Perfect Syllogisms and the Method of Minimal Representation

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***Abstract*—In this paper, the Method of Minimal Representation (MMR, i.e., an alternative diagrammatic technique to test the validity of syllogisms) is employed to differentiate perfect (first figure) from imperfect syllogisms (other figures). It demonstrates that the validity of perfect syllogisms can be exhibited applying lesser number of rules, which govern the proposed method. This is also in accordance with Aristotle's dictum of *ceteris paribus*, which assumes the superiority of a demonstration that is derived through fewer postulates or hypotheses. This paper is divided into four sections. The first section gives a brief exposition on the historical preliminaries of perfect syllogisms. The second section elaborates the method of minimal representation. In the penultimate section, we test first figure syllogisms and contrast them with other figures by the proposed method. In the concluding section, we demonstrate the primacy of first figure among all figures.**

I. Introduction

Syllogism, according to Aristotle, "is [a] discourse in which, certain things being stated, some other thing than that what is stated follows [out] of necessity from their being so" [McKeon 1946:66]. The expressions "certain things be stated" forms the premise(s) whereas "some other thing" refer to conclusion. In other words, the implicit assertion(s) in the premise(s) is made explicit through the conclusion. With reference to this premise to conclusion transformation, he called the first figure syllogisms, perfect.

In *Organon*, he said, "I call that a perfect syllogism[1] which needs nothing other than what has been stated to make it plain what necessarily follows; a syllogism is imperfect, if it needs either one or more propositions, which are indeed necessary consequences of the terms set down, but have not been expressly stated as premises"[Ibid:66]. The second and third figures are imperfect figures[2].

Imperfect syllogisms are not invalid syllogisms, rather 'potential syllogisms' where what he means by 'potentiality' is that the imperfect syllogisms are 'potentially perfect syllogisms' [Patzig 1968:46].That is to say that an imperfect syllogism can be transformed into a perfect syllogism. A more-imperfect syllogism takes more steps to transform into a perfect syllogism, whereas less-imperfect syllogism takes fewer steps.

Aristotle demonstrated the validity or invalidity of syllogisms using three methods. They are, namely, *ecthesis* (also expressed as *ekthesis*), *reductio ad absurdum* (also known as *reductio per impossible* or *reductio ad impossible*) and *reduction*. In the following subsections, I briefly explain these methods taking some examples.

A. Ecthesis

The proof by *ecthesis* is a method of exposition with which Aristotle validated a given syllogism. However, he has not used this method extensively. According to Smith, the proof by 'ecthesis or 'setting out methodology is used several times by Aristotle to provide an alternative deduction schema for completing certain syllogistic moods [Smith 1983: 224-232]. He also points, Aristotle never explains why he includes this alternative deductions and there is some debate about what exactly the procedure is.

Let us try to understand this procedure taking the example which Aristotle took. Let '**P** belong to all **M**' and '**S** belong to all **M**'. Then, '**P** belong to some **S**'. In order to prove this, Aristotle introduced an element of novelty by assuming, say '**c** (a member of the class)' which belong to **M**. Then '**c**' belongs to **P** and also '**c**' belongs to **S**. It means, that '**c**' belongs to both **P** and **S**. If '**c**' is a common member of **P** and **S**, then there is something common between **P** and **S**. This then entails that **P** belongs to some **S**.

Alexander maintains that the constant '**c**' is a singular term given by perception, and the proof by exposition (ecthesis) consists in a sort of perceptual evidence [Lukasiewicz 1955:60]. Lukasiewicz, like any other logician of his time has not accepted it as a proof[3], as perception is not a logical proof [Ibid 60]. In fact, he calls it a method outside the limits of the syllogistic [Ibid 45].

[1]In an interesting study, perfect syllogisms have been further divided into two sets [Lehman 1972]. However, in this paper, I have *not* considered that approach.

[2]The fourth figure that was later added by Galen is also imperfect.

[3]Until late twentieth century, diagrammatic representations or reasoning based on diagrams were not accepted as proofs. They were at most understood as schemes based on heuristics.

B. Reductio ad impossible

It is one of the oldest[4] method used in any system of reasoning. This is performed in two steps. In the first step, the negation of the conclusion is assumed. In the second step, the falsity (as a result of contradiction) of this negation of the conclusion is proved. If the negation of the original conclusion is proved to be false, then the original conclusion is proved true. This is called reductio ad absurdum or what Aristotle termed it as *reductio* ad impossible. Roy called it reductio *per* impossible [Roy 1958:].

Let us take an example from third figure, 'if **R** belongs to some **S**, and **P** to all **S**, **P** must belong to some **R**'. Aristotle says, that this can be shown valid using reduction, reductio ad impossible as well as ecthesis[5]. Let us examine *reductio* here. Suppose, **P** does not belong to any **R**. It means, **P** belongs to no **R**. Then, **R** belongs to no **P**. But **P** belongs to all **S** and **R** belongs to some **S**, thus, **P** must belong to some **R**. This makes our initial supposition false. Therefore, the given syllogism is valid.

The method of *reductio* was one of the prevalent methods of reasoning during those times. Aristotle has used it in order to show his systems coherence to the existing rationality as this expression many a times occurs in *Organon* that a syllogism is "possible to demonstrate it [or show its validity] *also* [using] reductio *per impossible*".

C. Reduction

This refers broadly to a method of transforming the moods of one figure to moods of another figure [Roy 1958:216]. More precisely, the transformation of second and third figure moods into first figure is reduction. A first figure valid syllogism is accepted as an axiom by Aristotle. It is a necessary truth and thus requires no further explanation. He shows that a form is valid by showing how to deduce its conclusion from its premises. A deduction is a series of steps leading from the premises to the conclusion, each of which is either an immediate inference[6] from the previous step or an inference from two previous steps.

A simple question which arises at this juncture: *How Aristotle shows a given syllogism valid by transforming the terms of the proposition?* He shows them valid by changing a syllogism of second or third figure to first figure. The next question then will be: *Why Aristotle transforms second or third figure syllogism into first figure?* It is because, the first figure is perfect figure and the valid syllogisms of this figure are perfect syllogisms. Hence to establish that the conclusion of a perfect syllogism follows from the premises, one should need to do no more than state the syllogism itself [Lear 1980:2].

[4]It is one of the oldest, if not the oldest method used as a proof. Pre-socratics have used it along with counter-example technique. However, it is difficult to comment on their precedence.

[5]Ecthesis is seldom used by Aristotle for proving validity of syllogisms.

[6]Conversion, Obversion and Contraposition (sometimes, Inversion as well) are the immediate inferences. They play a vital role in determining the validity of syllogisms. It is also contended that rules of immediate inferences were principally developed to serve the purpose of validation of syllogism using the method of reduction.

Logicians theorized various overtures to understand the nature of perfect syllogisms though there exists no exact logical analysis of the proofs Aristotle gives to reduce the imperfect syllogisms to the perfect [Lukasiewicz 1955:47]. Roy opines, the *dictum de omni et nullo* is directly applicable to first figure [Roy 1958:216]. Whereas, Patterson claims "this is misleading not because Aristotle nowhere explicitly formulates the *dictum* and casts it in the role of logical touchstone, but because the *dictum* itself should be seen as a reflection or encapsulation of Aristotelian convictions about the (small number of) ways in which one item can relate predicatively to another [Patterson 1993:375].

In a[nother] word[s], perfect syllogisms are 'self-evident' syllogism [Patzig 1968:45]. He further opines, that the defined 'necessity' (as per the definition of syllogism) not only occurs but also 'appears' or is transparent. In an imperfect syllogism, this defined 'necessity' undergoes certain operations before it 'appears' or becomes transparent. This observation of Patzig also supports the claim of Kneales that Aristotle's thought was guided by diagrams which makes this necessity 'appear' evident [Kneale & Kneale 1962:72]. Flannery has successfully provided a rationale for the notion of perfect syllogisms using diagrams [Flannery 1987:455-471].

Another interesting remark made by Roy is that 'reduction' reveals the essential unity of all forms of syllogistic inference (Roy 1958:218) as all the figures get transformed into the first figure. Flannery's rationale, Roy's remark along with Kneales' claim, points to an important fact that in order to cognize the idea of perfect syllogisms, one requires visual aid and clarification. Similarly, when Patzig says, that defined necessity 'appears' transparent, what he means is that the line of reasoning (in case of perfect syllogisms) creates a picture in our mind, which our rationality grasps. Thus, in what follows, I intend to understand this notion of perfect-*ness* with the help of a new diagrammatic technique to test the validity of syllogism, called the 'method of minimal representation'.

II. Method of Minimal Representation

Method of Minimal Representation[7] is an alternative diagrammatic scheme for testing syllogisms [Sharma 2008:412-415]. The purpose of developing this method is to differentiate traditional and modern valid syllogisms. Let us in brief, revisit the history of valid syllogism (in numbers) before explaining the proposed method.

A. Valid Syllogisms

Aristotle discusses fourteen syllogisms belonging to the first, second and third of the traditional figures [Patzig 1968:132] as valid. With the development of fourth figure and considering the weakened moods[8], twenty-four syllogisms are

[7]This method was originally presented as Method of Least Representation at the First World Congress on the Square of Opposition, Montreux, Switzerland (June 2007). Later, it was revised (based on the comments and suggestions), and was named as Method of Minimal Representation. See Sharma (2008, 2012).

[8]Weakened moods refer to those syllogisms, where we replace a universal conclusion with its sub-altern. For e.g., if **AAA**-1 is valid then **AAI**-1 must also be valid since **I** proposition is the sub-altern of **A** proposition.

traditionally valid. If we leave the weakened moods, then there are nineteen valid syllogisms. However, if we consider modern interpretation, only fifteen of these are valid.

The Aristotlelian methods (as discussed in the previous section) along with mnemonics (developed by Petrus Hispanus [Keynes 1906:329]) were used to test syllogisms. Formal syllogistic rules, which follows the late ninteenth and early twentieth century development in logic is widely used these days. Nonetheless, Euler circles (for traditional valid syllogisms) and Venn-Peirce system (for modern valid syllogisms) also gained currency for testing syllogisms from the standpoint of diagrams.

However, none of the above diagrammatic technique is able to demonstrate whether a syllogism is valid/invalid in both traditional and modern readings[9]. The motivation to develop a unified diagrammatic scheme for interpreting syllogisms in both traditional and modern readings came from here[10]. In the next subsection, I explain the method of drawing propositions and consequently syllogisms.

B. Drawing Propositions and Syllogisms

There are two basic shapes[11] that are employed in this method. They are, namely, rectangles and right angled triangles. These shapes act as initial tools for drawing a proposition. Apart from these, arrows are used to show the possible location of the shape of subject term of the proposition. We also label the rectangles or triangles for pinpointing them. The scheme for propositions is as under:

*1) Universal Affirmative Proposition (**A**) - All S is P:* Here, a small rectangle (S) is drawn from the right bottom edge of the larger rectangle (P) containing it. The arrow shows that the possibility of finding S is inside P only.

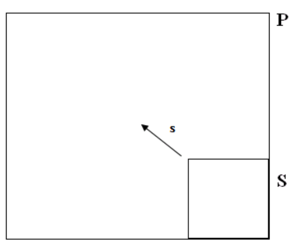

Fig. 1. **A** proposition

*2) Universal Negative Proposition (**E**) - No S is P:* Here, two disjoint rectangles (S and P), of equal areas, are drawn. The arrow shows that the possibility of finding S is outside P only.

[9]It may also be noted here that no standard digrammatic technique attempts to address the notion of perfect syllogism with its diagram.

[10]After explaining this diagrammatic technique, I with the help of an example clarify, how to use the same diagram for traditional and modern interpretation.

[11]I use 'shape' intentionally so as to distinguish it from 'figure'. 'Shape' means a geometrical structure used in the MMR whereas 'figure' in syllogistic is that which determines the position of middle term in a syllogism.

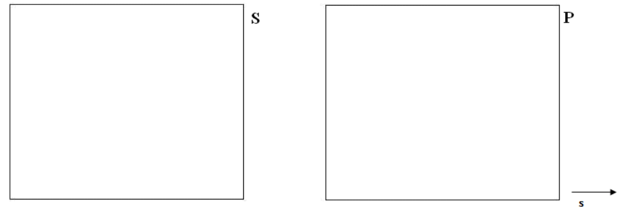

Fig. 2. **E** proposition

*3) Particular Affirmative Proposition (**I**) - Some S is P:* Here, a right-angled triangle (S) is drawn from the right bottom edge of the rectangle (P) containing it. The arrows suggest that the possibility of finding S is both inside as well as outside P.

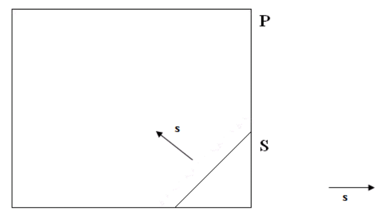

Fig. 3. **I** proposition

*4) Particular Negative Proposition (**O**) - Some S is not P:* Here, a right-angled triangle (S) is drawn from the right bottom edge of the rectangle (P) outside it. The arrows suggest that the possibility of finding S is both inside as well as outside P.

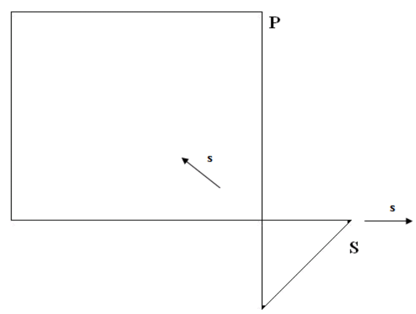

Fig. 4. **O** proposition

Summarizing the above:

- The Universal Propositions (**A** and **E**) are represented by a smaller rectangle inside a greater rectangle.
- The Particular Propositions (**I** and **O**) are represented by a right-angled triangle inside or outside a greater rectangle.
- In a Universal Affirmative Proposition (**A**), a rectangle is drawn from the right bottom edge approximately less than one-fourth of the area of greater rectangle containing it.
- In a Universal Negative Proposition (**E**), two disjoint

rectangles, of around equal areas, are drawn.

- In a Particular Affirmative Proposition (**I**), a right-angled triangle is drawn from the right bottom edge approximately less than one-fourth of the area of greater rectangle containing it.

- In a Particular Negative Proposition (**O**), a right-angled triangle is drawn from the right bottom edge approximately less than one-fourth of the area of greater rectangle outside it.

C. Additional Parameters

Apart from the above rules, certain additional conventions are also followed while drawing the propositions and eventually syllogisms. In these propositions, the subject class (smaller rectangle or right-angled triangle) is represented in the predicate class (greater rectangle) with less than one-fourth occupation. The prescription for drawing rectangles or right-angled triangles approximately less than one-fourth of the area of greater rectangle needs to be elucidated here. This convention has been included to nullify the possibility of any unnecessary overlapping of basic figures. This is the case because while representing syllogisms we may require drawing basic shapes (rectangles/right-angled triangles) inside a rectangle. Furthermore, to avoid any needless overlapping, we always draw the second premise in the syllogism from another edge of the greater rectangle. Let us take the following example:

Suppose, we have to draw All **P** is **M** and All **S** is **M** in a syllogism, then this is drawn as:

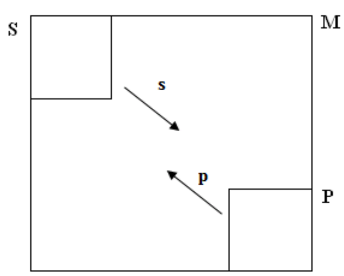

Fig. 5. All P is M and All S is M

Similarly, arrows become important for validating syllogisms as given under with the following example. Suppose, we have to draw No **P** is **M** and No **S** is **M** in a syllogism, then this is drawn as three disjoint rectangles as given below:

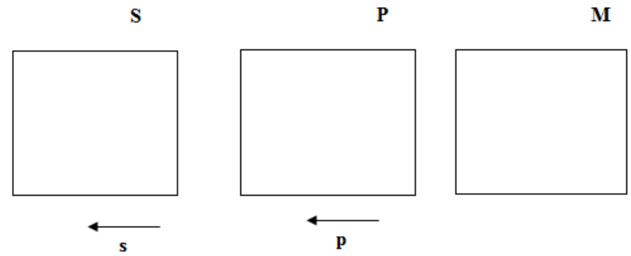

Fig. 6. No P is M and No S is M

If the conclusion says that No **S** is **P**, then it seems from the above that it holds. However, the arrows (s and p) depicts that **S** or **P** can be anywhere except inside **M**. Thus, **S** and **P** can be overlapping or can fully be contained inside. Therefore, we cannot conclude No **S** is **P**, and the given syllogism is invalid. In the next subsection, I discuss the inference rules briefly.

D. Inference Rules

Let us take an example to show how this method can test the validity of a syllogism in both traditional and modern understanding. *Darapti* i.e., **AAI**-3 has the following structure:
All **P** is **M**
All **S** is **M**
Therefore, Some **S** is **P**.
We first draw a small rectangle **M** inside the greater rectangle **P** along with arrows. Similarly, keeping the small rectangle **M** there, we draw another greater rectangle **S** as shown below:

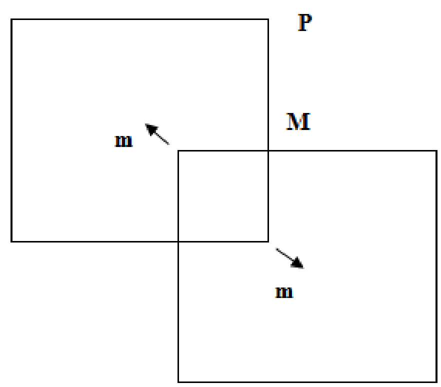

Fig. 7. All P is M and All S is M

In the traditional interpretation, we need to find that there shall be a part common part between **S** and **P**. In the diagram, it is portrayed by **M**. Therefore, **AAA**-3 is valid in the traditional interpretation. However, in the modern viewpoint, we need to have a right-angled triangle **S** inside rectangle **P**. If we examine the diagram In the next section, we test syllogisms with MMR in each first, second and third figure.

III. MMR AND PERFECT SYLLOGISMS

In this section, we take three examples from each figure to demonstrate the simplicity of first figure and the relative complexities associated with second and third figures.

A. MMR and the First Figure

Let us take **AAA**-1:
All **M** is **P**
All **S** is **M**

Therefore, all **S** is **P**.

We first draw the rectangle **M** inside the largest rectangle **P** as given in the first premise. We then draw rectangle **S** inside the larger rectangle **M**. We label the diagram appropriately and draw the arrows as per rules.

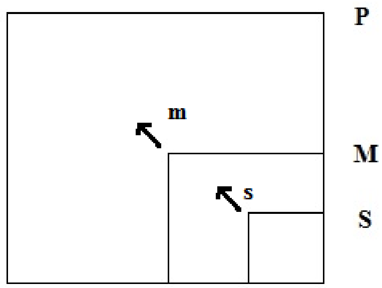

Fig. 8. All M is P and All S is M

B. MMR and the Second Figure

Let us take **AOO**-2:
All **P** is **M**
Some **S** is not **M**
Therefore, some **S** is not **P**.

In this, we first draw the smaller rectangle **P** inside greater rectangle **M**. The minor premise i.e., some **S** is not **M** poses a problem (to be integrated) in this diagram. In order to avoid any intersection, so as to preserve the conventions of minimal representation, we draw it in the following way:

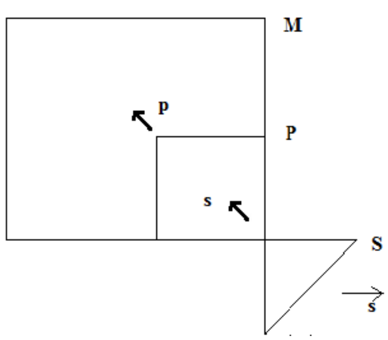

Fig. 9. All P is M and Some S is not M

C. MMR and the Third Figure

Considering **OAO**-2:
Some **M** is not **P**
All **M** is **S**
Therefore, some **S** is not **P**
The problem it (this syllogism) poses first is that which premise may be drawn first. If we draw the major premise first, we find it difficult to integrate the minor premise[12]. In order to tacle this difficulty, we draw the minor premise first[13]. Nonetheless, if we draw the minor premise first, then also it requires suitable accommodation (of inference rules and additional parameters) to integrate the major premise after that in this diagram. After taking all these into consideration, the given syllogism can be drawn in the following fashion:

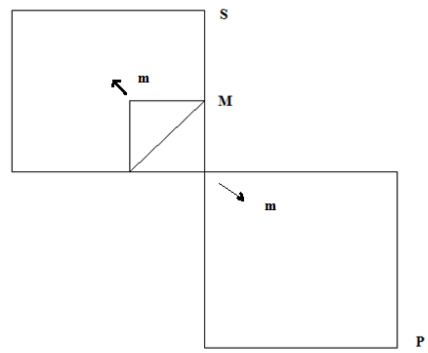

Fig. 10. Some M is not P and All M is S

Though, we draw the above diagram, putting the arrows in the right positions is also difficult. However, the above diagram can be drawn for the given syllogism using MMR considering the inference rules and additional parameters.

The above examples, show that the second and third figure[14] poses problems (as they are tangled) while drawing with the help of MMR.

IV. Conclusion

This article shows that the syllogisms of first figure are simple[15] (or easier) to draw using the proposed method. In the first figure, we use only the rules prescribed in *Drawing Propositions and Syllogisms*. However, the second and third figures are relatively complex and thus require (apart from the above rules) the usage of *Additional parameters*. Moreover, the syllogisms of second and third figures need careful considerations (to suitably incorporate both the major and minor premises along with the positioning of arrows) for its final diagram to be drawn. MMR depicts the clarity and simplicity of first figure and its line of reasoning. The clarity and simplicity of first figure makes it perfect.

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[12]In fact, it is not only difficult but impossible to draw the minor premise later, if we draw the major premise before.

[13]It may further be noted that drawing the minor premise first and then the major premise, is to use *additional parameters* as explained in **Section II-C**.

[14]I have *not* considered fourth figure syllogisms in this paper, in order to keep Aristotelian notion of figures intact. Nonetheless, we face similar difficulties (as in second and third figure) while drawing the fourth figure syllogisms as well.

[15]

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REFERENCES

[1] K.J. Flannery, A Rationale for Aristotle's Notion of Perfect Syllogisms, *Notre Dame Journal of Formal Logic*, 28, no. 3 , 1987: 455-471.

[2] J.N. Keynes, *Studies and Exercises in Formal Logic*, London:MacMillan and Co., 1906.

[3] W. Kneale and M. Kneale, *The Development of Logic*, Oxford:Clarendon Press, 1962.

[4] J. Lear, *Aristotle and Logical Theory*, Cambridge:Cambridge University Press, 1980.

[5] A. Lehman, Two sets of Perfect Syllogisms, *Notre Dame Journal of Formal Logic*, 1972: 425-429

[6] J. Lukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Oxford: Clarendon Press, 1972.

[7] R. McKeon, ed. *The Basic Works of Aristotle*, New York: Random House, 1941.

[8] R. Patterson, Aristotle‘s Perfect Syllogisms, Predication and the Dictum De Omni, *Synthese*,96, 1993: 359-378

[9] G. Patzig, *Aristotle's Theory of Syllogism*, Dodrecht: D. Reidel Publishing Company, 1968.

[10] B.N. Roy, *A Textbook of Deductive Logic*, Calcutta: Sarkar and Sons, 1958.

[11] S.S. Sharma, *Interpreting Squares of Opposition with the help of Diagrams*, In Square of Opposition: A General Framework, by J.Y. Beziau and G. Payette, 175-194. Bern: Peter Lang Publishing House, 2012.

[12] S.S. Sharma, Method of Minimal Representation: An Alternative Diagrammatic Technique to Test the Validity of Categorical Syllogisms, *Lecture Notes in Artificial Intelligence*, 2008: 412-414.

[13] R. Smith, Completeness of an Ecthetic Syllogistic, *Notre Dame Journal of Formal Logic*, 1983: 224-232.