress once more that we here assume an atemporal framework in which we consider reality only as it exists *at a single moment in time*. This means that we do not take into account the fact that a complex can have different components at different times. For example, I might lose one of my fingers but still my hand before and after the accident are the same complex. How things preserve their identity while undergoing changes in this way is a difficult and controversial subject. For discussions see for example [28, 12, 14].

Given the above axioms for the component-of relations, we can in fact prove that it has the property of extensionality. This is because, using ACP1, ACP2, ACP4, and ACP5, we can prove that two complexes are identical if and only if they have the same components (TCP1). Moreover using ACP6 we can prove in addition that two complexes are identical iff they have the same atomic components (TCP2).

$$
\begin{array}{ll}
TCP1 & (\exists z)(z <_{cp} x) \rightarrow \\
 & \quad (x = y \leftrightarrow (z)(z <_{cp} x \leftrightarrow z <_{cp} y)) \\
TCP2 & x = y \leftrightarrow (z)(At_{cp}\, z \rightarrow (z \leq_{cp} x \leftrightarrow z \leq_{cp} y))
\end{array}
$$

Notice that neither TCP1 nor TCP2 is derivable from the axioms for a partial ordering alone. Both are consequences of the partial ordering axioms *in conjunction* with the specific axioms which we added in order to characterize the component-of relation. Consequently, relations which are only characterized to be a partial ordering may or may not be extensional in the sense described above. Therefore omitting axioms ACP3-7 means leaving important properties of the relation in question unspecified.

To be sure, the principles discussed here are built implicitly into systems like the FMA or GALEN. The important point, however, is that in order to explicate relations like component-of it is critical to make such axioms explicit.

Theorem TCP2 is also interesting from a computational perspective. Clearly, when comparing complexes it is much easier to check the identity only of atomic components rather than of all components.

Parthood among masses

An important aspect of entities classified as *masses* is that they do not have any compositional structure. This means that parts can be carved out from the original mass in an arbitrary fashion. Consider, for example, body-substances like blood, plasma, urine, etc. They can be separated arbitrarily into quantities, for example, by pouring them into containers or – abstractly – by applying fiat boundaries [25]. According to the FMA [7] we can distinguish, for example, the blood in containers like my right ventricle, my artery, my coronary artery, and so on; we can apply fiat boundaries and distinguish the blood in the left part and the right part of my body or the blood in the upper and lower parts of my body. All these operations carve out parts or quantities of the original mass. (See also [9].)

We start the formal treatment of the parthood relation among masses by introducing the binary primitive $x <_M y$ which is interpreted as 'the mass x is a proper part of the mass y'. We then add the axioms for asymmetry and transitivity (referred to as AM1-2) together with definitions for the improper parthood relation and for overlap (referred to as D_{O_M} and $D_{\leq_M}$, respectively) along the lines discussed in the opening paragraphs of the section on complexes. In this section we omit the statement of those axioms and definitions. As discussed above we then can prove that $\leq_M$ is a partial ordering relation.

In contrast to the component-of relation, the parthood relation among masses does not form a tree structure. This is because partial overlap can occur between masses. Consider, for example, the relation of overlap between the blood in the left part of my body and the blood in the upper part of my body. They partly overlap since they share a common quantity of blood, namely the blood in the upper left part of my body, but neither is part of the other. Consequently, we cannot have the no-proper-overlap principle (NPO) as an axiom or theorem in our theory of $<_M$.

On the other hand we clearly need the weak supplementation principle (WSP) to be an axiom or theorem of such a theory, since WSP ensures that there cannot be a mass that has a single proper part. Adding WSP as an axiom to this theory, however, is insufficient if we want to be able to identify and to distinguish masses in terms of their proper parts by means of a principle of extensionality similar to the one for complexes discussed above. (For details on why this is the case see [23].)

In order to characterize $\leq_M$ beyond its structure as a partial ordering we add an axiom to the effect that if x is not a part of y then there exists a z such that z is part of x and z does not overlap y (AM3).

$$AM3 \quad \neg x \leq_M y \rightarrow (\exists z)(z \leq_M x \wedge \neg\, O_M\, zy)$$

To see that AM3 is a sensible axiom consider the blood in my heart and the blood in my left ventricle. Clearly, the former is not a part of the latter. Moreover, the blood in my heart has parts, for example the blood in my right ventricle, which do not overlap with the blood in the left ventricle.

Using AM3 we can then prove the $<_M$ -counterpart of the weak supplementation principle (WSP) as a theorem (TM1), which then ensures that there cannot be a mass that has a single proper part. Using AM4 we can also prove that two masses are identical if and only if they have the same proper parts (TM2).

$$\begin{array}{ll} TM1 & x <_M y \rightarrow (\exists z)(z <_M y \wedge \neg\, O_M\, zx) \\ TM2 & (\exists z)(z <_M x) \rightarrow \\ & \quad (x = y \leftrightarrow (z)(z <_M x \leftrightarrow z <_M y)) \end{array}$$

Consequently, the property of extensionality holds for $<_M$.

The theory governing the compositional structure of masses, formed by AM1-3 together with the definitions for $\leq_M$ and O_M, is known in the literature as extensional mereology [23].

Containment

Consider Figure 2. Here we have a sequence of nested containers: my heart, containing a certain quantity of blood; my pericardial sac containing my heart; my thorax containing, among other anatomical entities, my pericardial sac. As pointed out above, containment understood in this sense is irreflexive, asymmetric, and transitive.

In our theory of containment we now introduce a binary primitive $x <_{ct} y$, which is interpreted as 'the entity x is contained in the entity y' together, with the axioms of asymmetry and transitivity (referred to as ACT1-2). We also add the usual definitions for overlap O_{ct} and for improper containment which includes identity $\leq_{ct}$, exactly analogous to those in the opening paragraphs of the section on complexes. We then can prove that $\leq_{ct}$ is a partial ordering relation.

Notice that, in contrast to the case of masses and complexes, we cannot here have the weak supplementation principle (WSP) either as an axiom or as a theorem in a theory of containment. This is because there are examples of containers with only one contained entity: my brain is contained in my skull; my sister is carrying a single baby in her uterus; my pericadial sac contains my heart as the only entity, etc. Those examples would be ruled out by a theory which contained WSP.

On the other hand, our theory of containment should permit us to identify or distinguish containers – at a given point in time – by means of the entities they contain. We therefore add an axiom to the effect that if (i) x has at least one contained entity, and (ii) every entity contained in x is also contained in y, then x is contained in y (ACT3).

$$\begin{array}{ll} ACT3 & ((\exists z) z <_{ct} x \wedge \\ & \quad (z)(z <_{ct} x \rightarrow z <_{ct} y)) \rightarrow x \leq_{ct} y \end{array}$$

The idea of modeling containment using the axioms ACT1-3 is due to Brock Decker. For details see [5].

Using the definition of $\leq_{ct}$ and the axioms ACT1-3 we can now prove that two containers x and y are identical iff they are non-empty and they contain the same entities (TCT1):

$$\begin{array}{ll} TCT1 & ((\exists z) z <_{ct} x \wedge \\ & \quad (z)(z <_{ct} x \leftrightarrow z <_{ct} y)) \leftrightarrow x = y \end{array}$$

Consider Figure 2. Like complexes, containers form tree-like structures in the sense that (1) there is a maximal container and (2) containers do not partially overlap. The structure is tree-*like* since there can be containers with only a single contained entity and hence nodes with a single child node in the corresponding tree representation (as the one shown in Figure 3(b)).

Formally we define predicates for the root, $root_{ct}$, and for atoms, At_{ct}, in terms of $\leq_{ct}$ exactly analogous to the definitions $D_{root_{cp}}$ and $D_{At_{cp}}$ in the section on complexes.

$$\begin{array}{ll} D_{root_{ct}} & root_{ct}\, x \equiv (y)(y \leq_{ct} x) \\ D_{At_{ct}} & At_{ct}\, x \equiv \neg(\exists y)(y <_{ct} x) \end{array}$$

The root here is understood as the maximal container and atoms are understood as entities which themselves do not contain any other entities.

We then add axioms ACT4 and ACT5 in terms of $root_{ct}$, $<_{ct}$, $\leq_{ct}$ and O_{ct} exactly analogous to ACP3 and ACP4.

$$\begin{array}{ll} ACT4 & (\exists x) root_{ct}\, x \\ ACT5 & O_{ct}\, xy \rightarrow (x \leq_{ct} y \vee y <_{ct} x) \end{array}$$

Here ACT4 enforces the existence of a root container. ACT5 is an instance of the no-partial-overlap principle (NPO) and rules out the partial overlap of containers.

Finally we add axioms ACT6 and ACT7 enforcing the condition that the resulting tree-like containment structures are finite. ACT6+7 are the $<_{ct}$ -counterparts of ACP6+7.

$$\begin{array}{ll} ACT6 & (\exists y)(At_{ct}\, y \wedge y \leq_{ct} x) \\ ACT7 & \neg At_{ct}\, y \rightarrow (\exists x_1, \ldots, x_n)((\bigwedge_{1 \leq i \leq n} x_i <_{ct} y) \wedge \\ & \quad (z)(z <_{ct} y \rightarrow \bigvee_{1 \leq i \leq n} z = x_i)) \end{array}$$

Conclusions

The theories of the component-of, mass-part-of, and contained-in relations presented in this paper share the fact that they all are partial orderings and satisfy the principle of extensionality. (In Table 1 this is indicated by the + symbols.) That the principle of extensionality is satisfied means that at a given moment in time we can identify and distinguish masses in terms of their proper parts, complexes in terms of their components, and containers in terms of the entities they contain. The fact that this kind of reasoning is permitted, however is not implied by the underlying partial ordering structure. Other principles needed to be added in order to support this kind of reasoning. I showed that the same principles allow us to distinguish these relations formally.

relation	part. order	WSP	NPO	EXT
component-of	+	+	+	+
mass-part-of	+	+	–	+
contained-in	+	–	+	+

Table 1: Theories of partial ordering relations and their underlying principles.

The principles which allow us to distinguish the three relations are the weak supplementation principle (WSP) and the no-proper-overlap (NPO) principle. The former holds in the theories of the component-of and the mass-part-of relations but not in the theory of the contained-in relation (indicated by the – symbol). The weak supplementation principle in the theories of component-of and mass-part-of tells us that a mass or a complex cannot have a single proper part or component. The no-partial-overlap principle in the theories of component-of and contained-in tells us that there cannot be partial overlap among components of complexes and among containers.

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