Observer design and feedback controller synthesis with observer in idempotent semiring

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In this paper, we present an observer design and a feedback controller with observer for a discrete event system involving synchronization phenomena. These systems can be described by linear models in the idempotent semiring. The approach follows the same principle as the Luenberger observer used in continuous systems. Theoretical results are applied to an industrial process and simulation results are reported to show the effectiveness of these methods in the estimation for min max plus linear systems using Scilab.

idempotent semiring, observer, dioids, $(max, +)$ linear systems, feedback controller

1. INTRODUCTION

A discrete event system (DES) is a dynamic system whose behavior can be described by means of a set of time-consuming activities, performed according to a prescribed ordering. Events correspond to starting or ending some activity (Cassandras (1999); Cohen (1984)). These systems can represent a great number of processes characterized as being concurrent, asynchronous, distributed or parallel, such as flexible manufacturing systems, multiprocessor systems or transportation networks. If the concerned systems are characterized by delay and synchronization phenomena, the Timed Event Graphs (TEG) constitute interesting models. Timed Event Graphs are a subclass of timed Petri Net in which all places have a single transition upstream (A single upstream transition means that there is no competition in either consumption or supply of token in TEG) and a single one downstream (means that all potential conflicts in using tokens in places have been already arbitrated). This class of system plays an important role because of its deterministic temporal behavior.

In opposition to continuous systems, Timed Event Graphs are not modeled through differential or difference equations. An appropriate model is developed to describe the behavior of these systems and provide a framework for analytical techniques to meet the goals of design, control and performance evaluation. For about 30 years, a particular algebraic structure, called Dioids has motivated the elaboration of a new linear system theory (Baccelli (1992); Cuninghame-Green (1979); Cohen (1984)). This theory offers a striking analogy with conventional linear system theory such as state representation, transfer matrices, corrector synthesis and identification theory (Cohen (1999); Lhommeau (2003); Cottenceau (1999)).

In control theory, a state observer is a system that provides an estimate of the internal state of a given real system from measurements of the input and output of the real system. the observer was first proposed and developed in (Luenberger (1964, 1966)). Since these early papers, which concentrated on observers for purely deterministic continuous linear systems, observer theory has been extended by several researchers to include discrete event dynamic systems, in particular Timed Event Graph.

The observer design problem of Timed Event Graph has received much attention over the last few years. A first problem considered is to estimate state in presence of disturbances for max-min plus linear system initially developed by (Laurent (2010)). Here,

the main approach is based on the dioid of series $\mathcal{M}_{in}^{ax}[\gamma, \delta]$. A second objective is to use an observer to feedback controller in order to obtain a desired behavior.
In this paper, the approaches are applied to an industrial process. Simulation results using Scilab are reported.
The article is organized as follows. In section 2 we recall basic notions and results about idempotent semiring and residuation theory (Cohen (1998a)). A brief description of the industrial plant is given, and we then introduce the modeling of Timed Event Graph in the dioid of formal series $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ in section 3. In section 4, the observer is designed by analogy with the classical Luenberger observer for linear systems and controller synthesis with observer is obtained by considering residuation theory which allows the inversion of mapping in section 5. Finally, an example of production process is given in section 6.

2. PRELIMINARIES

In this section we give the notations and some algebraic tools concerning the dioid and residuation theories.

2.1. Definitions

Definition 1 *(Monoid). A Monoid is a set D, endowed with an internal law noted $\oplus$, which is associative and has a neutral element, denoted ϵ, $\forall a \in D, a \oplus \epsilon = \epsilon \oplus a = a$.*

Definition 2 *(Dioid or idempotent semiring). A dioid $(D, \oplus, \otimes)$ is an algebraic structure, endowed with two internal operations, denoted by $\oplus$ and $\otimes$. The operation $\oplus$ is associative, commutative and idempotent, that is $a \oplus a = a$. The operation $\otimes$ is associative (but not necessarily commutative), and distributive at left with respect to $\oplus$: $\forall\ a, b, c \in D, (a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$, and at right: $\forall\ a, b, c \in D, a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$. The neutral elements of $\oplus$ and $\otimes$ are represented by ϵ and e respectively, and ϵ is absorbing for $\otimes$: $\forall\ a \in D, a \otimes \epsilon = \epsilon \otimes a = \epsilon$*

One says that the dioid is commutative provided that the law $\otimes$ is commutative.

Definition 3 *(Complete dioid). A dioid D is said to be complete if it is closed for infinte for infinite sums and if the product disitributes over infinte sums. A dioid is said complet*

2.1.1. Example 1.
($(\max, +)$algebra).$\overline{R}_{\max} = (\mathbf{R} \cup \{-\infty\} \cup \{+\infty\}, \max, +)$ is a commutative dioid with zero element ϵ equal to $-\infty$, and the unit element e equal to 0. We adopt the usual notation, so that the symbol $\oplus$ stands for the max operation, and $\otimes$ stands for the addition. Notice that $\epsilon \otimes (+\infty) = (-\infty) + (+\infty) = \epsilon = (-\infty)$ in $\overline{R}_{\max}$.

2.1.2. Example 2.
($(\min, +)$algebra).$\overline{R}_{\min} = (\mathbf{R} \cup \{-\infty\} \cup \{+\infty\}, \min, +)$ is also a commutative dioid, for which ϵ equals to $+\infty$, and e equals to 0. We shall denote $\oplus$ the min operation in the sequel, and the symbol $\otimes$ will stand for the addition. Notice that $\epsilon \otimes (-\infty) = (+\infty) + (-\infty) = \epsilon = (+\infty)$ in $\overline{R}_{\min}$.

2.1.3. Remark.
Most of the time the symbol $\otimes$ will be omitted as in conventional algebra, moreover $a^i = a \otimes a^{i-1}$ and $a^0 = e$.

Definition 4 *(Order relation). A set D is said to be ordered if there exisists a binary relation $\preceq$ such that the following conditions are satisfied for all a, b and c in D:*

- *Reflexive: every element is in relation with itself ($a \preceq a$);*
- *Antysymmetric: if $a \preceq b$ and $b \preceq a \Rightarrow a = b$.*
- *Transitive: if $a \preceq b$ and $b \preceq c \Rightarrow a \preceq c$.*

In a dioid, the relation $\preceq$ associated with $\max$ application is an oreder relation which correspond to the usual order $\leq$, $a \preceq b \Leftrightarrow b = a \oplus b \Leftrightarrow a \leq b$.
The relation $\preceq$ associated with $\min$ application is an oreder relation which correspond to the reverse of the usual order $\geq$, $a \preceq b \Leftrightarrow b = a \oplus b \Leftrightarrow a \geq b$.

Definition 5 *(Majorant and minorant). Let $(D, \preceq_D)$ be an ordered set, $C \subset D$ a non-empty subset of D, and a, $b \in C$.*

- *An element $x \in D$ satisfying $\forall b \in C$, $b \preceq x$ is called majorant of set C.*
- *An element $y \in D$ satisfying $\forall b \in C$, $y \preceq b$ is called minorant of set C.*

In particular, if the upper bound (i.e. the least majorant) or/and lower bound (i.e. the greatest minorant) of set a, b exist, we denote them by $a \vee b$ and $a \wedge b$, respectively.

2.2. Matrix dioid

Let $(D, \oplus, \otimes)$ be a given dioid, and denote $D^{n \times n}$ the set of square $n \times n$ matrices with entries over D. The sum and the product over D extend as usually over $D^{n \times n}$ as follows:

$$(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$$

$$(A \otimes B)_{ij} = \bigoplus_{k=1}^{n} (A_{ik} \otimes B_{kj}).$$

One can see that $(D^{n\times n}, \oplus, \otimes)$ is a dioid. The neutral matrix for the law $\oplus$ is the matrix with entries equal to ϵ, the identity matrix for the law $\otimes$ is the matrix with entries equal to e on the diagonal and ϵ elsewhere. Notice that the products of matrices in $\overline{R}_{\max}$ and in $\overline{R}_{\min}$ are not equal, and do not equal the usual sum of matrices.

***Theorem 1** (Kleene star operator). Over a complete dioid D, the implicit equation $x = ax \oplus b$ admits $x = a^*b$ as least solution. where $a^* = \bigoplus_{i\in N} a^i$*
In the following this operator will sometimes be represented by the mapping K : $D \to D$,$x \mapsto x^*$. Furthermore, letting $a, b \in D$, Kleene star operator satisfies the following well known properties : :

$$(a^*)^* = a^*, a^*a = aa^*, a(ba)^* = (ab)^*a \qquad (1)$$

$$(a \oplus b)^* = (a^* \oplus b)^* = (a \oplus b^*)^* = (a^* \oplus b^*)^* \qquad (2)$$

Thereafter, the operator $a^+ = \bigoplus_{i\in N^+} a^i = aa^* = a^*a$ is also considered, it satisfies the following properties:

$$a^+ \preceq a^*, (a^+)^* = a^*, (ab^*)^+ = a(a \oplus b)^* \qquad (3)$$

Inversion of mappings is an important issue in many control applications. Unfortunately, in general manner, mappings defined over idempotent semiring do not admit inverse. However the residuation theory allows to characterize the solution set of an inequality such as $f(x) \preceq b$. The reader may consult Cohen (1998a) to obtain a complete presentation of this theory.

***Definition 6** (Isotone mapping). f is an isotone mapping if it preserves order, that is, $a \preceq b \Rightarrow f(a) \preceq f(b)$.*

***Definition 7** (Residuated mapping). An isotone mapping $f : D \to C$, where D and C are ordered sets, is a residuated mapping if for all $b \in C$ there exists a greatest element x that satisfies the inequality $f(x) \preceq b$. This greatest element is denoted by $f^\sharp(b)$ and mapping $f^\sharp$ is called the residual of f. Dually, if there exists a least element x for the inequality $f(x) \succeq b$, it is denoted by $f^\flat(b)$. Mapping $f^\flat$ is called the dual residual of f.*

***Corollary 1** The mappings $L_a : x \mapsto a \otimes x$ and $R_a : x \mapsto x \otimes a$ defined over a complete idempotent semiring D are both residuated Cohen (1998a). Their residuals are isotone mappings denoted respectively by $L_a^\sharp(x) = a \circ x$ and $R_a^\sharp(x) = x \not{o} a$, were $\circ$ and $\not{o}$ are the left and right residuation respectively.*

***Theorem 2** The mappings $x \mapsto a \circ x$ and $x \mapsto x \not{o} a$ satisfy the following properties:*

$$a \circ a = (a \circ a)^*, \qquad a \not{o} a = (a \not{o} a)^*, \qquad (4)$$

$$a(a \circ (ax)) = ax, \qquad ((xa) \not{o} a)a = xa, \qquad (5)$$

$$b \circ a \circ x = (ab) \circ x, \qquad x \not{o} a \not{o} b = x \not{o} (ba), \qquad (6)$$

$$a^* \circ (a^*x) = a^*x, \qquad (a^*x) \not{o} a^* = a^*x, \qquad (7)$$

$$(a \circ x) \wedge (a \circ y) = a \circ (x \wedge y), \qquad (8)$$

$$(x \not{o} a) \wedge (y \not{o} a) = (x \wedge y) \not{o} a. \qquad (9)$$

The sum, the product and the residuation of matrices are defined after the sum, product and the residuation of scalars in D.

3. TIMED EVENT GRAPH

A Timed Event Graph (TEG) is a subclass of timed Petri Net where each place has a single input transition and a single output transition. For more details about Petri net see David and Alla (1997).

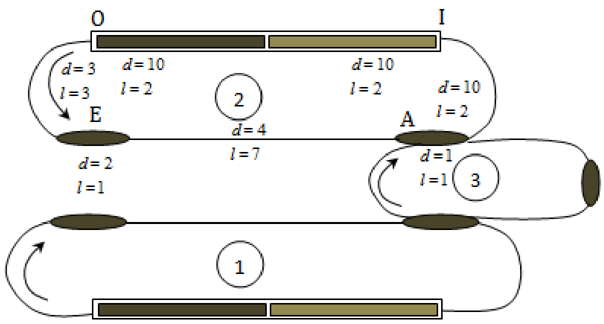

***Figure 1:** Production process*

3.1. Plant description

The process we study here (see Martinez (2003); Amari (2004)) is composed of three conveyor belts connected by loops. the parts are made on an extruding machine in loop 3. Loop 1 and loop 2 are both similar one to each other. they are dedicated to a thermal processing of the parts. Loop 3 processes parts that are conveyed on pallets to one of the other loops. we study loop 2 Figure 2 (identical process for loop1). Parts arrive from loop 3 at point A and an operator fixes them to point I. Here they enter inside the furnace. This element is a channel divided into two sections. Inside the former section parts are

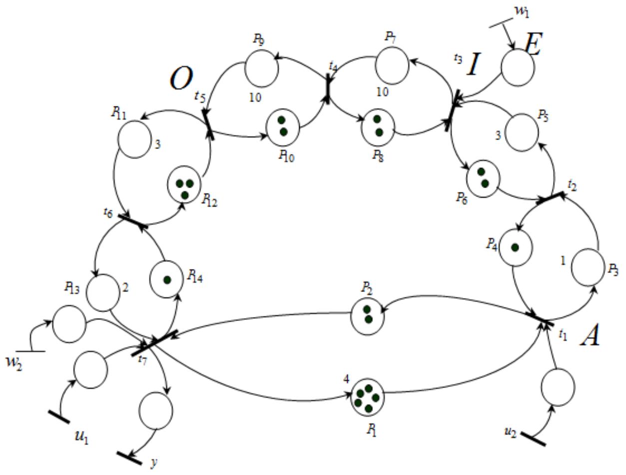

Figure 2: *Petri Net of the loop2*

heated and they are next cooled down inside the latter. Once, pallets come outside the furnace (point O), they are transferred to a second operator who removes parts from the pallets. Thus, parts are taken away at point E according to the external resources. Finally, the free pallets are released and transfer to point A.

The main problem is to achieve the thermal treatment on loop 1 or loop 2 without major failures. In figure 1, d and l are assumed to be the durations of operations and the conveyor capacities respectively. This physical process (loop2) is modelled thanks to a TEG. Transition u_1 models parts arrivals from loop3, u_2 models the necessity of a resource to carry the terminated part and Transition y represents the departure of an achieved part. Figure 2 shows a model of the plant.

3.2. Timed Event Graph description in dioids

Timed Event Graph can be expressed by linear relations over some dioids Cohen (1999). By associating with each transition x a dater *function, in which $x(k)$ is equal to the date when which the firing numbered k occurs, it is possible to obtain a linear state representation in $\overline{\mathbf{R}}_{max}$. there is another representation of TEG in $\overline{\mathbf{R}}_{min}$, a function of time t, corresponding to the cumulated number of firings of the transition at time t. such a function is called a* counter. *A two-dimensional representation of input-output maps called $\mathcal{M}_{in}^{ax}[\gamma,\delta]$ is considered here Cohen (1998b), where γ is an indeterminate which may also be considered as the backward shift operator in the event domain, and δ is the backward shift operator in the time domain. this property means that each entry can be written as an expression of the form $s = p \oplus qr^*$ in which p and q are polynomials in (γ,δ) which represent the transient behavior and the repeated pattern respectively. whereas r is a monomial $\gamma^{\nu}\delta^{\tau}$ which reproduces the pattern q along the slope $\frac{\nu}{\tau}$.*

Considering the Timed Event Graph in Figure2. The dynamic behavior of this system can be expressed as follow: $x_1(k) = \max(x_7(k-5)+4, x_2(k-1), u_1(k))$, $x_2(k) = \max(x_1(k)+1, x_3(k-2))$, $x_3(k) = \max(x_2(k)+3, x_4(k-2), w_1(k))$, $x_4(k) = \max(x_3(k)+10, x_5(k-2))$, $x_5(k) = \max(x_4(k)+10, x_6(k-3))$, $x_6(k) = \max(x_5(k)+3, x_7(k-1))$, $x_7(k) = \max(x_6(k)+2, x_1(k-2), u_2(k), w_2(k))$, $y(k) = x_7(k)$.

In terms of Max Plus notation, we obtain the following linear equations: $x_1(k) = 4 \otimes x_7(k-5) \oplus x_2(k-1) \oplus u_1(k)$, $x_2(k) = 1 \otimes x_1(k) \oplus x_3(k-2)$, $x_3(k) = 3 \otimes x_2(k) \oplus x_4(k-2) \oplus w_1(k)$, $x_4(k) = 10 \otimes x_3(k) \oplus x_5(k-2)$, $x_5(k) = 10 \otimes x_4(k) \oplus x_6(k-3))$, $x_6(k) = 3 \otimes x_5(k) \oplus x_7(k-1)$, $x_7(k) = 2 \otimes x_6(k) \oplus x_1(k-2) \oplus u_2(k) \oplus w_2(k))$, $y(k) = x_7(k)$.

consequently, The transitions are related as follows over $\mathcal{M}_{in}^{ax}[\gamma,\delta]$:

$x_1 = \gamma^5\delta^4 x_7 \oplus \gamma x_2 \oplus u_1$, $x_2 = \delta x_1 \oplus \gamma^2 x_3$, $x_3 = \delta^3 x_2 \oplus \gamma^2 x_4 \oplus w_1$, $x_4 = \delta^{10} x_3 \oplus \gamma^2 x_5$, $x_5 = \delta^{10} x_4 \oplus \gamma^3 x_6$, $x_6 = \delta^3 x_5 \oplus \gamma x_7$, $x_7 = \delta^2 x_6 \oplus \gamma^2 x_1 \oplus u_2 \oplus w_2$, $y = x_7$.

We obtain the following state space representation over $\mathcal{M}_{in}^{ax}[\gamma,\delta]$:

$$x = Ax \oplus Bu \oplus Rw = A^*Bu \oplus A^*Rw \tag{10}$$

$$y = Cx = CA^*Bu \oplus CA^*Rw \tag{11}$$

With: $A = \begin{pmatrix} \epsilon & \gamma & \epsilon & \epsilon & \epsilon & \epsilon & \gamma^5\delta^4 \\ \delta & \epsilon & \gamma^2 & \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \delta^3 & \epsilon & \gamma^2 & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \delta^{10} & \epsilon & \gamma^2 & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \delta^{10} & \epsilon & \gamma^3 & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon & \delta^3 & \epsilon & \gamma \\ \gamma^2 & \epsilon & \epsilon & \epsilon & \epsilon & \delta^2 & \epsilon \end{pmatrix}$,

$$B = \begin{pmatrix} e & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & e \end{pmatrix}, \quad R = \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & \epsilon \\ e & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & \epsilon \\ \epsilon & e \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix},$$

$C = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e, \end{pmatrix}$, $u^t = \begin{pmatrix} u_1 & u_2 \end{pmatrix}$, $w^t = \begin{pmatrix} w_1 & w_2 \end{pmatrix}$.

where u, y and x are respectively the input, output and state vector. w represents uncontrollable inputs, each entry of w corresponds to a transition which disable the firing of internal transition of the graph, and then decreases the performance of the system. A, B, C and R are given matrices over $\mathcal{M}_{in}^{ax}[\gamma,\delta]$. CA^*B is the input/output transfer matrix and CA^*R is the disturbance/output transfer matrix.

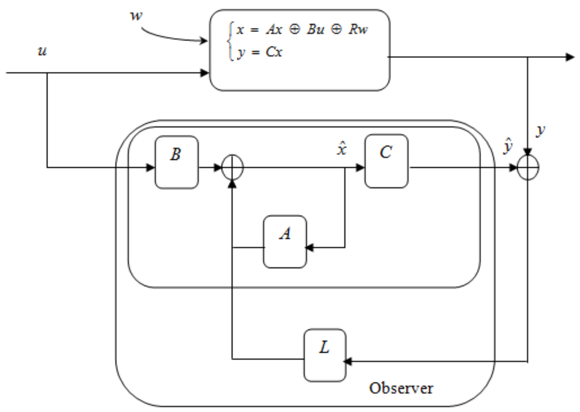

Figure 3: *observer structure*

3.3. Periodicity, causality and asymptotic slope

Definition 8 *(Periodicity) A series $s \in \mathcal{M}_{in}^{ax}[\gamma,\delta]$ is said to be periodic if it can be written as $s = p \oplus q(\gamma^{\nu}\delta^{\tau})$ with p and q two polynomials and $\nu, \tau \in N$. A matrix is said to be periodic if all its entries are periodic.*

Definition 9 *(Causality) A series $s \in \mathcal{M}_{in}^{ax}[\gamma,\delta]$ is causal if $s = \epsilon$. The set of causal elements of $\mathcal{M}_{in}^{ax}[\gamma,\delta]$ has a complete dioid structure denoted by $\mathcal{M}_{in}^{ax+}[\gamma,\delta]$*

Definition 10 *(asymptotic slope) The asymptotic slope of a periodic series $s = p \oplus q(\gamma^{\nu}\delta^{\tau})^*$ denoted $\sigma_\infty(s)$ is defined as the ratio $\sigma_\infty(s) = \nu/\tau$*

Let s_1 and s_2 be two periodic series such that ν_1, ν_2 $\neq 0$ et τ_1, $\tau_2 \neq 0$), then

$$\sigma_\infty(s_1 \oplus s_2) = \min(\sigma_\infty(s_1), \sigma_\infty(s_2)),$$

$$\sigma_\infty(s_1 \otimes s_2) = \min(\sigma_\infty(s_1), \sigma_\infty(s_2)),$$

If $\sigma_\infty(s_1) \leq \sigma_\infty(s_2)$ then

$$\sigma_\infty(s_2 \circ s_1) = \sigma_\infty(s_1)$$

otherwise $s_2 \circ s_1 = \epsilon$.

4. OBSERVER DESIGN

The system evolves according to its state vector equations. Or, In many systems of practical importance, the entire state vector is not available for measurement. When faced with this difficulty , a solution is to provide an estimate of the internal state of the given plant from measurements of the input and output of the real system. By analogy with the classical Luenberger observer Luenberger (1964, 1966), we present an observer design (inspired from the work of (Laurent (2010)) for Timed Event Graph modeled in $\mathcal{M}_{in}^{ax}[\gamma,\delta]$. Figure3 depicts the observer structure whose equations are:

$$\hat{x} = A\hat{x} \oplus Bu \oplus L(\hat{y} \oplus y) = A\hat{x} \oplus Bu \oplus LC\hat{x} \oplus LCx \quad (12)$$

$$\hat{y} = C\hat{x} \quad (13)$$

Where L is an observer gain matrix.
Using equation (10), we obtain the following structure:
$\hat{x} = A\hat{x} \oplus Bu \oplus LC\hat{x} \oplus LC(A^*Bu \oplus A^*Rw)$
$\hat{x} = (A \oplus LC)\hat{x} \oplus Bu \oplus LCA^*Bu \oplus LCA^*Rw$
$\hat{x} = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Bu \oplus (A \oplus LC)^*LCA^*Rw$
By applying Kleene star properties we are:

$$(A \oplus LC)^* = A^*(LCA^*)^* \quad (14)$$

Replacing (14) in previous equation, we obtain:
$\hat{x} = A^*(LCA^*)^*Bu \oplus A^*(LCA^*)^*LCA^*Bu \oplus A^*(LCA^*)^*LCA^*Rw$
and by recalling that $(LCA^*)^*LCA^* = (LCA^*)^+$, this equation may be written as follows:
$\hat{x} = A^*(LCA^*)^*Bu \oplus A^*(LCA^*)^+Bu \oplus A^*(LCA^*)^+Rw$
Equation (3) yields: $(LCA^*)^+ \leq (LCA^*)^*$
Then the observer equation may be written as follows:

$$\hat{x} = A^*(LCA^*)^*Bu \oplus A^*(LCA^*)^+Rw$$

$$\hat{x} = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw \quad (15)$$

The objective, is to calculate the greatest observation matrix L such that the estimated vector $\hat{x}$ be as close as possible to state x, under the constraint $\hat{x} \leq x$, formally it can be written :

$$(A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw \leq A^*Bu \oplus A^*Rw$$

Or equivalently:

$$(A \oplus LC)^*B \leq A^*B \quad (16)$$

$$(A \oplus LC)^*LCA^*R \leq A^*R \quad (17)$$

Lemma 1 *The greatest matrix L such that $(A \oplus LC)^*B \leq A^*B$ is given by :*

$$L_1 = (A^*B)\phi(CA^*B) \quad (18)$$

Lemma 2 *The greatest matrix L such that $(A \oplus LC)^*LCA^*R \leq A^*R$ is given by :*

$$L_2 = (A^*R)\phi(CA^*R) \quad (19)$$

The greatest observer matrix such that $\hat{x} \leq x$ is:

$$L = L_1 \wedge L_2$$

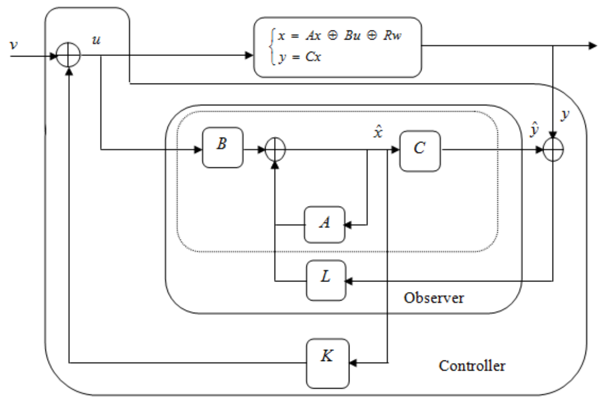

Figure 4: *Controller using observer*

5. FEEDBACK CONTROLLER SYNTHESIS WITH OBSERVER

Now, we consider a problem of controller synthesis with an observer, for TEG, in an objective of reference model matching. This problem can be described in the following way: Taking a TEG of which one knows the transfer matrix, we estimate a state and we compute a controller which leads to a closed loop system whose behavior is as close as possible to the given reference model G_{ref}.
The input output transfet function is expressed by:

$$y = Hu, \ \ such \ \ that \ \ H = CA^*B$$

The observer equation is given by:

$$\hat{x} = (A \oplus LC)^*Bu$$

Acontroller denoted K is added between $\hat{x}$ and u, the input is described by:

$$u = K\hat{x} \oplus v = (K(A \oplus LC)^*B)^*v \qquad (20)$$

Then, the system equation is given by:

$$x = A^*Bu = A^*B(K(A \oplus LC)^*B)^*v$$

$$y = Cx = CA^*B(K(A \oplus LC)^*B)^*v \qquad (21)$$

Firstly, we calculate the observer matrix $L_{opt} = (A^*B)\o(CA^*B)$. Then, within the framework of feedback controller synthesis, we have to find for a given G_{ref}, a greatest K_{opt} with respect to the residuation theory.

$$K_{opt} = H \circ G_{ref}\o((A \oplus L_{opt}C)^*B) \qquad (22)$$

Proof 1 *We calculate the greatest solution K_{opt} in order that the controlled system (with estimate state) will behave as close as possible to a given reference model.*

$$CA^*B(K(A \oplus L_{opt}C)^*B)^* \preceq G_{ref}$$

Using the residuation properties, the following equivalences are given :

$$CA^*B(K(A \oplus L_{opt}C)^*B)^* \preceq G_{ref}$$

$$\Leftrightarrow (K(A \oplus L_{opt}C)^*B)^* \preceq CA^*B \circ G_{ref}$$

$$\Leftrightarrow K(A \oplus L_{opt}C)^*B \preceq CA^*B \circ G_{ref}$$

$$\Leftrightarrow K \preceq CA^*B \circ G_{ref}\o(K(A \oplus L_{opt}C)^*B) = K_{opt}$$

6. ILLUSTRATION

In order to illustrate results presented previously, we consider a Timed Event Graph depicted in figure, 2. Transitions w_1 and w_2 represent uncontrollable inputs. each one corresponds to a transition which delays or disables the firing of internal transition of the graph. In our example, w_1 corresponds to a failure at entry inside the furnace, the beginning of the thermal process, is modeled by x_3, the firing of this transition is reported to time 25 instead of time 15. w_2 means that the operator removes parts from the pallets at time 45 instead of time 39. Then, the state are delayed by disturbances whose trajectories are as follows :

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \gamma^3\delta^{25} \\ \gamma^2\delta^{45} \end{pmatrix}$$

Using minmaxgb Toolbox for Scilab (see http://www.maxplus.org/), the observer matrix is given by:

$$L_{11} = \gamma^5\delta^4 \oplus \gamma^6\delta^6 \oplus \gamma^7\delta^8 \oplus \gamma^8\delta^{10} \oplus \gamma^9\delta^{12} \oplus (\gamma^{10}\delta^{37} \oplus \gamma^{11}\delta^{39} \oplus \gamma^{12}\delta^{47} \oplus \gamma^{13}\delta^{49} \oplus \gamma^{14}\delta^{57})[\gamma^5\delta^{33}]^*$$

$$L_{21} = \gamma^5\delta^5 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^9 \oplus \gamma^8\delta^{11} \oplus \gamma^9\delta^{18} \oplus (\gamma^{10}\delta^{38} \oplus \gamma^{11}\delta^{40} \oplus \gamma^{12}\delta^{48} \oplus \gamma^{13}\delta^{50} \oplus \gamma^{14}\delta^{58})[\gamma^5\delta^{33}]^*$$

$$L_{31} = \epsilon \oplus (\gamma^5\delta^8 \oplus \gamma^6\delta^{10} \oplus \gamma^7\delta^{18} \oplus \gamma^8\delta^{20} \oplus \gamma^9\delta^{28})[\gamma^5\delta^{33}]^*$$

$$L_{41} = \epsilon \oplus (\gamma^5\delta^{18} \oplus \gamma^6\delta^{20} \oplus \gamma^7\delta^{28} \oplus \gamma^8\delta^{30} \oplus \gamma^9\delta^{38})[\gamma^5\delta^{33}]^*$$

$$L_{51} = \gamma^4 \oplus (\gamma^5\delta^{28} \oplus \gamma^6\delta^{30} \oplus \gamma^7\delta^{38} \oplus \gamma^8\delta^{40} \oplus \gamma^9\delta^{48})[\gamma^5\delta^{33}]^*$$

$$L_{61} = \gamma \oplus \gamma^2\delta^2 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^6 \oplus (\gamma^5\delta^{31} \oplus \gamma^6\delta^{33} \oplus \gamma^7\delta^{41} \oplus \gamma^8\delta^{43} \oplus \gamma^9\delta^{51})[\gamma^5\delta^{33}]^*$$

$$L_{71} = e \oplus \gamma\delta^2 \oplus \gamma^2\delta^4 \oplus \gamma^3\delta^6 \oplus \gamma^4\delta^8 \oplus (\gamma^5\delta^{33} \oplus \gamma^6\delta^{35} \oplus \gamma^7\delta^{43} \oplus \gamma^8\delta^{45} \oplus \gamma^9\delta^{53})[\gamma^5\delta^{33}]^*$$

and $\hat{y}$ is equal to y:

$$\hat{y} = \delta^{29} \oplus \gamma\delta^{31} \oplus \gamma^2\delta^{45} \oplus \gamma^3\delta^{50} \oplus \gamma^4\delta^{52} \oplus \gamma^5\delta^{62} \oplus \gamma^6\delta^{64} \oplus (\gamma^7\delta^{78} \oplus \gamma^8\delta^{83} \oplus \gamma^9\delta^{88} \oplus \gamma^{10}\delta^{95} \oplus \gamma^{11}\delta^{98})[\gamma^5\delta^{33}]$$

The simulation results are Shown in figure (Fig. 5, Fig. 6). The estimated states of Timed Event Graph $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6, \hat{x}_7$ are compared to the actual state. $x_1, x_2, x_3, x_4, x_5, x_6, x_7$. We remark a little difference between some state and corresponding estimated state : $\hat{x}_3$ and x_3, $\hat{x}_4$ and x_4, $\hat{x}_5$ and x_5, $\hat{x}_6$ and x_6. it implies that the disturbances applied for transition x_3 reduce the system performances, but these disturbances are not acknowledge by the observer. it is assumed that the model and the initial state correspond to the fastest behavior, ie. $\hat{y}$ and y are equivalent, and there are equality between asymptotic slope of the state and the one of estimated state, $\sigma_\infty(\hat{y}) = \sigma_\infty(y) = 5/33$.

Consider a problem of controller synthesis with observer, we propose to compute a greatest K so that the system has a transfer relation close to a given reference transfer G_{ref}.

The objective of the reference model is to impose a desired behavior G_{ref} to a given system H, then in our example, we consider $G_{ref} = H$.Using Scilab, we calculate H:

$$H_{11} = \delta^{29} \oplus \gamma^1\delta^{31} \oplus \gamma^2\delta^{39} \oplus \gamma^3\delta^{41} \oplus \gamma^4\delta^{49} \oplus (\gamma^5\delta^{37} \oplus \gamma^{11}\delta^{39}[\gamma^5\delta^{33}$$

$$H_{12} = \gamma^0\delta^0 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^4 \oplus \gamma^3\delta^6 \oplus \gamma^4\delta^8 \oplus (\gamma^5\delta^{33} \oplus \gamma^6\delta^{35} \oplus \gamma^7\delta^{43}$$
$$\oplus \gamma^8\delta^{45} \oplus \gamma^9\delta^{53}[\gamma^5\delta^{33}]^*$$

For L_{opt} obtained, we compute the greatest realizable feedback K_{opt} using the expression:

$$K_{opt} = H \circ H \phi((A \oplus L_{opt}C)^* B)$$

therefore, we can compute the controller and the output of the system:

$$u = (K(A \oplus LC)^* B)^* v$$

$$y = CA^* B(K(A \oplus LC)^* B)^* v$$

Assume that: $Gkopt = CA^* B(K(A \oplus LC)^* B)^*$

$$Gkopt_{11} = \delta^{29} \oplus \gamma^1\delta^{31} \oplus \gamma^2\delta^{39} \oplus \gamma^3\delta^{41}$$
$$\oplus \gamma^4\delta^{49} \oplus (\gamma^5\delta^{37} \oplus \gamma^{11}\delta^{39}[\gamma^5\delta^{33}]^*.$$

$$Gkopt_{12} = \gamma^0\delta^0 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^4 \oplus$$
$$\gamma^3\delta^6 \oplus \gamma^4\delta^8 \oplus (\gamma^5\delta^{33} \oplus \gamma^6\delta^{35} \oplus \gamma^7\delta^{43}$$
$$\oplus \gamma^8\delta^{45} \oplus \gamma^9\delta^{53}[\gamma^5\delta^{33}]^*.$$

For any TEG, it is possible to preserve its own transfer with either a greatest realizable feedback control. We have: $Gkopt \preceq Gref = H$

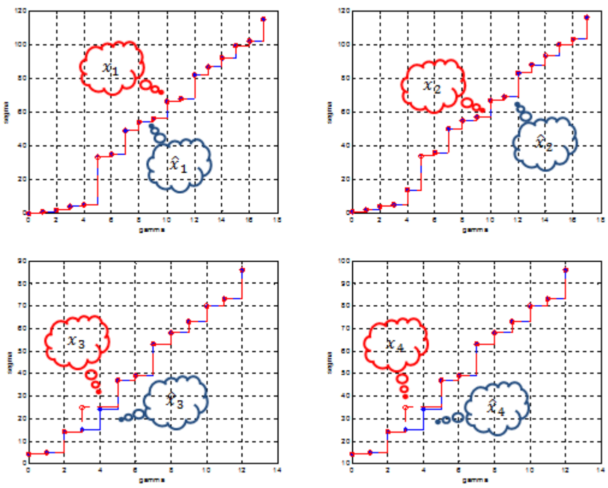

Figure 5: Estimated state of x_1, x_2, x_3 and x_4

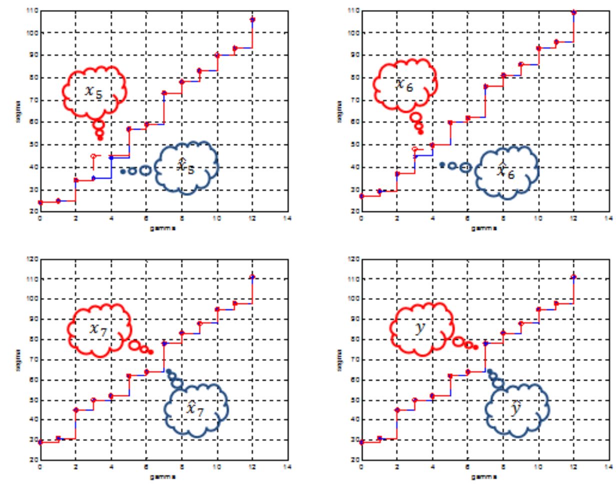

Figure 6: Estimated state of x_5, x_6, x_7 and y

7. CONCLUSION

In this paper, we have applied to an industrial process an observer design and the synthesis of controller with observer. this system is modeled by Timed Event Graph which can be described by linear equations in idempotent semiring. Firstly, The estimation state in presence of perturbation based on Luenberger observer, is considered, and the effectiveness of this method is shown by simulation results given by the use of Scilab. Afterwards, A synthesis of controller with observer for TEG in a model reference is presented, we have shown that it is possible to preserve its own transfer with a greatest realizable feedback control.

REFERENCES

Amari, S., Demongodin, I. and Loiseau, J.J. (2004) Sizing, cycle time and plant control using dioid algebra. Springer, pp.77-85.

Baccelli, F., Cohen, G., Olsder, G. and Quadrat, J. (1992) Synchronization and Linearity: An Algebra for Discrete Event Systems. Wiley and Sons.

Cassandras, C.G. and Lafortune, S. (1992) Introduction to discrete event systems. Kluwer Academic.

Cohen, G., Moller, P., Quadrat, J.P. and viot, M. (1984)Linear system theory for discrete event systems. In 23rd IEEE Conf. on Decision and Control, Las Vegas, Nevada.

Cohen, G. (1998a) Residuation and applications,In Algébres Max-plus et applications en informatique, Ecole de printemps d'informatique thèorique, Noirmoutier, France.

Cohen, G. (1998b)Two-dimentional domain representation of timed event graphs,In algébres Max-Plus et applications en informatique et automatique, Ecole de printemps d'informatique théorique, Noirmoutier, France.

Cohen, G., Gaubert, S. and Quadrat, J. (1999) Max-plus algebra and system theory,Where we are and where to go now, Annuel Reviews in control,(23),pp. 207-219.

Cottenceau, B. Contribution á la commande des systèmes á événements discrets: Synthése de correcteurs pour les graphes d'événements temporisés dans les dioïdes. Phd thesis, Angers University.

Cuninghame-Green, R.A. Minimax Algebra. (1979) Number 166 in Lecture notes in Economics and Mathematical Systems, Springer.

David, R. and Alla, H. (1997) Du Grafcet aux réseaux de Petri. Ed. Hermes, 2e édition revue et augmentée.

Houssin, L. (2003) Contribution á l'étude des systèmes événements discrets dans l'algébre des dioïdes. Applications aux systémes de transport. Rapport de stage de DEA, DEA Automatique et informatique Appliquée, Nantes-Angers, France.

Laurent, H., Carlos, A.M., Bertrand, C. and Mehdi, L. (2010) observer design for $(\max,+)$*linear systems. IEEE Transactions on Automatic Control, VOL.55.*

Lhommeau, M. (2003) Etude de systèmes á événements discrets dans l'Algébre $(\max,+)$*: Synthèse de correcteurs robustes dans un dioïde d'intervalles, Synthèse de correcteurs en présence de perturbation.Phd Thesis, Angers University.*

Luenberger, D G. (1964) Observing the State of a Linear System , IEEE Trans. on Military Electronics , Vol. MIL-8,pp.74-80, April 1964.

Luenberger, D G. (1964) Observers for multivariable Systems, IEEE Trans. on Automatic Control , Vol. AC-II,NO.2, pp.190-197, April 1966.

Martinez, C. and Castagna, P. (2003) Sizing of an industrial plant using tight time constraints using complementary approaches: $(\max,+)$ *algebra and computer simulation. Simulation Modelling Practice and Theory, Elsevier, Vol. 11,pp.75-88.*