Singular Propositions and their Negations in Diagrams

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Abstract— **This paper deals with the visual representation of negation involving particular propositions. The underlying consideration depends on the three basic desirable aspects of visual representation viz. simplicity, visual clarity and expressiveness [9]. For incorporation of constants in diagrams we discuss Venn-i (2004), Swoboda's diagrams (2005) and Spider diagrams with constants (2005). We also discuss representation of negation in these diagrams. To depict negation in Venn-i the concept of absence is brought in from the conceptual schema of Indian philosophy. The advantage of Venn-i over spider diagram is discussed. The notion of absence naturally calls for the concept of an open universe. A brief discussion on open universe is presented at the end.**

Keywords- **singular propositions, negation, absence, simplicity, open universe**

I. Introduction

Negation plays a crucial role in all logics [12]. The notion of inconsistency and contradiction are primarily understood in terms of negation. Although there is the notion of absolute inconsistency that does not involve negation and that turns out to be equivalent to inconsistency in classical logic, for a pictorial representation absolute inconsistency is not useful. In diagram logic inconsistency is attributed to a diagram whereas contradiction is a relation between diagrams. Of course two diagrams are contradictory if and only if the conjoined diagram is inconsistent. A set of propositions is absolutely inconsistent if any proposition what so ever follows from it. This notion originally proposed by Peirce, is presently called 'explosiveness' and has gained much importance after the advent of paraconsistent logics [6]. In classical logic one deals with the following three types of basic propositions:

(a) a is P: (P a) where a represents an individual,
(b) all S is P: ($\forall$x (Sx $\rightarrow$ Px)),
(c) some S is P: ($\exists$x (Sx ΛPx)).

The latter two types and their negations constitute Aristotle's categorical propositions which were used specifically for syllogistic reasoning. Types (b) and (c) are known as universal affirmative and particular affirmative propositions respectively. 'Not all S is P' (equivalently 'some S is non-P') and 'Not some S is P'(equivalently 'No S is P') are particular negative and universal negative respectively. The negation of a universal proposition is a particular proposition and vice versa and the negation of an affirmative proposition is a negative proposition and vice versa. In this sense we can say that the class of categorical propositions is closed with respect to negation.

Traditionally singular propositions like 'a is P' is considered to have the syllogistic form (b) as "All a's are P". But sentences of the form (b) admit both contradictories and contraries, while sentences of the form (a) have only contradictories. The two sentences (a) and (b) behave differently under negation. So their logical forms are to be considered different. Less attention has been paid to this fact. For an ongoing debate on the issue we refer to the site: http://www.tandfonline.com/doi/abs/10.1080/0004840538520 0151. Negation of sentences like Pa makes the distinction between (a) and (b) more transparent. We are however interested in the representation of sentences of the form (a) and their negations rather than the debate. From this standpoint it would be clear that (a) should be considered differently from (b) if the desirable parameters of diagrams viz. simplicity, visual clarity and expressiveness are to be taken into account. It should be mentioned that the above three qualities of diagrammatic representation is to be understood only informally.

It is interesting to note that although representation of logical propositions through diagrams and reasoning thereby is an issue that has engaged pioneering logicians like Euler L (1707-1783), Venn J (1834-1923) and Peirce C S (1839-1914) for more than three centuries it is only recently that researchers are paying serious attention to representation of propositions of type (a) as well as their negations. Diagram systems, as alternative system of reasoning faces the challenge of incorporating all the basic items that are involved in logical reasoning.

In section 2 diagrams involving individuals are presented. In section 3 negations of diagrams involving individuals are discussed in detail. Section 4 deals with open universe. Section 5 consists of some concluding remarks.

II. Diagrams with Individuals

It is already mentioned that sentences of the form P*a* or singular propositions were treated as universal propositions. In this approach instead of individuals, the subject term was viewed as a singleton set. We have also stated before that this treatment is somewhat problematic. Representation of individuals was however not dealt with in the diagrams of Euler[11], Venn – Peirce[7,2], Shin[15] and Hammer[3]. Propositions like ‘Socrates is not mortal’ cannot be expressed in any of these diagrams.

Recent incorporation of individuals in diagrams may be discussed now from the perspective of Venn-i [8](2004), Swoboda’s system [13](2005) and Spider diagram[4] (2005).

Spider diagrams are extensions of Venn II diagrams [15]. In spider diagrams centrally connected clusters of nodes called spiders are used for representing individuals.

Spiders are of two types: existential spiders expressed by round nodes and constant spiders expressed by square nodes. Existential spiders are similar to the sign x introduced by Peirce the difference being in the nature of connectivity. Constant spiders are labelled and are similar to constants in first order logic. The habitat of a spider is represented by placing nodes at different regions and joining them in pairs by lines all originating from one of the nodes. Distinct spiders represent distinct elements. Following is an example:

Constant spider: ‘Web is either a dog and not a cat or Web is a cat and not a dog’ (Fig. 1)

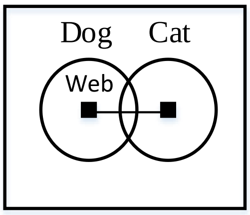


Fig. 1

Thus in spider the singular proposition P*a* will be represented by:

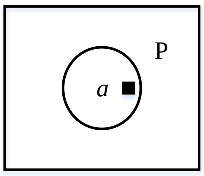


Fig.2

Swoboda’s (2005) system is again an extension of Venn II system by incorporating constants.

Following is an example representing ‘Jill is either in home and not in school or Jill is either in school and not in home’.

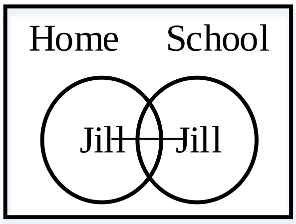


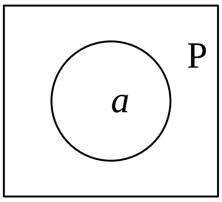
Fig.3

As a matter of fact, Fig. 1 and Fig. 3 are similar.

II.I Representation of individual in Venn-i

In Venn-i we adopted Venn-Peirce convention of expressing universal and particular propositions. A rectangle is used to represent the universe. Besides, Venn-i introduces two additional diagrammatic objects: one is constant symbol representing individual and the other absence of individual. An individual’s presence in a region is represented by placing the individual’s name in that region (Fig.4). Absence of an individual a is represented by placing $\bar{a}$ in that region (Fig. 16). Additionally, broken lines are introduced to express possibility (intended exclusive disjunction) of an individual’s presence in different regions.

Before entering into the discussion on representation of negation we shall make some general observations. An individual is assumed to occur along with some properties or predications (positively or negatively). In diagrammatic representations of Venn-i, Spider or Swoboda this means that each individual ‘*a*’ shall occur within the region of a closed curve or its complementary region in the universe. On the other hand a closed curve can be drawn without any individual depicted in the rectangle. The simplest picture with individual hence, would contain one closed curve and one individual. Thus there are two possibilities: Fig.4 or Fig.5



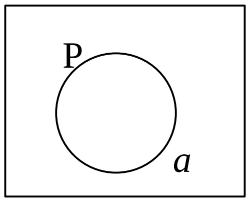


Fig.4 Fig.5

These pictures may be extended with one individual and two predicates giving rise to pictures in the series:

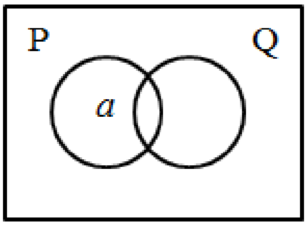


Fig.6

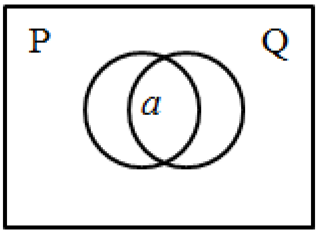


Fig.7

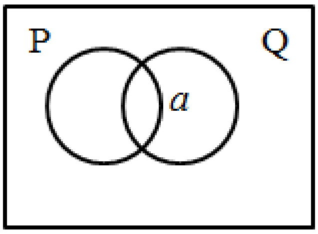


Fig.8

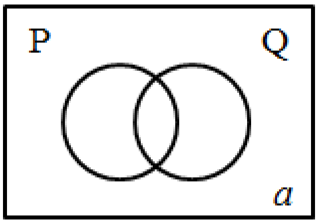


Fig.9

A further extension with more than one individuals and more predicates is possible in a natural way. In spider diagrams the pictures would remain the same except in that they would use square dots for each individual along with their names. The difference will be noticed in depicting their negations which will be treated in section III. But we need to say a few words about the representation of the absence of an individual. The connection between absence and negation will be discussed in the following section.

So long in the diagram literature there was no symbol to represent the absence of an individual. Placing individual in the complement would indicate its absence from the class as free ride. In the context of closed universe i.e when the discourse is limited within a fixed universe represented by the rectangle in diagrams, mark of absence goes along with simplicity of representation. Say for example, when the teacher marks the attendance of the students and a student is not found in the class, puts absence mark against the student meaning there by that the student is not present in the class — the teacher is thus depicting the absence of the particular student.

a with an upper bar ($\bar{a}$) placed within M represents literally that absence of *a* belongs to M that is, not that *a* belongs to M. Thus absence of an individual is used for negating some predication about the individual. In the system Venn-i this semiotic device should be considered as an additional means to represent negation.

It is to be noted that when the paper was published in 2004, the diagram system Venn-i considered only classical negation. The negative statement ~($a \in$M) has been taken as equivalent with $\bar{a} \in$ M. There have been two diagrammatic representations of negative statements of the above form viz. by using $\bar{a}$ in M and by using broken lines showing the possible alternative locations of *a* in the complement M^C in the universe. The basic advantage had been simplicity and directness of visual representation. The completeness proof had been carried out on the basis of this equivalence. However, during subsequent developments we have noticed a deeper significance of the use of the symbol '$\bar{a}$' and have preferred to shift from this equivalence. The subtle difference between the two will be discussed in the following section.

For a formal presentation of the alphabet and formation rules we refer to [10].

A cue to the depiction of absence by a symbol directly may be traced in the knowledge system of ancient India. The Indian logicians (Nyaya Vaisesika thinkers) admit a distinct ontological category called abhāva (absence) with a view to accounting for negative statements [14]. It is important to note that absence was also considered to be real. The fact that an absence is always an absence of some entity shows that absence as a category presupposes the existence of positive entities. Absence has to be admitted as the object of negative form of cognition.

Russell in his "Philosophy of Logical Atomism" maintains similar view when he considers two kinds of atomic facts: positive atomic fact and negative atomic fact. To quote from o Russell ".. I think you will find it better to take negative facts as ultimate. Otherwise you will find it so difficult to say what it is that corresponds to a proposition. When, e.g., you have a false positive proposition, say 'Socrates is alive', it is false because of the non-correspondence between Socrates being alive and the state of affair. A thing cannot be false except because of a fact, so that you find it extremely difficult to say what exactly happens when you make a positive assertion that it is false, unless you are going to admit negative facts".([1] p.214). That the absence of Socrates belongs to the collection of living humans may be considered as a negative fact. In Venn-i this fact is directly depicted.

III. Negation of Diagrams with Individuals

We now consider negation of diagrams involving individuals. In Spider diagrams P*a* is represented as Fig.10

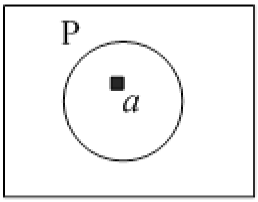


Fig.10

~P(a) is represented as Fig.11

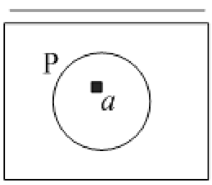


Fig.11

which is equivalent to Fig.12

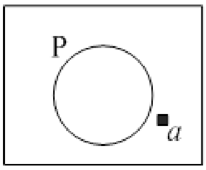


Fig.12

Extension of this with two predicates would be P(a)&Q(a) (vide Fig.13) and

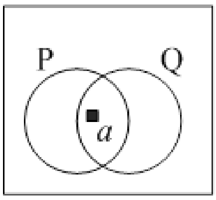


Fig.13

~(P(a)&Q(a)) as Fig.14

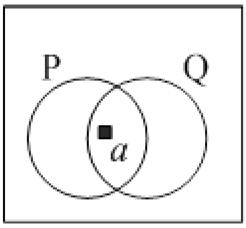


Fig.14

which is equivalent to Fig. 15

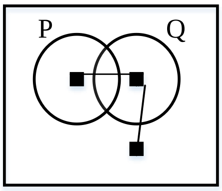


Fig. 15

In Venn-i P(a) is represented as Fig.4 and ~P(a) as Fig. 16

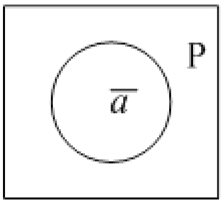


Fig.16

which is equivalent to Fig.5

With two predicates P(a)&Q(a) it would be Fig.17

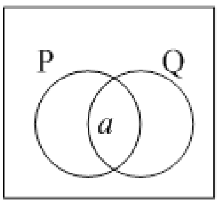


Fig.17

And ~ (P(a)&Q(a)) would be the following Fig.18

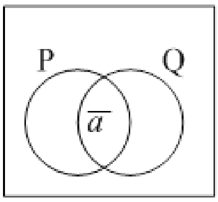


Fig. 18

which is equivalent to Fig.19

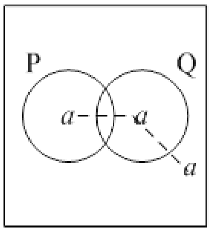


Fig.19

One may wonder what might be the advantage of representing negation of singular proposition by using $\bar{a}$-like diagrammatic entities. Let us consider the following cases:

Case1. Negation of the content $a \in S \cap P \cap M$ in Spider would be

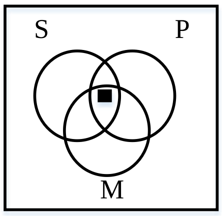


Fig. 20

which is equivalent to: Fig. 21

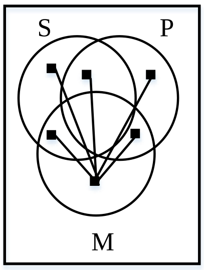


Fig. 21

The first diagram does not convey information to our cognition immediately. While the second one is quite complex, the complexity will increase with the increase in the number of predicates and individuals. Whereas in Venn-i its representation of the same information is Fig.22 which says directly that 'a' is absent in $S \cap P \cap M$.

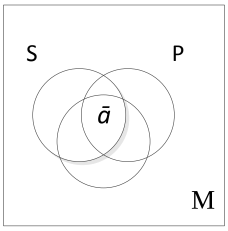


Fig. 22

In Venn-i there is an equivalent representation with dotted lines.

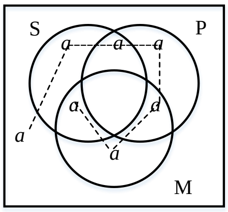


Fig.23

This diagram is visually more elegant than the spider (Fig. 21) since one has to intersect lesser number of bordering curves. One may argue that the visual complexity of the figure in our diagram will increase if the location of absence in our diagram will increase i.e if the absence of '*a*' is to be depicted in many zones. But since we have at our disposal the sign to represent presence too, it is possible to have a trade-off and decide which diagram to take.

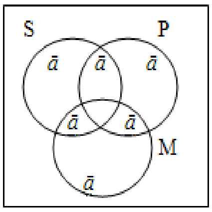


Fig. 24

Thus Fig.24 may be replaced by Fig 25 since *a* is absent in more zones than its possible presence.

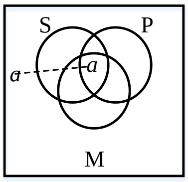


Fig. 25

Case 2.

With increase in the number of individuals the picture loses its visual clarity in spider diagrams. For example, one can compare the following diagrams Fig.26 of spider and Fig. 27 of Venn-i.

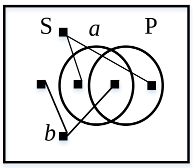


Fig.26

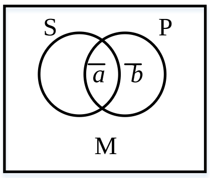


Fig.27

Case 3.

In order to express both *a* and $\bar{a}$ in the same region the representation in Venn-i (Fig.28) and Spider(Fig.29) can be compared

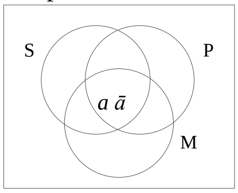


Fig.28

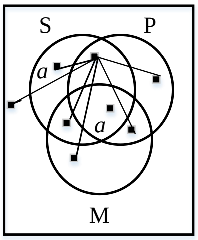


Fig.29

Apart from visual simplicity of Fig.28 over Fig. 29, the direct cognition of contradiction in the first diagram may also be taken into account.

IV. MODIFICATION OF THE INTERPRETATION OF ABSENCE IN VENN-I

So long we have interpreted $\bar{a} \in$ P equivalently with $a \in$ P^C. But we have mentioned in the introduction that such a representation has a deeper significance.

In the modified version we assume that absence of *a* in the set M does not necessarily imply that *a* belongs to the complement of M with respect to the universe although *a* is in M implies that absence of *a* viz. $\bar{a}$ is in the complement of M. Thus from Fig.4, in Venn-i modified follows Fig.30.

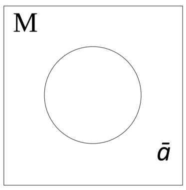


Fig.30

but not conversely.

Also from Fig. 5 follows Fig.16 not conversely.
The main idea behind this unidirectionality lies in the following consideration. From the fact of $a \notin$M it is outright inferred in classical set theory and logic that a belongs to the complement of M irrespective of the fact whether a is locatable in the complement M or not. Actual locatability is our concern. So if we do not see a in M, we do not infer that a is in the complement of M until or unless a becomes locatable in the complement of M. For a more detailed discussion we refer to [8, 10].

At this point the notion of absence gains significance and now there are two kinds of negation viz. classical negation and absence. Let us state our conventions regarding individuals and their absence. First, an individual a cannot occur in more than one regions in a diagram but $\bar{a}$ can occur. However, in case of closed universe $\bar{a}$ cannot occur in all regions because in that case a would be a non-entity. Secondly, there is no notion like absence of absence, there is no diagrammatic object like a with double bar in the system Venn-i. This feature particularly makes absence different from intuitionistic negation although there is a flavour of constructivity in the above mentioned aspect of locatability.

With this standpoint one can see that there emerge two types of negations in Venn-I modified:

The modified interpretation of $\bar{a}$ is compatible with the notion of open universe to be discussed in Section V.

V. OPEN UNIVERSE

The representation of absence gives way to the notion of open universe [10], where the description of the universe admits to be incomplete. A reading of the information given in Fig. 16 may be thus: we do not see a in P but know not where. In the context of *open universe* the notion of absence becomes more significant. Absence of a in P does not necessarily imply a is in the complement of P since the complement is not known because the universe is open.

In depicting open universe, diagrammatically [10], there is no rectangle outside the closed curve indicating that the domain of discourse is not fixed. Objects here continuously appear and disappear; at one instant it is existent and may be nonexistent at another instant. Here, $a \notin$A does not necessarily imply $a \in$B for some B since, although 'a' had been an object of the universe, or because of the universe being in flux, 'a' may have disappeared altogether. In the classical case B is A^C. This means that it becomes meaningful to assert the law of excluded middle $a \in$A or $a \notin$A although the latter does not entail that $a \in A^C$. In fact, A^C is not at all determined since there is no fixed universe or even if fixed initially, it is subject to change. Absence of an individual here in this room does not entail her presence outside, she might nt be locatable or she might have disappeared altogether. It should be realized that once open universe is accepted, classical negation fails to operate since there exists no notion of absolute complementation. In such a situation, negation of $\bar{a} \in$P turns into the presence of the absence of a in P. Absence in Venn-i (closed universe) draws complement as free ride. But Venn-i^o (the system for open universe [10]) does not allow the free ride. What is given is exactly an individual not appearing in a set. The open universe and absence would change the ontology of the Euler Venn diagram.

Incorporation of absence leads towards the admission of the third possibility viz. the 'know not' situation. Introduction of *open universe* along with the absence of a particular will render the diagrammatic system more natural language friendly in the sense that we will be able to talk about fictitious objects, like ghosts or fairies, unidentified objects of science fiction like UFO or life in other planets etc.

VI. CONCLUSION

In conclusion we present a summary. We have attempted to represent negation (of sentences of type Pa) by the absence of a ($\bar{a}$) in P. This representation gives more visual clarity. This representation pertains to the philosophical position of considering absence as a positive category similar to *abhāva* of Indian philosophy and Russell's negative fact. Diagrams with absence of individuals represent negation in a way term negation is used in logic with the exception that here negation (absence) is placed with the subject term which is the name of an individual. In the context of open universe, representation of negation by absence seems to be an essentiality. A more formal treatment of the notion of absence is presented in [10] however a formal way of measuring the clutter of a diagram and comparing various systems in terms of these definitions are still open issues.

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