Reasoning about directions in an egocentric spatial reference frame

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***Abstract*—Within qualitative spatial reasoning, spatial objects having directions have been abstracted in different ways; directed line segments, oriented points etc. have been used. In certain applications, it becomes necessary to reason about directions of objects without focusing on the dimensionality of these objects. We present a framework for representation of and reasoning about directions in a qualitative way within an egocentric spatial reference frame. Qualitative direction has been separated from spatial location and dimensionality of spatial objects and as such, the formalism may be used to represent qualitative direction in a dimension-independent way. The formalism uses fewer numbers of base relations than existing formalisms. Further granularity can be refined easily. Qualitative direction relations separated from spatial location and dimensionality can be used to express spatio-temporal patterns of directional entities using a regular grammar.**

***Keywords*—QSR; Qualitative direction; Qualitative Direction Algebra; Composition.**

I. Introduction

Everyday reasoning involving spatial and temporal attributes is driven through qualitative abstractions rather than complete quantitative knowledge. Qualitative abstractions are an integral part of our conceptualization of space and time [1]. Quantitative information is precise and accurate. However, such precise information may not be cognitively meaningful at times. For example, in the domain of traffic analysis, we can measure the change in direction of a moving car at certain intervals. This quantitative data is precise, but we will not be able to extract much high level knowledge from it. Instead, a qualitative expression like "the car is approaching me from the opposite direction" will convey more information. Further, space and time are inextricably linked. For any commonsense theory of spatial representation and reasoning, space and spatial change need to be interwoven! Commonsense theories of space and spatial change for cognitive agents need to be qualitative rather than quantitative.

Space, in our commonsense knowledge, is characterized by many different attributes. Within Qualitative Spatial Reasoning (QSR), spatial characteristics are abstracted in a qualitative way. Aspects of space that have been treated in a qualitative way are spatial orientation, distance, direction, shape and size etc [2]. In this paper, we have taken up one such spatial aspect i.e. qualitative direction. Qualitative direction plays an important role when we think of analyzing patterns formed by directed objects. Such an object may be stationary or moving. For example, vehicles moving on a highway leads to different spatio-temporal patterns. In this case, we take the direction of the vehicle along its front. Each vehicle may find the qualitative direction of other vehicles relative to its own direction of motion. For example, as shown in Figure 1, we have different scenario (from top to bottom) such as 'car in *front* of truck'; 'car to *left* of truck' and 'car *behind* truck'. At any instance of time, these directions form a pattern that can be analyzed to extract higher level semantics. Having seen the above direction relations sequence (from top to bottom) we understand a *composite* pattern 'truck *overtake* car'.

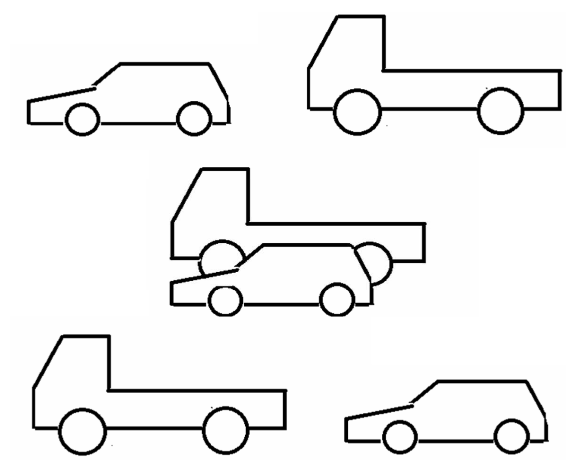

Fig. 1. Each vehicle may find the qualitative direction of other vehicles relative to its own direction of motion. Such qualitative direction relations form the basis of describing motion patterns such as illustrated above.

In QSR literature, different formalisms have been used for representing directions of spatial objects in a qualitative way. In some works, spatial objects have been abstracted as directed dimensionless points. In others, directed line segments have been used [2]. An interesting issue here is about the representation of direction of spatial objects extended in space. If we treat objects in two or three dimensional forms, then we would not be able to abstract these as points or lines. One needs to separate the issue of representation from the issue of dimensionality. Definitions of qualitative direction relations should not be influenced by underlying dimensions. For example, in dipole relation algebra [3], spatial objects are abstracted as directed line segments having a start and an end point. In defining qualitative relations between two dipoles, the location of end points with respect to a line (whether on the left or on the right side) is considered. Therefore, this formalism will not be suitable if our spatial objects are rectangles instead of lines. Similarly, in oriented point algebra [4], spatial objects are abstracted as dimensionless points having directions. The direction of a point sets up a coordinate system and orientation labels like $Front$, $Back$ etc. can be defined with respect to this coordinate system. As higher dimensional objects are extended in space, meaning of orientation labels like $Front$, $Back$ etc. will be quite different. We can cite a formalism called rectangular cardinal directions [5] where spatial orientation of rectangles having sides parallel to the axes of projection is treated. Oriented point algebra will not be suitable here because the objects under consideration are extended in space in two dimensions. In order to have a dimension independent representation of qualitative direction, we should not bring in the spatial orientation of any part of the abstracted object into our definition.

For the Qualitative Direction Algebra (QDA) proposed in this work, we separate qualitative direction from the issues of spatial location and dimensionality. We propose a direction model that does not use spatial location labels for representing qualitative direction. We have used angular measurements between directions for representing our qualitative direction relations. Direction relation labels, that we have proposed, are closer to our cognitive perception of object locomotion. A Jointly Exhaustive and Pairwise Disjoint (JEPD) set of binary qualitative relations has been proposed for representing and reasoning with qualitative directions. The issue of spatio-temporal continuity of these base relations has also been addressed. For constraint based reasoning, this set of relations needs to be closed under composition and converse. We have proposed algorithms for finding the converse of a single relation and also to find the composition of two base relations. An interesting characteristic of this formalism is that the granularity can be refined depending on requirement. In QSR, continuous input is discretized and qualitative abstractions are introduced. For example, let us consider distance as a spatial aspect. Distance of one object from another will have continuous numeric values. From a qualitative view point, we can discretize these values into three zones and label these as $close$, $near$ and far. If necessity arises, we can further subdivide the $close$ range and introduce two labels, namely, $veryclose$ and $close$. In doing so, we are refining our granularity in such a way that finer changes in numerical distance can be represented. In a similar way, in the formalism proposed in this paper, it is possible to move to finer granularity so that smaller changes in angular direction can be represented. The algorithms proposed for finding converse and composition can handle refinement of granularity.

II. Egocentric Reference Frame

A spatial reference frame is a coordinate system with respect to which the qualitative direction labels are defined. Spatial reference frame is an important issue when we want to represent qualitative direction. Tversky advocates that people's spatial mental models use only two basic perspectives - locating elements relative to one another from a point of view or locating an element to a higher order environmental feature or reference frame [6]. The first of these corresponds to an egocentric frame of reference and the second corresponds to an allocentric frame of reference. When two objects are moving in a two dimensional plane, their directions can be specified in different ways. If the plane has a large geographical extent, then it is possible that we may use the north-south-east-west reference system. We may say that one is heading north while the other is heading south-west. If the scope is confined to a piece of paper, one may use the X-Y coordinate system for this. Here, we are using an external reference system and this type of frame of reference is named as allocentric [7] or extrinsic [8]. On the other hand, qualitative direction can be represented in a relative way. For this, the direction of one object is taken as a reference and the direction of others is stated with respect to this reference. This direction typically depends on factors like topology, size, shape etc. This reference direction is not static; it may change with time as the object moves or rotates. Such a frame of reference is termed as egocentric [7] or intrinsic [8]. In deictic frame of reference [8], the concept of direction is defined by an external observer. Here, the direction in which two or more objects are moving is specified from the point of view of an external observer. In this paper, we have handled qualitative direction in an egocentric spatial reference frame.

III. Qualitative Direction Algebra

A. Defining the JEPD Set of Direction Relations

In order to explain the qualitative direction relations, we need to introduce a few definitions. The direction in which an object is headed is specified by a straight line. In a two dimensional plane, the direction of this straight line expresses the direction of the object. This straight line, used for specifying the direction of an object, has been termed as a direction line. For finding binary qualitative direction relations, at first we need the direction lines for each object under consideration.

Definition. *A Direction Line is a directed line segment in a 2-D plane having direction dir and magnitude m.*

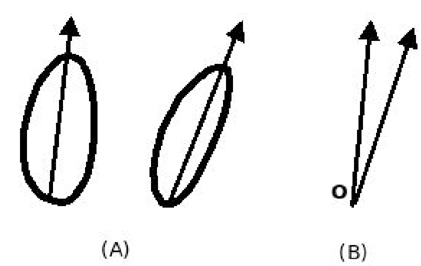

Fig. 2. Objects with their intrinsic directions. Direction lines of each object made to originate at the same point leading to a direction region.

For convenience, we assume that the start point for all these direction lines is same. This can be done because the direction line represents the direction logically and translation of these lines do not change the direction of the object. Moreover, spatial locations of the objects are noway important to us. As an illustration, in Figure 2, we have shown two objects whose directions are indicated by arrowheads (part A). In part (B) of the same figure, two direction lines are drawn parallel to the direction of the two objects. We will use the notation dir_A to mean the direction line of the spatial object A. When we make the direction lines originate at the same point, we have regions between two consecutive direction lines.

Definition. *Direction Region : Let l_1 and l_2 be two direction lines having directions dir_1 and dir_2 respectively and having a point of intersection o. Let θ be the angle between dir_1 and dir_2 in an anticlockwise direction. Then, a direction region defines a set of direction lines that originate at o and the direction of any such line is bounded by the angle θ from dir_1 in an anticlockwise direction.*

The concept of direction regions will be used in algorithms for finding composition and converse of base relations of the qualitative direction algebra. We have the task for arriving at a set of binary qualitative direction relations from the angles measured between their direction lines. QSR discretizes the continuous domain and introduces abstractions. We may tend to think that QSR is same as fuzzy approximations; but there is an important difference. Categories in fuzzy approach are approximations of real values, while categories in QSR depends on application requirement [2].

We start with four qualitative abstractions and name these as $Same$, $Opposite$, LR and RL. These abstractions are derived from the angular displacements between direction lines. Our qualitative relations for direction are binary. So, the direction of one of the objects is taken as the reference direction. The direction of the other object is taken as the primary direction and a qualitative relation expresses the relationship of the primary with respect to the reference.

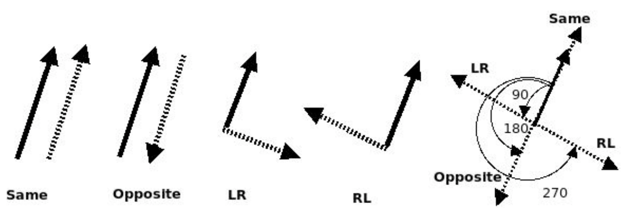

Fig. 3. Direction relations based on four qualitative abstractions.

Let A and B be two spatial objects. When we say $A\ Same\ B$, we mean that the angle between dir_A and dir_B is zero. For uniformity, we will measure all angles counterclockwise. Meaning of $A\ Opposite\ B$ is that the angle between dir_A and dir_B is 180 degrees. When an object A moves along dir_A in a two dimensional plane, its course of motion divides the plane equally into two parts. Assume that the direction line dir_B, corresponding to some object B, intersects dir_A at right angle in a left to right direction. Then, the resulting qualitative relation is named as LR. An identical case in the right to left direction is termed as RL. We have a set of four binary qualitative direction relations here. These relations are illustrated in Figure 3.

QD_8: QDA with granularity eight

The level of granularity is too coarse and change in direction is represented when it crosses a threshold of 90 degrees. These are the major direction relations that we are going to refine further. Naturally, one would like to divide these right angles equally so that granularity is refined. An important aspect is to keep the angular span same for all relations. We identify two direction regions, namely, a $+$ region and a $-$

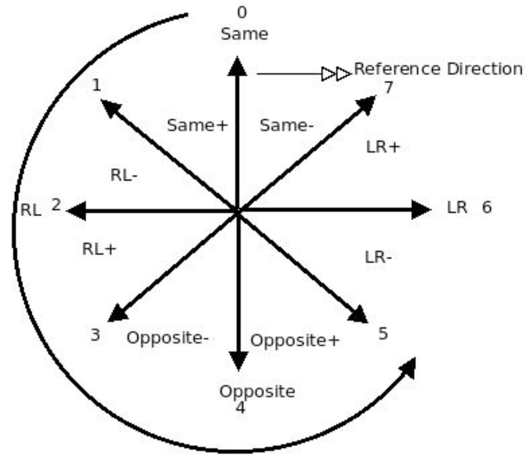

Fig. 4. QD_8: Pictorial representation of eight major direction relations.

region. The $+$ region is direction region with an angular span of 45 degrees measured counterclockwise from one of the major direction relations introduced earlier. The $-$ region is direction region with an angular span of 45 degrees measured clockwise from one of the major direction relations. Since each major direction relation now will give rise to one $+$ region and one $-$ region, we will have eight such direction regions in total. These can be considered as qualitative direction relations, because the direction line of the primary may fall along any of these. For naming these relations, we use a simple notation where we append a $+$ or $-$ symbol to the name of a major direction label. For example, the major relation $Same$ results in two relations, namely, $Same+$ and $Same-$. Let $A\ Same\ B$ and let B be the reference object. In the relation $Same$, we know that the angle between dir_A and dir_B is zero. In $Same+$, the direction line dir_A lies within an angular span of 45 degrees measured counterclockwise from dir_B. Similarly, in $Same-$, the direction line dir_A lies within an angular span of 45 degrees measured clockwise from dir_B. These twelve base relations are enumerated in Table I. The angular spans for all the relations are listed. All these angles are measured in anticlockwise direction from the direction line of the reference object. In Figure 4, these relations are shown pictorially. There are eight equal divisions of the full angular span of 360 degrees. We choose to use this number of divisions to express the granularity of the algebra. So, the base relations enumerated in Table I are for a QDA with granularity eight. We denote this as QD_8.

QD_{16}: Refining the Granularity

The twelve base relations listed in Table I, can record change in direction when the direction of the primary object crosses discrete boundaries at integral multiples of 45 degrees. In many applications, it may be necessary to record change at finer intervals. In our proposed formalism, this can be done very easily. In this section, we will show one level of refinement. The same process can be repeated to arrive at even finer granularity of base relations.

For refinement, we equally divide the $+$ and $-$ direction regions. For example, if we divide the $+$ region for which the angle range is $]0, 45]$, we obtain two direction regions of span 22.5 degrees each. The region $[0, 22.5]$ is denoted by

TABLE I. DIRECTION RELATIONS OF QD_8.

Sl. No.	Base Relation	Angle Range	Converse of Base Relation	Sl. No.	Direction Relation	Angle Range	Converse of Base Relation
1	$Same$	$[0, 0]$	$Same$	7	lr	$[270, 270]$	rl
2	$Same+$	$]0, 45]$	$Same-$	8	$lr+$	$]270, 315]$	$rl-$
3	$Same-$	$]315, 360[$	$Same+$	9	$lr-$	$]225, 270[$	$rl+$
4	$Opposite$	$[180, 180]$	$Opposite$	10	rl	$[90, 90]$	lr
5	$Opposite+$	$]180, 225]$	$Opposite-$	11	$rl+$	$]90, 135]$	$lr-$
6	$Opposite-$	$]135, 180[$	$Opposite+$	12	$rl-$	$]45, 90[$	$lr+$

the symbol $+^1$ and the region $]22.5, 45]$ is denoted by $+^2$. As a result, we get twenty four base relations that are listed in Table II. This time, change in direction is noticed after a threshold of 22.5 degrees. The same thing can be done to the $-$ region and the resulting direction regions will be $-^1$ and $-^2$. These refined relations are shown in Figure 5. Now, there are 16 equal divisions of the full span of 360 degrees and accordingly, we have QD_{16}.

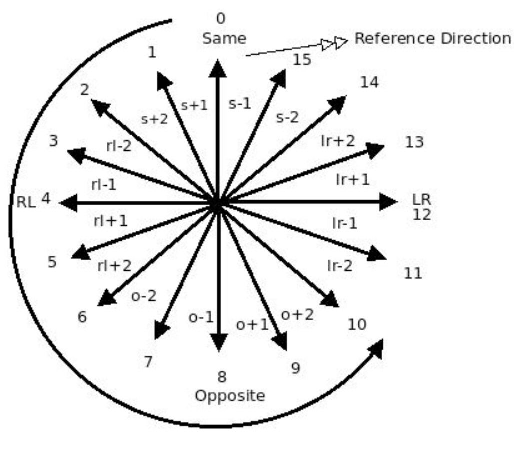

Fig. 5. QD_{16}: Direction relations one level refined.

In order to apply constraint based reasoning to a set of spatial relations, we develop a partition scheme for the objects in the domain under consideration [9] and arrive at a set of Jointly Exhaustive Pairwise Disjoint (JEPD) base relations. General relations are obtained by taking the power set of base relations, with top, bottom, union, intersection and complement of relations defined in the set theoretic way [9]. Moreover, an identity relation and a converse operation on base relations must be provided. For the set of base relations introduced earlier, $Same$ is the identity relation. Each relation is cosed under converse operation. The converses for the base relations were listed in Table I and in Table II.

B. Finding Converse and Composition

For finding the direction relation that holds between directions of two spatial objects, one of these objects is considered as a reference. A direction line parallel to the direction of the reference object is drawn and this direction line can be designated as line 0. The direction relation wheel can now be drawn with respect to this line according to the granularity level under consideration. Then, the direction line corresponding to the direction of the primary object is drawn. The direction region in which this line falls tells us the direction relation of the primary with respect to the reference. All angles are measured counterclockwise and direction relations in terms of angle ranges have been listed before.

We present an algorithm for finding the converse of any qualitative direction relation. Let us assume that $A\ dr\ B$, where A and B are spatial objects and dr is the direction relation holding between their directions. Intuitively, for finding the converse of dr, we should know the number of rotations we should give to the direction line of A to get back to the direction line of B. This is because of the fact that the converse expresses the relation of B with respect to A. So, for finding the converse relation, the direction line of A will be taken as line 0. Moreover, we should know the relation that results after these many rotations. An algorithm is presented below for finding the converse of a qualitative direction relation.

```
Algorithm Converse(R,m)
R is the relation whose converse has to
be returned and m is the granularity
BEGIN
1.n:= Calc_Rot_Conv(R)
2.If (R=='Same'||R=='Opposite'||R=='LR'
            || R=='RL') Then
 Begin
 Conv_Rel := Find_Rel(m-n,m-n)
 End
 Else
 Begin
 Max_rot:= n
 Min_rot:= n-1
 Conv_Rel:=Find_Rel(m-Max_rot,m-Min_rot)
```

Function $Calc\_Rot\_Conv$ returns the number of rotations needed to align the direction line of the primary object with that of the reference. The bottom and top lines for the relation are retrieved into local variables p and q. p denotes the index of the bottom line and q denotes the index of the top line. Since the function returns $m - p$, we understand that the maximum required number of rotations is returned by the function. This returned value gets stored in the local variable n inside the function $Converse$. If the relation is one of $Same$, $Opposite$, LR or RL , then we know that the direction line of the primary will not fall in a direction region. It will align with one of the lines (at one of the angles 90 , 180 , 270 or 360 degrees measured counterclockwise)in the direction wheel. Then, it is easy to see that we will have to give $m - n$ number of rotations to the direction line of the primary object. For example, let us consider QD_8 and let $A\ Opposite\ B$ hold. Then, the direction line of the primary aligns with the line at an angle of 180 degrees in the direction wheel. The value of $< p, q >$ will be $< 4, 4 >$. The value returned by $Calc\_Rot\_Conv$ will be 4. Inside the function $Converse$, a call will be made as $Find\_Rel(8-4, 8-4)$. The relation whose bottom and top lines are $(4, 4)$ is $Opposite$. So, the converse of $Opposite$ is computed as $Opposite$.

TABLE II. REFINED DIRECTION RELATIONS OF QD_{16}.

Sl. No.	Base Relation	Angle Range	Converse of Base Relation	Sl. No.	Base Relation	Angle Range	Converse of Base Relation
1	$Same$	$[0, 0]$	$Same$	13	lr	$[270, 270]$	rl
2	$Same+^1$	$]0, 22.5]$	$Same-^1$	14	$lr+^1$	$]270, 292.5]$	$rl-^1$
3	$Same+^2$	$]22.5, 45]$	$Same-^2$	15	$lr+^2$	$]292.5, 315]$	$rl-^2$
4	$Same-^1$	$]337.5, 360[$	$Same+^1$	16	$lr-^1$	$]247.5, 270[$	$lr+^1$
5	$Same-^2$	$]315, 337.5]$	$Same+^2$	17	$rl-^2$	$]225, 247.5]$	$lr+^2$
6	$Opposite$	$[180, 180]$	$Opposite$	18	rl	$[90, 90]$	$lr+$
7	$Opposite+^1$	$]180, 202.5]$	$Opposite-^1$	19	$rl+^1$	$]90, 112.5]$	$rl-^1$
8	$Opposite+^2$	$]202.5, 225]$	$Opposite-^2$	20	$rl+^2$	$]112.5, 135]$	$rl-^2$
9	$Opposite-^1$	$]157.5, 180[$	$Opposite+^1$	21	$rl-^1$	$]67.5, 90[$	$rl+^1$
10	$Opposite-^2$	$]135, 157.5]$	$Opposite+^2$	22	$rl-^2$	$]45, 67.5]$	$rl+^2$

For a discussion of other type of relations, let us consider $RL+$. The bottom and top lines for this relation will be returned as $< p, q >$ = $< 2, 3 >$. The value returned from $Calc\_Rot\_Conv$ will be $8 - 2$ i.e. 6. Inside the function $Converse$, Max_rot will be 6 and Min_rot will be 5. This time, there will be a call like $Find\_Rel(8 - 6, 8 - 5)$ i.e. $Find\_Rel(2, 3)$. The relation for which dir_{bottom} is 2 and dir_{top} is 3 is $RL+$. So, the converse of $RL+$ is $RL+$. An outline of the $Calc\_Rot\_Conv$ function is given below.

```
Algorithm Calc_Rot_Conv(R,m)
The function Get_Lines gives the bottom
and top direction lines associated with
the relation R
BEGIN
1. < p , q> := Get_Lines(Rel)
2. Return m-p
END
```

We assume that the function $Find\_Rel$ retrieves the appropriate relation from a hash table depending on the pair of integers passed to it . Every direction relation can be represented by a pair of integers (i, j) where i is the integer corresponding to dir_{bottom} and j is the integer corresponding to dir_{top}. For example, when the pair $(0, 0)$ is passed, the retrieved relation is $Same$, when $(0, 1)$ is passed, the retrieved relation is $Same+$ and so on. An outline of the algorithm for the $Find\_Rel$ function is given below. The algorithm takes care of the fact that sometimes (while computing composition of relations) the second argument can be two more than the first and the local variables c and d are used to control this. The algorithm will return a quadruple of relations. Let us assume that this quadruple is of the form $< R, Q, S, T >$. In this quadruple, only non null entries are meaningful. For example, if we call $Find\_Rel(2, 2)$, then only one relation is returned and this is available in R. If the call is like $Find\_Rel(4, 5)$, then also a single relation is returned in R. The parameters Q , S and T become meaningful when max is $min + 2$.

```
Algorithm Find_Rel( min , max)
1. c:=d:=-1 ;  R:= Q := S := T := Null
2. If (min==max) Then
3. Begin
4.  R := From_Hash(min,min)
5.  Return <R,Q,S,T>
6. End
7. Else If (max==min+1) Then
8. Begin
9.  R := From_Hash(min,max)
10. Return <R,Q,S,T>
11.Else If (max==min+2) Then
12.Begin
13. <a,b> := <min,min+1>
14. If <min+1==0 || min+1==2
15.   || min+1==4 || min+1==6> Then
16.   <c,d> := <min+1 , min+1>
17.   <e,f> := <min+1 , max>
18.    Q := Get_From_Hash(a,b)
19.   If (c!=-1) Then
20.    Begin
21.      S := From_Hash(c,d)
22.      T := From_Hash(e , f)
23.    End
24. Return <R,Q,S,T>
```

For constraint based reasoning, composition of base relations is an important issue. An algorithm for composition of base relations is presented. Let A , B and C be three objects such that A $Rel1$ B and B $Rel2$ C hold. We want to find $Rel1 \circ Rel2$, where $\circ$ denotes set theoretic composition. For this, direction relation wheel is drawn with respect to the direction of B. Since the relation of A to B is already known, we can identify the direction line or direction region for A. The remaining task is to fix C in the wheel. $Rel2$ is given. But that expresses the relation of B with respect to C. We take the converse of $Rel2$ and identify the direction line or region for C with respect to B. To find the composition we need to compute the number of rotations required to align direction of C with that of A and find the resulting relation. For any relation Rel, let us denote the lower line of its direction region as Rel_{Bottom} and the corresponding upper line as Rel_{Top}. Then, any relation can be expressed as an ordered pairs of the form $(Rel_{Bottom}, Rel_{Top})$. For example, in the Figure 4, the relation $Opposite+$ can be specified as $(4, 5)$ and $Same$ as $(0, 0)$.

```
Algorithm Compose(Rel1,Rel2,m)
This algorithms computes composition of
rel1 and rel2. m is the granularity
1. S:=Converse( Rel2 , m)
2. <p,q >:= Get_Lines(S)
3. <r,s>:= Get_Lines(Rel1)
4. <min,max>:=Calc_Rot_Comp(<p,q>,<r,s>)
5. Find_Rel( min , max )
```

C. Conceptual Neighbourhood

Conceptual dependency of a spatial relation defines a set of relations that may hold after this relation whenever a change is recorded. For example, if at any point of time $Same$ is the qualitative direction relation that holds between directions of two objects, then it is not possible that this relation will change to $Opposite$ whenever change in direction is noted. After the relation $Same$, the possible relations that may hold can be either $Same+$ or $Same-$. The relations that may hold after the current relation are termed as its conceptual neighbour(s). Conceptual neighbours dictate how one relation can or cannot change. This gives rise to a notion of spatio-temporal continuity which can be exploited in many applications. Conceptual neighbours are generally expressed by a graph where nodes represent relations and edges are drawn from a node to its conceptual neighbours. In Figure 6, conceptual dependency of 12 base relations for QD_8 is shown.

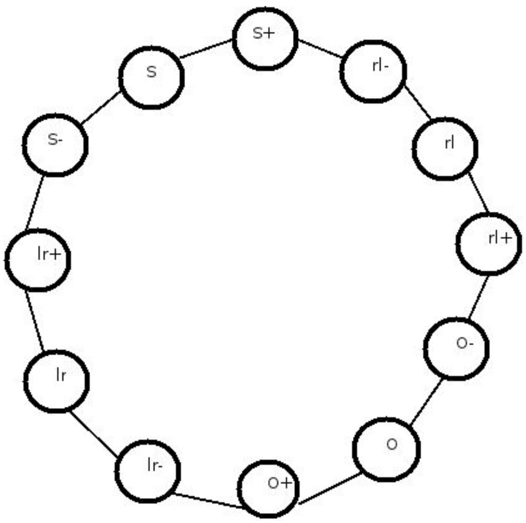

Fig. 6. Conceptual neighbourhood diagram for QD_8.

IV. QDA and Motion Pattern Analysis

Qualitative direction relations can be used to represent and reason about spatio-temporal patterns of directional entities. One such example is motion pattern analysis. For such applications, formalisms are required for representation and recognition of patterns in an input stream.

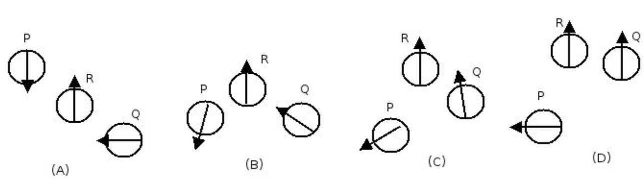

Fig. 7. Motion events of a set of directed points.

Figure 7 present a synthetic example where three objects are involved in a motion pattern recognition application. One of the objects is stationary and has a direction along its egocentric orientation. The other two objects are non stationery and changes in their egocentric headings. R is the stationery object and it is taken as the reference object. We want to represent the motion sequence of P and Q with respect to R. The motion sequence of P with respect to R can be expressed as $Opposite, Opposite-, RL+, RL$ and that of Q can be stated as $RL, RL-, Same+, Same$. Such patterns can be expressed using a regular grammar and a parser can be used to recognize the pattern in an input stream.

Elsewhere a general technique has been developed to combine QSR with formal grammars for recognition of motion events among multiple spatial entities [10]. Use of formal grammars as a recognition technique allows creation of hierarchies of conceptual abstractions; motion events that one expects in an input stream are expressed by writing programs in this language. Such programs are parsed using context free grammar and interpreted using regular grammar. Successful interpretation is equivalent to recognition of the events expressed in the program.

V. Final Comments

In this paper, a qualitative direction algebra is proposed for representation and reasoning about directions of spatial objects in a dimension independent way. Qualitative spatial relations for QD_8 and QD_{16} have been defined.

Existing formalisms combine dimensionality into definition of relations and as a result of this, such representation become unsuitable when dimension scales up. In the proposed formalism, it is easy to move to a finer granularity. This finer granularity is realized using fewer base relations than existing formalisms. A limitation of this approach might be the fact that it ascertains the directions in a deterministic way. At any point of time, it is assumed that we know the angles between the directions with certainty. Future work includes study of formal properties of these calculi.

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