Dual Aspects of Abduction and Induction

Flavio Zelazek

Department of Philosophy, Sapienza University of Rome, Italy
flavio.zelazek@gmail.com

Abstract. A new characterization of abduction and induction is proposed, which is based on the idea that the various aspects of the two kinds of inference rest on the essential features of increment of (so to speak, extensionalized) comprehension and, respectively, of extension of the terms involved. These two essential features are in a reciprocal relation of duality, whence the highlighting of the dual aspects of abduction and induction. Remarkably, the increment of comprehension and of extension are dual ways to realize, in the limit, a 'deductivization' of abduction and induction in a similar way as the Closed World Assumption does in the case of the latter.

Keywords: abduction, induction, extension, comprehension, duality, closed world assumption.

1 The Uses of Abduction

The analysis of abduction is a topic of very much interest, albeit not a central one, in the fields of philosophy of science and AI. The former often relates abduction to reasoning from effects to cause and to *Inference to the Best Explanation* (IBE), while the latter exploits it in *Abductive Logic Programming* (ALP) [11], closely related to *Inductive Logic Programming* (ILP) [14], which in turn is a powerful tool for Machine Learning.

In the rich subfield of logic programming, the formal characterization of abduction is of particular interest, especially in relation with induction (here the fundamental reference is [5]) and deduction. Typically, abduction is explicated in terms of *default reasoning* and *Negation as Failure* (NAF), which in turn is related to the *completion* technique, which allows talking of abduction in deductive terms.[1]

Now, 20 years after the birth of ALP, there is an impressive proliferation of new frameworks and techniques meant to represent abductive reasoning, so that it seems by now really hard to have precise and yet unifying characterizations of it like those just mentioned. Maybe what is missing is just a simple and general – yet rigorous – enough logical characterization of abduction which encloses the common characters it has across so many diverse approaches and techniques.[2]

[1] For the relevant references see e.g. [7].

[2] In fact the need for unifying frameworks, typical of philosophy and science, is growing also in the fields of computer science and AI: consider, for example, the recent works by Kowalski and Sadri, like [12].

As is well known, the first logical analysis of abduction (and the term itself) has been given in the works of Charles Sanders Peirce. His vast speculation on abduction has been subdivided by Fann [4] into two periods: an early one, from 1859 to 1890, and a later one from 1891 to 1914; to these periods correspond, by and large, the two different conceptions of abduction which have been called by Flach and Kakas [6] the *syllogistic theory* and the *inferential theory*.

What has remained of the first theory, mainly in the AI and logic programming literature, is the triad of examples relating deduction, induction, and abduction (2.623),[3] of which the pertinent one is the following:

$$\begin{array}{ll} \text{(Rule)} & \text{All the beans from this bag are white} \\ \text{(Result)} & \text{These beans are white} \\ \hline \text{(Case)} & \text{These beans are from this bag} \end{array} \qquad (\textsc{Abd}_1)$$

On the other hand, the later peircean theory of abduction is summarized by this notorious reasoning schema (5.189), to which a massive literature is dedicated, especially in the field of philosophy of science:

$$\begin{array}{l} \text{The surprising fact, } C\text{, is observed;} \\ \text{But if } A \text{ were true, } C \text{ would be a matter of course,} \\ \text{Hence, there is reason to suspect that } A \text{ is true.} \end{array} \qquad (\textsc{Abd}_2)$$

The "would be a matter of course" implies something like the fact that A is a *cause* – or at any rate an *explanation* – of C (as this is what generally accounts for C being 'naturally' expected, given A),[4] whence the matching of abduction with (backward) causal reasoning and/or IBE.

Despite the formal clarity of ($\textsc{Abd}_1$) and the informal intent of ($\textsc{Abd}_2$), often abduction is represented by this inference schema:

$$\frac{\begin{array}{l} M \to P \\ P \end{array}}{M}\ \textsc{Abd}_0$$

Now, presumably the 'rule' ($\textsc{Abd}_0$) is simply to be conceived as a forgetful interpretation of ($\textsc{Abd}_2$), and of this other schema (extracted from ($\textsc{Abd}_1$)), which is at the heart of ALP:

[3] I'm using the standard quotation format for the *Collected Papers* of C. S. Peirce [9]: (<*volume number*>.<*paragraph number(s)*>).

[4] Actually, the quoted expression could also be construed as the weakest claim that C is positively (even if spuriously) correlated to A; but this is usually not the case.

$$\frac{\forall x\,(M(x) \to P(x)) \qquad P(s_1)}{M(s_1)}\ \textsc{Abd}_{\mathrm{Q}}$$

But the fact is that the schema ($\textsc{Abd}_0$) 'forgets too much': if we want to characterize abduction as IBE, we must operate in a causal/explanatory framework; while if we want to talk of abduction in a purely logical way, e.g. when doing logic programming, we must at least use predicate logic. A formulation in terms of mere propositional logic like that of ($\textsc{Abd}_0$), besides being vaguely evocative, is not sufficient for neither approach; and, what is worst, it leaves the door open to easy criticisms against abduction in general (see note 8 below).

On the other side, it must be said that, often, many useful logical features of reasoning are captured in terms of pure propositional logic; so the problem is: can we characterize abduction within propositional logic in a less trivial and more illuminating way than ($\textsc{Abd}_0$) does? And can we extract from such a characterization any new (or, up to now, largely overlooked) feature of abductive reasoning? The answer to both questions is – I hope – positive, as will be shown in the next section.

2 Abduction as the Dual of Induction

The analysis of abduction – and of its relation of *duality* with induction – which is to follow is based on, and is an explication of, the works belonging to the early logical speculation of Peirce, which are framed in a syllogistic (and, later, probabilistic) setting.

Other works insisting on the duality between abduction and induction are [15], based on the early peircean theory like the present one; [1], which exploits the notion of *preferential entailment* and instead considers the later peircean conception of abduction; [2], which remarks that abduction and induction are related in the same way extension and intension are.[5]

I shall represent induction and abduction by the following inference schemata, which are dual to each other:

[5] Few remarks about the importance of the duality relation. It is a key concept in category theory; in fact, category theory itself seems to have been born to solve a duality problem, with Mac Lane's 1950 article *"Duality in Groups"* (cf. [3]). Moreover, duality is an 'hidden interest' of philosophers: think of Hempel and Oppenheim's attempt to define the *systematic power* ('content') of a theory as the notion which is dual to the one of *logical probability* ('range') in [10]. Finally, the concept of duality is a central one in recent logical research on proof theory and the foundations of theoretical computer science: see [8].

$$
\frac{\begin{array}{l} S_1 \vee \cdots \vee S_j \to M \\ S_1 \to P \\ \quad\vdots \\ S_j \to P \end{array}}{M \to P}\,\textsc{Ind}^* \qquad\qquad \frac{\begin{array}{l} M \to P_1 \wedge \cdots \wedge P_k \\ S \to P_1 \\ \quad\vdots \\ S \to P_k \end{array}}{S \to M}\,\textsc{Abd}^*
$$

These definitions arise as a way to connect the schema ($\textsc{Abd}_1$) above (p. 2), and the analogous schema for induction, to the earliest characterization of inducton and abduction given by Peirce (in (2.511), and also in (2.424–425)), in which *multiple subject/predicate classes* are explicitly inserted.

The natural means (suggested, more or less in clear terms, by Peirce himself in (2.461–516)) to move from a multiplicity of objects to a single class is simply to take their *union* or *intersection*,[6] if they are subjects or, respectively, predicates.

The enlargement of a class by adding members to an union, and the shrinkage of a class by adding members to an intersection, are respectively the extensional counterparts of the concepts of *breadth* and *depth* of a term (i.e. a class), which Peirce investigates thoroughly in (2.407 ff.). They are akin to the concepts of *extension* and *comprehension*,[7] and to the aforementioned notions of *range* and *content* examined in the early hempelian analysis of scientific explanation (see note 5).

The central aspect in the present approach is the idea that induction and abduction are essentially based upon, respectively, the enlargement and the shrinkage of classes, all the other features being derived from these. Console and Saitta [2] make a similar claim, identifying as the logical processes governing induction and abduction those of *generalization* and *specialization*, which are at the base of the theory of Machine Learning.

To see how the inductive and abductive inferences work in this conceptual framework, consider Fig. 1 and Fig. 2, which illustrate what happens in (the set-theoretical versions of) the inferences ($\textsc{Ind}^*$) and ($\textsc{Abd}^*$) with $j = k = 2$. Briefly, an inductive inference is the more confirmed/plausible the more *subjects* it examines, up to the ideal point when the subject class has been enlarged so much that it coincides with the middle term class; at which point all the tests $S_j \to P$ are passed, so that $M \setminus P = \varnothing$ (the shaded area, i.e. the potential

[6] I take for granted the shifting from predicate logic to set notation and then to propositional logic, in the case of categorical propositions of the form "A" (universal affermative); so that $\forall x\ (A(x) \to B(x))$ becomes $A \subseteq B$, which in turn can be written as $A \to B$. Taking $\to, \neg, \vee, \wedge$ instead of $\subseteq, \setminus, \cup, \cap$ (as indeed happens in the schemata ($\textsc{Ind}^*$) and ($\textsc{Abd}^*$)) is really a way to let propositional logic speak for the relevant logical features of objects otherwise represented by predicate logic.

[7] Things are actually more complicated: Peirce makes a distinction between *essential*, *substantial* and *informed* breadth and depth, each subjected to different rules; and he is critical precisely of the careless flattening of these concepts on those of extension and comprehension.

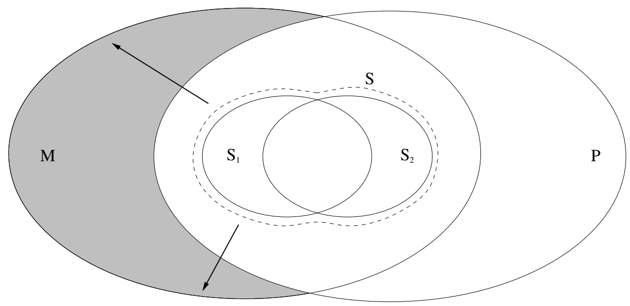

Fig. 1. Induction

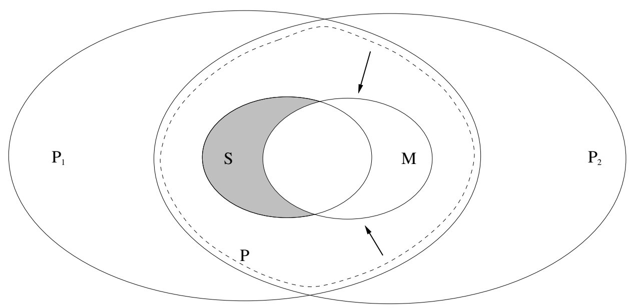

Fig. 2. Abduction

counterexample space for the conclusion, is empty), and the arrow in the first premise of (IND*) can be inverted, so that a deduction is obtained.

Dually, an abductive inference is the more confirmed/plausible the more *properties* of subjects it considers, up to the ideal point when the predicate class has been shrunk so much that it coincides with the middle term class. That being the case, all the tests $S \to P_k$ are passed, so that $S \setminus M = \varnothing$ (again, the shaded area, i.e. the potential counterexample space, is empty), and the arrow in the first premise of (ABD*) can be inverted, so as to get a deduction.

This reversal of arrows in induction and abduction is linked with *completion* or *circumscription* techniques, so basically with the *Closed World Assumption*. On this point an interesting analysis has been made by Lachiche [13].

3 Conclusions and Further Research

The present view of induction and abduction is an ‘incremental’ one: we increase the number of tests up to the limiting point in which all of the relevant subjects (induction) or all of the relevant properties (abduction) have been tested; even before reaching this ideal point, we get more and more confident about our conclusion as we proceed further.[8] In the opposite direction, we have as limiting/trivial cases, respectively, single-case induction and (ABD_Q), as in both these inference forms only a single test is taken into consideration.

In this work I have considered implications (i.e. inclusions) as categorical, and tests as yes/no questions; it may be interesting, then, to extend this approach in order to capture probabilistic features as well. This in turn is needed if one wants to define some measure of confirmation/plausibility and, possibly, to implement the new inference figures in a logic programming system.

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[8] Thus it can be avoided, or at least minimized, the problem of (ABD_0) committing the *fallacy of affirming the consequent*: an effect can have, e.g., two competing causes/explanations; but it is improbable that for a conjunction of effects (i.e. for a more detailed description of the effect in question – or, in the limit, for its 'maximally detailed', or 'complete', description) both of the same two explanations remain available. I wish to thank a reviewer for having pointed this matter.