SAT-based Bounded Model Checking for Weighted Deontic Interpreted Systems (Extended Abstract) ⋆

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Abstract. We present WECTL*KD, a weighted branching time temporal logic to specify knowledge, and correct functioning behaviour in multi-agent systems (MAS). We interpret the formulae of the logic over models generated by weighted deontic interpreted systems (WDIS). Furthermore, we investigate a SAT-based bounded model checking (BMC) technique for WDIS and for WECTL*KD.

1 Introduction

The formalism of *interpreted systems* (IS) [4] provides a useful framework to model multi-agent systems (MASs) [13], and to verify various classes of temporal and epistemic properties. The formalism of *deontic interpreted systems* (DIS) [7] is an extension of ISs, which makes possible reasoning about temporal, epistemic and correct functioning behaviour of MASs. An important assumption in this line of models is that there are no costs associated to agents' actions. The formalism of *weighted deontic interpreted systems* (WDISs) [16] extends DISs to make the reasoning possible about not only temporal, epistemic and deontic properties, but also about agents quantitative properties. In particular, in the Kripke model of WDIS each transition is labelled by a pair: a joint action and a positive integer value that represents the cost of acting agents.

The basic idea in SAT-based bounded model checking (BMC) [1, 9] is to translate the existential model checking problem for a modal (e.g., temporal, epistemic, deontic) logic [2, 13] to the propositional satisfiability problem. In particular, in BMC we first represent a counterexample, whose length is bounded by some integer k, by a propositional formula, and then check the resulting propositional formula with a specialised SAT-solver. If the formula in question is satisfiable, then the SAT-solver returns a satisfying assignment that can be converted into a concrete counterexample. Otherwise, the bound k is increased and the process repeated; we increases k until either a witness is found, the problem becomes intractable, or some pre-known upper bound is reached.

To model check the requirements of MASs various extensions of temporal logics [3] with epistemic [4], beliefs [6], and deontic [7] components have been proposed. In this paper we aim at completing the picture of applying the SAT-based BMC techniques to MASs by looking at the existential fragment of the weighted CTL*KD (i.e. weighted CTL* extended with epistemic and deontic components), interpreted over the *weighted deontic interpreted systems* (WDISs). The proposed BMC encoding is based on the

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BMC encoding introduced in [16, 21, 24]. Namely, in [24] a propositional encoding of the BMC problem for ECTL* and for standard Kripke models has been introduced. The method has been experimentally evaluated. Next, in [16] weighted deontic interpreted systems (WDIS) and a propositional encoding of the BMC problem for WECTLKD and for WDIS have been introduced. Finally, in [21] a BMC method for WELTLK and for weighted interpreted systems has been introduced and experimentally evaluated.

The rest of the paper is organised as follows. In Section 2 we introduce WDIS together with its Kripke model. In Section 3 we define the WECTL*KD language together with the bounded semantics. In Section 4 we provide a SAT-based BMC method for WECTL*KD and for WDIS. In the last section we conclude the paper.

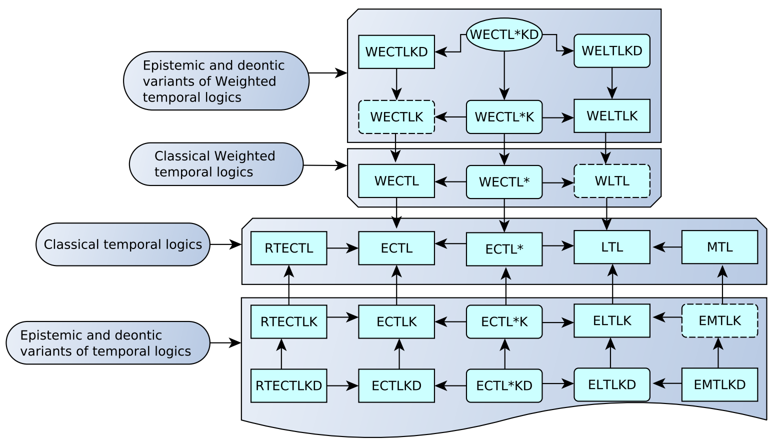

Fig. 1. Classical temporal and weighted logics with discrete semantics, and their epistemic and deontic extensions. The dedicated SAT-based BMC methods have been defined for the logics placed in rectangles. The logics for which SAT-BMC methods can be easily inferred from the dedicated one are placed in "dashed" rectangles.

Related work. Figure 1 provides diagram showing the relations between classical temporal logics, weighted temporal logics, and their epistemic and deontic extensions, and indicates for which logic a SAT-based BMC method (SAT-BMC for short) has been developed. For classical temporal logics with discrete semantics over Kripke models SAT-BMC has been defined in [1] for LTL, in [10, 23] for ECTL, in [14, 24] for ECTL*, in [22] for RTECTL, and in [12] for MTL. For classical weighted temporal logics with discrete semantics over weighted Kripke models generated, for example, by simply timed systems, SAT-BMC has been defined for WECTL [20] only. For epistemic and deontic variants of classical temporal logics with semantics over Kripke models generated by (deontic) interpreted systems SAT-BMC has been defined in [9, 5] for ECTLK, in [15] for ECTLKD, in [11, 8] for ELTLK, in [19] for RTECTLK, in [18] for RTECTLKD, and in [17] for EMTLKD (this method provides obviously a SAT-BMC solution for EMTLK). There is no paper about SAT-BMC for ECTL*K and for ECTL*KD. These missing methods can however be easily designed as a fusion of SAT-BMC methods for ECTL* and for ECTLK / ECTLKD. For epistemic and deontic variants of classical weighted temporal logic with semantics over Kripke models generated by (deontic) weighted interpreted systems SAT-BMC has been defined in [21] for WELTLK (this method provides obviously a SAT-BMC solution for WELTL), and in [16] for WECTLKD (this method provides obviously a SAT-BMC solution for WECTLK).

2 Weighted Deontic Interpreted Systems

Let $Ag = \{1, \ldots, n\}$ be the non-empty and finite set of agents. We assume that a MAS consists of n agents and a special agent $\mathcal{E}$ that represent the environment in which the agents operate. Next, we assume that a given MAS is modelled by the *weighted deontic interpreted system* (WDIS), in which each agent $\mathbf{c} \in Ag \cup \{\mathcal{E}\}$ is modelled using a non-empty set $L_\mathbf{c} = \mathcal{G}_\mathbf{c} \cup \mathcal{R}_\mathbf{c}$ of *local states* such that $\mathcal{G}_\mathbf{c}$ is a non-empty set of *faultless (green)* states and $\mathcal{R}_\mathbf{c}$ is a set of *faulty (red)* states, a non-empty set $\iota_\mathbf{c} \subseteq L_\mathbf{c}$ of *initial* states, a non-empty set $Act_\mathbf{c}$ of *possible actions*, a *protocol function* $P_\mathbf{c} : L_\mathbf{c} \to 2^{Act_\mathbf{c}}$ that defines rules according to agents operate, a (partial) *evolution function* $t_\mathbf{c} : L_\mathbf{c} \times Act \to L_\mathbf{c}$ with $Act = Act_1 \times \cdots \times Act_n \times Act_\mathcal{E}$ (each element of Act is called a *joint action*), a *weight function* $d_\mathbf{c} : Act_\mathbf{c} \to \mathbb{N}$, and a *valuation function* $\mathcal{V}_\mathbf{c} : L_\mathbf{c} \to 2^{\mathcal{PV}}$ that assigns to each local state a set of propositional variables that are assumed to be true at that state. Further, we do not assume that the sets $Act_\mathbf{c}$ are disjoint for all $\mathbf{c} \in Ag \cup \{\mathcal{E}\}$.

Now for a given set of agents Ag, the environment $\mathcal{E}$ and a set of propositional variables $\mathcal{PV}$, we define the *weighted deontic interpreted system* as a tuple WDIS $=$ $(\{\iota_\mathbf{c}, L_\mathbf{c}, \mathcal{G}_\mathbf{c}, Act_\mathbf{c}, P_\mathbf{c}, t_\mathbf{c}, \mathcal{V}_\mathbf{c}, d_\mathbf{c}, \}_{\mathbf{c} \in Ag \cup \{\mathcal{E}\}})$. For a given WDIS we define:

- a set of all *possible global states* $S = L_1 \times \ldots \times L_n \times L_\mathcal{E}$ such that $L_1 \supseteq \mathcal{G}_1, \ldots, L_n \supseteq \mathcal{G}_n, L_\mathcal{E} \supseteq \mathcal{G}_\mathcal{E}$; by $l_\mathbf{c}(s)$ we denote the local component of agent $\mathbf{c} \in Ag \cup \{\mathcal{E}\}$ in a global state $s = (\ell_1, \ldots, \ell_n, \ell_\mathcal{E})$;
- a *global evolution function* $t : S \times Act \to S$ as follows: $t(s, a) = s'$ (or $s \xrightarrow{a} s'$) iff for all $\mathbf{c} \in Ag$, $t_\mathbf{c}(l_\mathbf{c}(s), a) = l_\mathbf{c}(s')$ and $t_\mathcal{E}(l_\mathcal{E}(s), a) = l_\mathcal{E}(s')$;
- a *weighted model* (or *a model*) as a tuple $M = (\iota, S, T, \mathcal{V}, d)$, where
  - $\iota = \iota_1 \times \ldots \times \iota_n \times \iota_\mathcal{E}$ is the set of all possible initial global state;
  - S is the set of all possible global states as defined above;
  - $T \subseteq S \times Act \times S$ is a transition relation defined by the global evolution function as follows: $(s, a, s') \in T$ iff $s \xrightarrow{a} s'$. We assume that the relation T is total, i.e., for any $s \in S$ there exists $s' \in S$ and an action $a \in Act \setminus \{(\epsilon_1, \ldots, \epsilon_n, \epsilon_\mathcal{E})\}$ such that $s \xrightarrow{a} s'$;
  - $\mathcal{V} : S \to 2^{\mathcal{PV}}$ is the valuation function defined as $\mathcal{V}(s) = \bigcup_{\mathbf{c} \in Ag \cup \{\mathcal{E}\}} \mathcal{V}_\mathbf{c}(l_\mathbf{c}(s))$.
  - $d : Act \to \mathbb{N}$ is a "joint" weight function defined as follows: $d((a_1, \ldots, a_n, a_\mathcal{E})) = \sum_{\mathbf{c} \in Ag \cup \{\mathcal{E}\}} d_\mathbf{c}(a_\mathbf{c})$; note that this definition is reasonable, since we are interested in MASs, in which transitions carry some cost;
- an indistinguishability relation $\sim_\mathbf{c} \subseteq S \times S$ for agent $\mathbf{c}$ as follows: $s \sim_\mathbf{c} s'$ iff $l_\mathbf{c}(s') = l_\mathbf{c}(s)$;
- a deontic relation $\propto_\mathbf{c} \subseteq S \times S$ for agent $\mathbf{c}$ as follows: $s \propto_\mathbf{c} s'$ iff $l_\mathbf{c}(s') \in \mathcal{G}_\mathbf{c}$.

A *path* in M is an infinite sequence $\pi = s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} s_2 \xrightarrow{a_3} \ldots$ of transitions. For such a path, and for $j \leqslant m \in \mathbb{N}$, by $\pi(m)$ we denote the m-th state s_m, by π^m we denote the m-th suffix of the path π, which is defined in the standard way: $\pi^m = s_m \xrightarrow{a_{m+1}} s_{m+1} \xrightarrow{a_{m+2}} s_{m+2} \ldots$. Next, by $\pi[j..m]$ we denote the finite sequence $s_j \xrightarrow{a_{j+1}} s_{j+1} \xrightarrow{a_{j+2}} \ldots s_m$ with $m-j$ transitions and $m-j+1$ states, and by $D\pi[j..m]$ we denote the (cumulative) weight of $\pi[j..m]$ that is defined as $d(a_{j+1}) + \ldots + d(a_m)$ (hence 0 when $j = m$). By $\Pi(s)$ we denote the set of all the paths starting at $s \in S$, and by $\Pi = \bigcup_{s^0 \in \iota} \Pi(s^0)$ we denote the set of all the paths starting at initial states.

3 The logic WECTL*KD

Our specification language, which we call WECTL*KD, extends ECTL* [3] with cost constraints on temporal modalities and with epistemic and deontic modalities. More precisely, the basic modalities of WECTL*KD consist of the path quantifier E (for some path) followed by a temporal-epistemic-deontic formula, which is built up from: propositional variables; the boolean operators ($\wedge$-conjunction, $\vee$-disjunction, $\neg$-negation); the temporal modalities (X_I-weighted next step, U_I-weighted until, R_I-weighted release, G_I-weighted always, and F_I-weighted sometime); the epistemic modalities $\overline{\mathrm{K}}_{\mathbf{c}}$ (for "agent $\mathbf{c}$ does not know whether or not"), $\overline{\mathrm{D}}_\Gamma$, $\overline{\mathrm{E}}_\Gamma$, and $\overline{\mathrm{C}}_\Gamma$ (for the dualities to the standard group epistemic modalities representing distributed knowledge in the group Γ, everyone in Γ knows, and common knowledge among agents in Γ); the deontic modalities ($\overline{\mathcal{O}}_{\mathbf{c}}$ and $\underline{\widehat{\mathrm{K}}}_{\mathbf{c}_1}^{\mathbf{c}_2}$ representing the *correctly functioning circumstances of agents*).

Syntax of WECTL*KD. Let $p \in \mathcal{PV}$ be a propositional variable, $\mathbf{c}, \mathbf{c}_1, \mathbf{c}_2 \in Ag$, $\Gamma \subseteq Ag$, and I be an interval in $\mathbb{N} = \{0, 1, 2, \ldots\}$ of the form: $[a, b)$ and $[a, \infty)$, for $a, b \in \mathbb{N}$ and $a \neq b$. We have the following syntax for WECTL*KD. We inductively define a class of state formulae (interpreted at states) and a class of path formulae (interpreted along paths) by the following grammar:

$$\varphi ::= \mathrm{true} \mid \mathrm{false} \mid p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathrm{E}\phi \mid \overline{\mathrm{K}}_{\mathbf{c}}\phi \mid \overline{\mathrm{E}}_\Gamma\phi \mid \overline{\mathrm{D}}_\Gamma\phi \mid \overline{\mathrm{C}}_\Gamma\phi \mid \overline{\mathcal{O}}_{\mathbf{c}}\phi \mid \underline{\widehat{\mathrm{K}}}_{\mathbf{c}_1}^{\mathbf{c}_2}\phi$$
$$\phi ::= \varphi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \mathrm{X}_I\phi \mid \phi \mathrm{U}_I \phi \mid \phi \mathrm{R}_I \phi$$

where φ is a state formula and ϕ is a path formula. WECTL*KD consists of the set of state formulae generated by the above grammar.

The derived basic temporal path modalities for *weighted eventually* (F_I) and *weighted globally* (G_I) are defined as follows: $\mathrm{F}_I\phi ::= \mathrm{true}\mathrm{U}_I\phi$, and $\mathrm{G}_I\phi ::= \mathrm{false}\mathrm{R}_I\phi$.

Note that the combination of weighted temporal, epistemic and deontic operators allows us to specify how agent's knowledge or correctly functioning circumstances of agents evolve over time and how much they cost.

Semantics of WECTL*KD. The semantics of WECTL*KD formulae is determined with respect to a model, defined in Section 2. In the semantics we assume the following definitions of epistemic relations: $\sim_\Gamma^E \stackrel{def}{=} \bigcup_{\mathbf{c} \in \Gamma} \sim_{\mathbf{c}}$, $\sim_\Gamma^C \stackrel{def}{=} (\sim_\Gamma^E)^+$ (the transitive closure of $\sim_\Gamma^E$), $\sim_\Gamma^D \stackrel{def}{=} \bigcap_{\mathbf{c} \in \Gamma} \sim_{\mathbf{c}}$, where $\Gamma \subseteq Ag$.

Definition 1. *Let M be a model, s a state of M, π a path in M, and $m \in \mathbb{N}$. For a state formula α over $\mathcal{PV}$, the notation $M, s \models \alpha$ means that α holds at the state*

*s in the model M. Similarly, for a path formula φ over $\mathcal{PV}$, the notation $M, \pi \models \varphi$ means that φ holds along the path π in the model M. Moreover, let $p \in \mathcal{PV}$ be a propositional variable, α, β be state formulae of WECTL*KD, and φ, ψ be path formulae of WECTL*KD. The relation $\models$ is defined inductively as follows:*

$M, s \models \mathrm{true}$, $M, s \not\models \mathrm{false}$, $M, s \models p$ *iff* $p \in \mathcal{V}(s)$, $M, s \models \neg p$ *iff* $p \notin \mathcal{V}(s)$,
$M, s \models \alpha \wedge \beta$ *iff* $M, s \models \alpha$ *and* $M, s \models \beta$,
$M, s \models \alpha \vee \beta$ *iff* $M, s \models \alpha$ *or* $M, s \models \beta$,
$M, s \models \overline{\mathrm{K}}_{\mathbf{c}}\alpha$ *iff* $(\exists \pi \in \Pi)(\exists i \geqslant 0)(s \sim_{\mathbf{c}} \pi(i)$ *and* $M, \pi^i \models \alpha)$,
$M, s \models \overline{Y}_{\Gamma}\alpha$ *iff* $(\exists \pi \in \Pi)(\exists i \geqslant 0)(s \sim^{Y}_{\Gamma} \pi(i)$ *and* $M, \pi^i \models \alpha)$, $Y \in \{\mathrm{D}, \mathrm{E}, \mathrm{C}\}$,
$M, s \models \overline{\mathcal{O}}_{\mathbf{c}}\alpha$ *iff* $(\exists \pi \in \Pi)(\exists i \geqslant 0)(s \propto_{\mathbf{c}} \pi(i)$ *and* $M, \pi^i \models \alpha)$,
$M, s \models \underline{\widehat{\mathrm{K}}}^{\mathbf{c_2}}_{\mathbf{c_1}}\alpha$ *iff* $(\exists \pi \in \Pi)(\exists i \geqslant 0)(s \sim_{\mathbf{c_1}} \pi(i)$ *and* $s \propto_{\mathbf{c_2}} \pi(i)$ *and* $M, \pi^i \models \alpha)$,
$M, s \models \mathrm{E}\varphi$ *iff* $(\exists \pi \in \Pi(s))(M, \pi^0 \models \varphi)$,
$M, \pi^m \models \alpha$ *iff* $M, \pi(m) \models \alpha$,
$M, \pi^m \models \varphi \wedge \psi$ *iff* $M, \pi^m \models \varphi$ *and* $M, \pi^m \models \psi$,
$M, \pi^m \models \varphi \vee \psi$ *iff* $M, \pi^m \models \varphi$ *or* $M, \pi^m \models \psi$,
$M, \pi^m \models \mathrm{X}_I\varphi$ *iff* $D\pi[m..m+1] \in I$ *and* $M, \pi^{m+1} \models \varphi$,
$M, \pi^m \models \varphi \mathrm{U}_I \psi$ *iff* $(\exists j \geqslant m)\big(D\pi[m..j] \in I$ *and* $M, \pi^j \models \psi$ *and* $(\forall m \leqslant i < j) M, \pi^i \models \varphi\big)$,
$M, \pi^m \models \varphi \mathrm{R}_I \psi$ *iff* $(\exists j \geqslant m)\big(D\pi[m..j] \in I$ *and* $M, \pi^j \models \varphi$ *and* $(\forall m \leqslant i \leqslant j)$ $M, \pi^i \models \psi\big)$ *or* $(\forall j \geqslant m)(D\pi[m..j] \in I$ *implies* $M, \pi^j \models \psi)$.

A WECTL*KD state formula α is *true* in the model M, denoted by $M \models \alpha$, iff for some $s \in \iota$, $M, s \models \alpha$, i.e., α holds at some initial state of M. The *model checking problem* asks whether $M \models \alpha$.

Bounded semantics of WECTL*KD. A *bounded semantics* for WECTL*KD is the basis of the translation of *bounded model checking problem* to the satisfiability of propositional formulae problem (i.e., SAT-problem) that is defined in the next section. To define the bounded semantics we need to represent infinite paths of a model in a special way. To this aim, as usually, we define the notions of *k-paths* and *loops*.

Definition 2. *Let M be a model, $k \in \mathbb{N}$ a bound, and $0 \leqslant l \leqslant k$. A k-*path *π_l is a pair (π, l), where π is a finite sequence $s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_k} s_k$ of transitions. A k-path π_l is a* loop *if $l < k$ and $\pi(k) = \pi(l)$.*

Note that if a k-path π_l is a loop, then it represents the infinite path of the form uv^{ω}, where $u = (s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_l} s_l)$ and $v = (s_{l+1} \xrightarrow{a_{l+2}} \dots \xrightarrow{a_k} s_k)$. $\Pi_k(s)$ denotes the set of all the k-paths of M that start at s, and $\Pi_k = \bigcup_{s^0 \in \iota} \Pi_k(s^0)$.

As in the definition of bounded semantics we need to define the satisfiability relation on suffixes of k-paths, we denote by π^m_l the pair (π_l, m), i.e. the k-path π_l together with the designated starting point m, where $0 \leqslant m \leqslant k$. Further, let s be a state and π_l be a k-path. For a state formula α over $\mathcal{PV}$, the notation $M, s \models_k \alpha$ means that α is k-true at the state s in the model M. Similarly, for a path formula φ over $\mathcal{PV}$, the notation $M, \pi^m_l \models_k \varphi$, where $0 \leqslant m \leqslant k$, denotes that the formula φ is k-true along the suffix $\pi(m) \xrightarrow{a_{m+1}} \pi(m+1) \xrightarrow{a_{m+2}} \dots \xrightarrow{a_k} \pi(k)$ of π.

Definition 3. *Let M be a model, s a state of M, π_l a k-path in M, $0 \leqslant m \leqslant k$, $p \in \mathcal{PV}$ a propositional variables, α, β state formulae of WECTL*KD, and φ, ψ path formulae of WECTL*KD. The relation $\models_k$ is defined inductively as follows:*

$M, s \models_k \mathrm{true}$, $M, s \not\models_k \mathrm{false}$, $M, s \models_k p$ *iff* $p \in \mathcal{V}(s)$, $M, s \models_k \neg p$ *iff* $p \notin \mathcal{V}(s)$,

$M, s \models_k \alpha \wedge \beta$ *iff* $M, s \models_k \alpha$ *and* $M, s \models_k \beta$,

$M, s \models_k \alpha \vee \beta$ *iff* $M, s \models_k \alpha$ *or* $M, s \models_k \beta$,

$M, s \models_k \overline{\mathrm{K}}_{\mathbf{c}}\alpha$ *iff* $(\exists \pi_l \in \Pi_k)(\exists 0 \leqslant j \leqslant k)(M, \pi_l^j \models_k \alpha$ *and* $s \sim_{\mathbf{c}} \pi(j))$,

$M, s \models_k \overline{Y}_{\Gamma}\alpha$ *iff* $(\exists \pi_l \in \Pi_k)(\exists 0 \leqslant j \leqslant k)(M, \pi_l^j \models_k \alpha$ *and* $s \sim_{\Gamma}^{Y} \pi(j))$, *where* $Y \in \{\mathrm{D}, \mathrm{E}, \mathrm{C}\}$,

$M, s \models_k \overline{\mathcal{O}}_{\mathbf{c}}\alpha$ *iff* $(\exists \pi_l \in \Pi_k)(\exists 0 \leqslant j \leqslant k)(M, \pi_l^j \models_k \alpha$ *and* $s \propto_{\mathbf{c}} \pi(j))$,

$M, s \models_k \underline{\widehat{\mathrm{K}}}_{\mathbf{c_1}}^{\mathbf{c_2}}\alpha$ *iff* $(\exists \pi_l \in \Pi_k)(\exists 0 \leqslant j \leqslant k)(M, \pi_l^j \models_k \alpha$ *and* $s \sim_{\mathbf{c_1}} \pi(j)$ *and* $s \propto_{\mathbf{c_1}} \pi(j))$,

$M, s \models_k \mathrm{E}\varphi$ *iff* $(\exists \pi_l \in \Pi_k(s))(M, \pi_l^0 \models_k \varphi)$,

$M, \pi_l^m \models_k \alpha$ *iff* $M, \pi(m) \models_k \alpha$,

$M, \pi_l^m \models_k \varphi \wedge \psi$ *iff* $M, \pi_l^m \models_k \varphi$ *and* $M, \pi_l^m \models_k \psi$,

$M, \pi_l^m \models_k \varphi \vee \psi$ *iff* $M, \pi_l^m \models_k \varphi$ *or* $M, \pi_l^m \models_k \psi$,

$M, \pi_l^m \models_k \mathrm{X}_I\varphi$ *iff* $(m < k$ *and* $D\pi[m..m+1] \in I$ *and* $M, \pi_l^{m+1} \models_k \varphi)$ *or* $(m = k$ *and* $l < k$ *and* $\pi(k) = \pi(l)$ *and* $D\pi[l..l+1] \in I$ *and* $M, \pi_l^{l+1} \models_k \varphi)$,

$M, \pi_l^m \models_k \varphi \mathrm{U}_I \psi$ *iff* $(\exists m \leqslant j \leqslant k)(D\pi[m..j] \in I$ *and* $M, \pi_l^j \models_k \psi$ *and* $(\forall m \leqslant i < j)M, \pi_l^i \models_k \varphi)or(l < m$ *and* $\pi(k) = \pi(l)$ *and* $(\exists l < j < m)(D\pi[m..k] + D\pi[l..j] \in I$ *and* $M, \pi_l^j \models_k \psi$ *and* $(\forall l < i < j)M, \pi_l^i \models \varphi$ *and* $(\forall m \leqslant i \leqslant k)M, \pi_l^i \models_k \varphi))$,

$M, \pi_l^m \models_k \varphi \mathrm{R}_I \psi$ *iff* $(D\pi[m..k] \geqslant right(I)$ *and* $(\forall m \leqslant j \leqslant k)(D\pi[m..j] \in I$ *implies* $M, \pi_l^j \models_k \psi))$ *or* $(D\pi[m..k] < right(I)$ *and* $\pi(k) = \pi(l)$ *and* $(\forall m \leqslant j \leqslant k)(D\pi[m..j] \in I$ *implies* $M, \pi_l^j \models_k \psi)$ *and* $(\forall l \leqslant j \leqslant k)(D\pi[m..k] + D\pi[l..j] \in I$ *implies* $M, \pi_l^j \models_k \psi))$ *or* $(\exists m \leqslant j \leqslant k)(D\pi[m..j] \in I$ *and* $M, \pi_l^j \models_k \varphi$ *and* $(\forall m \leqslant i \leqslant j)M, \pi_l^i \models_k \psi)$ *or* $(l < m$ *and* $\pi(k) = \pi(l)$ *and* $(\exists l < j < m)(D\pi[m..k] + D\pi[l..j] \in I$ *and* $M, \pi_l^j \models_k \varphi$ *and* $(\forall l < i \leqslant j)M, \pi_l^i \models \psi$ *and* $(\forall m \leqslant i \leqslant k)M, \pi_l^i \models_k \psi))$.

A WECTL*KD state formula α is k-true in M, denoted $M \models_k \varphi$, iff for some $s \in \iota$, $M, s \models_k \varphi$. The *bounded model checking problem* asks whether there exists $k \in \mathbb{N}$ such that $M \models_k \varphi$.

Lemma 1. *Let M be a model, s a state of M, and α a WECTL*KD state formula.*

- *for some $k \in \mathbb{N}$, if $M, s \models_k \alpha$, then $M, s \models \alpha$.*
- *if $M, s \models \alpha$, then $M, s \models_k \alpha$ for some $k \in \mathbb{N}$.*

The following theorem follows from Lemma 1 and it states that for a given model M and a formula α there exists a bound k such that the model checking problem can be reduced to the bounded model checking problem.

Theorem 1. *Let M be a model and α be a WECTL*KD state formula. Then, for some $s \in \iota$, $M, s \models \alpha$ iff $M, s \models_k \alpha$ for some $k \in \mathbb{N}$.*

4 Bounded Model Checking

In the section we present a propositional encoding of the bounded model checking problem for WECTL*KD and for weighted deontic interpreted systems (WDIS).

Let $M = (\iota, S, T, \mathcal{V}, d)$ be a model, α a WECTL*KD state formula, and $k \geqslant 0$ a bound. The BMC encoding relies on defining the following propositional formula: $[M, \alpha]_k := [M^{\alpha,\iota}]_k \wedge [\alpha]_{M,k}$, which is satisfiable if and only if $M \models_k \alpha$ holds.

The definition of $[M^{\alpha,\iota}]_k$ assumes that the states, the joint actions of M, and the sequence of weights associated to the joint actions are encoded symbolically, which is possible, since both the set of states and the set of joint actions are finite. Formally, let $\mathbf{c} \in Ag \cup \{\mathcal{E}\}$. Then, each state $s = (\ell_1, \ldots, \ell_n, \ell_{\mathcal{E}}) \in S$ is represented by a *symbolic state* which is a vector $\mathbf{w} = (\mathtt{w}_1, \ldots, \mathtt{w}_n, \mathtt{w}_{\mathcal{E}})$ of *symbolic local states*. Each symbolic local state $\mathtt{w}_{\mathbf{c}}$ is a vector of propositional variables (called *state variables*) whose length depends on the number of green and red local states of agent $\mathbf{c}$. Next, each joint action $a = (a_1, \ldots, a_n, a_{\mathcal{E}}) \in Act$ is represented by a *symbolic action* which is a vector $\mathbf{a} = (\mathtt{a}_1, \ldots, \mathtt{a}_n, \mathtt{a}_{\mathcal{E}})$ of *symbolic local actions*. Each symbolic local action $\mathtt{a}_{\mathbf{c}}$ is a vector of propositional variables (called *action variables*) whose length depends on the number of actions of agent $\mathbf{c}$. Next, each vector of weights associated to a joint action is represented by a *symbolic weight* which is a vector $\mathbf{d} = (\mathtt{d}_1, \ldots, \mathtt{d}_n, \mathtt{d}_{\mathcal{E}})$ of *symbolic local weights*. Each symbolic local weight $\mathtt{d}_{\mathbf{c}}$ is a vector of propositional variables (called *weight variables*), whose length depends on the weight functions $d_{\mathbf{c}}$.

Further, we assume that SV, AV and WV denote, respectively, the set of all the state variables, the set of all the action variables, and the set of all the weight variables such that $SV \cap AV = \emptyset$, $SV \cap WV = \emptyset$, and $AV \cap WV = \emptyset$. Next, we assume that $SV(\mathbf{w})$, $SV(\mathtt{w}_{\mathbf{c}})$, $AV(\mathbf{a})$, $AV(\mathtt{a}_{\mathbf{c}})$, and $WV(\mathbf{d})$ denote, respectively, the set of all the state variables occurring in the symbolic state $\mathbf{w}$, the set of all the state variables occurring in the local symbolic state $\mathtt{w}_{\mathbf{c}}$ of agent $\mathbf{c}$, the set of all the action variables occurring in the symbolic action $\mathbf{a}$, the set of all the action variables occurring in the local symbolic action $\mathtt{a}_{\mathbf{c}}$ of agent $\mathbf{c}$, and the set of all the weight variables occurring in the symbolic weight $\mathbf{d}$. Furthermore, we assume that NV denotes the set of propositional variables (called the *natural variables*) such that $SV \cap NV = \emptyset$, $AV \cap NV = \emptyset$, and $WV \cap NV = \emptyset$. Moreover, by $\mathbf{u} = (\mathtt{u}_1, \ldots, \mathtt{u}_y)$ we denote a vector of natural variables of length $y = max(1, \lceil log_2(k+1) \rceil)$, which we call a *symbolic number*, and by $NV(\mathbf{u})$ we denote the set of all the natural variables occurring in $\mathbf{u}$. Furthermore, we assume that:

- $PV = SV \cup AV \cup WV \cup NV$.
- $lit : \{0,1\} \times PV \to PV \cup \{\neg q \mid q \in PV\}$ is a function defined as: $lit(1, q) = q$ and $lit(0, q) = \neg q$.
- $V : PV \to \{0,1\}$ is a *valuation of propositional variables* (a *valuation* for short).
- r_w denotes the length of symbolic state, i.e. $\mathbf{w} = (\mathtt{w}_1, \ldots, \mathtt{w}_n, \mathtt{w}_{\mathcal{E}}) = (\mathtt{w}_1, \ldots, \mathtt{w}_{r_w})$.
- r_a denotes the length of a symbolic action, i.e. $\mathbf{a} = (\mathtt{a}_1, \ldots, \mathtt{a}_n, \mathtt{a}_{\mathcal{E}}) = (\mathtt{a}_1, \ldots, \mathtt{a}_{r_a})$,
- $r_d = r_{d1} + \ldots + r_{d(n+1)}$ denotes the length of a symbolic weight, i.e. $\mathbf{d} = (\mathtt{d}_1, \ldots, \mathtt{d}_n, \mathtt{d}_{\mathcal{E}}) = (\mathtt{d}_1^1, \ldots, \mathtt{d}_{r_{d1}}^1, \ldots, \mathtt{d}_1^{n+1}, \ldots, \mathtt{d}_{r_{d(n+1)}}^{n+1})$, where $r_{d1}, \ldots, r_{d(n+1)}$ denote lengths of local symbolic weights.

For every $r_w, r_a, r_d \in \mathbb{N}_+$, each valuation V induces the functions $\mathbf{S} : SV^{r_w} \to \{0,1\}^{r_w}$, $\mathbf{A} : AV^{r_a} \to \{0,1\}^{r_a}$, $\mathbf{W} : WV^{r_d} \to \mathbb{N}$, and $\mathbf{J} : NV^y \to \mathbb{N}$ defined in the following way: $\mathbf{S}((\mathsf{w}_1,\ldots,\mathsf{w}_{r_w})) = (V(\mathsf{w}_1),\ldots,V(\mathsf{w}_{r_w}))$, $\mathbf{A}((\mathsf{a}_1,\ldots,\mathsf{a}_{r_a})) = (V(\mathsf{a}_1),\ldots,V(\mathsf{a}_{r_a}))$, $\mathbf{W}((\mathsf{d}^1_1,\ldots,\mathsf{d}^1_{r_{d1}},\ldots,\mathsf{d}^{n+1}_1,\ldots,\mathsf{d}^{n+1}_{r_{d(n+1)}})) = \sum_{j=1}^{n+1}\sum_{i=1}^{r_{dj}} V(\mathsf{d}^j_i)\cdot 2^{i-1}$, $\mathbf{J}((\mathtt{u}_1,\ldots,\mathtt{u}_y)) = \sum_{i=1}^{y} V(\mathtt{u}_i)\cdot 2^{i-1}$.

Let $\mathbf{w}$ and $\mathbf{w}'$ be two different symbolic states such that $SV(\mathbf{w}) \cap SV(\mathbf{w}') = \emptyset$, $\mathbf{d}$ a symbolic weight, $\mathbf{a}$ a symbolic action, and $\mathbf{u}$ a symbolic number. We assume definitions of the following auxiliary Boolean formulae:

- $p(\mathbf{w})$ is a Boolean formula over $SV(\mathbf{w})$ that is true for a valuation V iff $p \in \mathcal{V}(\mathbf{S}(\mathbf{w}))$. It encodes a set of states of M in which $p \in \mathcal{PV}$ holds.
- $I_s(\mathbf{w}) = \bigwedge_{i=1}^{r_w} lit(s[i], \mathsf{w}_i)$. This Boolean formula is defined over $SV(\mathbf{w})$, and it encodes the state s of the model M.
- $H(\mathbf{w},\mathbf{w}') = \bigwedge_{i=1}^{r_w} \mathsf{w}_i \Leftrightarrow \mathsf{w}'_i$. This Boolean formula is defined over $SV(\mathbf{w}) \cup SV(\mathbf{w}')$, and it encodes equality of two symbolic states, i.e. it expresses that the symbolic states $\mathbf{w}$ and $\mathbf{w}'$ represent the same states.
- $H_{\mathbf{c}}(\mathbf{w},\mathbf{w}')$ is a Boolean formula over $SV(\mathbf{w}) \cup SV(\mathbf{w}')$ that is true for each valuation $V \in \{0,1\}^{SV}$ such that V satisfies $H_{\mathbf{c}}(\mathbf{w},\mathbf{w}')$ iff $\mathbf{S}(\mathbf{w}) \sim_{\mathbf{c}} \mathbf{S}(\mathbf{w}')$; it expresses that the local states of agent $\mathbf{c}$ are the same in the symbolic states $\mathbf{w}$ and $\mathbf{w}'$.
- $\mathcal{H}_a(\mathsf{a}_{\mathbf{c}})$ for $\mathbf{c} \in Ag \cup \{\mathcal{E}\}$ and $a \in Act_{\mathbf{c}} \cup \{\varepsilon\}$ is a Boolean formula over $AV(\mathsf{a}_{\mathbf{c}})$ that is true for each valuation $V \in \{0,1\}^{AV}$ such that V satisfies $\mathcal{H}_a(\mathsf{a}_{\mathbf{c}})$ iff $\mathbf{A}(\mathsf{a}_{\mathbf{c}}) = a$.
- $\mathcal{A}(\mathbf{a}) = \bigwedge_{a \in Act}(\bigwedge_{\mathbf{c}\in Ag(a)} \mathcal{H}_a(\mathsf{a}_{\mathbf{c}}) \vee \bigwedge_{\mathbf{c}\in Ag(a)} \mathcal{H}_\varepsilon(\mathsf{a}_{\mathbf{c}}))$, where $Ag(a) = \{\mathbf{c} \in Ag \cup \{\mathcal{E}\} \mid a \in Act_{\mathbf{c}}\}$. This formula is defined over $AV(\mathbf{a})$, and it encodes that each symbolic local action $\mathsf{a}_{\mathbf{c}}$ of $\mathbf{a}$ has to be executed by each agent in which it appears.
- $\mathcal{T}_{\mathbf{c}}(\mathtt{w}_{\mathbf{c}}, (\mathbf{a},\mathbf{d}), \mathtt{w}'_{\mathbf{c}})$ is defined over $SV(\mathtt{w}_{\mathbf{c}}) \cup SV(\mathtt{w}'_{\mathbf{c}})$, and is true for a valuation V iff $t_{\mathbf{c}}(\mathbf{S}(\mathtt{w}_{\mathbf{c}}), \mathbf{A}(\mathbf{a})) = \mathbf{S}(\mathtt{w}'_{\mathbf{c}})$. This Boolean formula encodes the local evolution function of agent $\mathbf{c}$.
- $\mathcal{T}(\mathbf{w}, (\mathbf{a},\mathbf{d}), \mathbf{w}') = \bigwedge_{\mathbf{c}\in Ag\cup\{\mathcal{E}\}} \mathcal{T}_{\mathbf{c}}(\mathtt{w}_{\mathbf{c}}, (\mathbf{a},\mathbf{d}), \mathtt{w}'_{\mathbf{c}}) \wedge \mathcal{A}(\mathbf{a})$. This Boolean formula is defined over $SV(\mathbf{w}) \cup SV(\mathbf{w}') \cup AV(\mathbf{a}) \cup WV(\mathbf{d})$, and it encodes the transition relation of the model M.
- $\mathcal{N}_j^{\sim}(\mathbf{u})$ is a formula over $NV(\mathbf{u})$ that is true for a valuation V iff $j \sim \mathbf{J}(\mathbf{u})$, where $\sim \in \{<, >, \leqslant, =, \geqslant\}$. This formula encodes that the value j is in the arithmetic relation $\sim$ with the value represented by the symbolic number $\mathbf{u}$.

Further, in order to define $[M^{\alpha,\iota}]_k$ we need to specify the number of k-paths of the model M that are sufficient to validate α. Let $p \in \mathcal{PV}$. To calculate the number, we define the following auxiliary function f_k : WECTL*KD $\to \mathbb{N}$: $f_k(\mathrm{true}) = f_k(\mathrm{false}) = 0$, $f_k(p) = f_k(\neg p) = 0$, $f_k(\varphi \wedge \phi) = f_k(\varphi) + f_k(\phi)$, $f_k(\varphi \vee \phi) = max\{f_k(\varphi), f_k(\phi)\}$, $f_k(\mathrm{X}_I\varphi) = f_k(\varphi)$, $f_k(\varphi \mathrm{U}_I \phi) = k \cdot f_k(\varphi) + f_k(\phi)$, $f_k(\varphi \mathrm{R}_I \phi) = (k+1) \cdot f_k(\phi) + f_k(\varphi)$, $f_k(\overline{\mathrm{C}}_\Gamma\varphi) = f_k(\varphi) + k$, $f_k(Y\varphi) = f_k(\varphi) + 1$, for $Y \in \{\overline{\mathrm{K}}_{\mathbf{c}}, \overline{\mathrm{D}}_\Gamma, \overline{\mathrm{E}}_\Gamma, \overline{\mathcal{O}}_{\mathbf{c}}, \underline{\widehat{\mathrm{K}}}^{\mathbf{c}_2}_{\mathbf{c}_1}, \mathrm{E}\}$.

Furthermore, we define the j-th symbolic k-path $\boldsymbol{\pi}_j$ as the sequence of *symbolic transitions*: $(\mathbf{w}_{0,j} \xrightarrow{\mathbf{a}_{1,j},\mathbf{d}_{1,j}} \mathbf{w}_{1,j} \xrightarrow{\mathbf{a}_{2,j},\mathbf{d}_{2,j}} \ldots \xrightarrow{\mathbf{a}_{k,j},\mathbf{d}_{k,j}} \mathbf{w}_{k,j}, \mathbf{u})$, where $\mathbf{w}_{i,j}$ are symbolic states, $\mathbf{a}_{i,j}$ are symbolic actions, $\mathbf{d}_{i,j}$ are symbolic weights, for $0 \leqslant i \leqslant k$ and $1 \leqslant j \leqslant f_k(\alpha)$, and $\mathbf{u}$ is the symbolic number, and we define the following auxiliary Boolean formulae. Let $\mathbf{w}$ and $\mathbf{w}'$ be two different symbolic states, $\mathbf{d}$ a symbolic weighs, $\mathbf{a}$ a symbolic action, $\mathbf{u}$ a symbolic number, I an interval in $\mathbb{N}$ of the form: $[a,b)$ and $[a,\infty)$, for $a,b \in \mathbb{N}$ and $a \neq b$, and $right(I)$ denote the right end of the interval I.

- $\mathcal{L}_k^l(\boldsymbol{\pi}_n) := \mathcal{N}_l^{=}(\mathbf{u}_n) \wedge H(\mathbf{w}_{k,n}, \mathbf{w}_{l,n})$.
- $\mathcal{B}_k^I(\boldsymbol{\pi}_n)$ is defined over $\bigcup_{i=1}^{k} WV(\mathbf{d}_{i,n})$, and is true for a valuation V iff $\sum_{i=1}^{k} \mathbf{W}(\mathbf{d}_{i,n}) \leqslant right(I)$. This Boolean formula encodes that the weight represented by the sequence $\mathbf{d}_{1,n}, \ldots, \mathbf{d}_{k,n}$ is less than or equal to $right(I)$.
- $\mathcal{D}_{a,b}^I(\boldsymbol{\pi}_n)$ for $a \leqslant b$: if $a < b$, then the formula encodes that the weight represented by the sequence $\mathbf{d}_{a+1,n}, \ldots, \mathbf{d}_{b,n}$ belongs to the interval I, i.e. the formula is true for a valuation V iff $\sum_{i=a+1}^{b} \mathbf{W}(\mathbf{d}_{i,n}) \in I$; otherwise, i.e. if $a = b$, then $\mathcal{D}_{a,b}^I(\boldsymbol{\pi}_n)$ is true for a valuation V iff $0 \in I$.
- $\mathcal{D}_{a,b;c,d}^I(\boldsymbol{\pi}_n)$ for $a \leqslant b$ and $c \leqslant d$:
  1. if $a < b$ and $c < d$, then the formula encodes that the weight represented by the sequences $\mathbf{d}_{a+1,n}, \ldots, \mathbf{d}_{b,n}$ and $\mathbf{d}_{c+1,n}, \ldots, \mathbf{d}_{d,n}$ belongs to the interval I, i.e. the formula is true for a valuation V iff $\sum_{i=a+1}^{b} \mathbf{W}(\mathbf{d}_{i,n}) + \sum_{i=c+1}^{d} \mathbf{W}(\mathbf{d}_{i,n}) \in I$;
  2. if $a = b$ and $c < d$, then the formula encodes that the weight represented by the sequence $\mathbf{d}_{c+1,n}, \ldots, \mathbf{d}_{d,n}$ belongs to the interval I, i.e. the formula is true for a valuation V iff $\sum_{i=c+1}^{d} \mathbf{W}(\mathbf{d}_{i,n}) \in I$;
  3. if $a < b$ and $c = d$, then the formula encodes that the weight represented by the sequence $\mathbf{d}_{a+1,n}, \ldots, \mathbf{d}_{b,n}$ belongs to the interval I, i.e. the formula is true for a valuation V iff $\sum_{i=a+1}^{b} \mathbf{W}(\mathbf{d}_{i,n}) \in I$;
  4. if $a = b$ and $c = d$, then $\mathcal{D}_{a,b;c,d}^I(\boldsymbol{\pi}_n)$ is true for a valuation Viff $0 \in I$.

The formula $[M^{\alpha,\iota}]_k$, which encodes the unfolding of the transition relation of the model M $f_k(\alpha)$-times to the depth k, is defined as follows:

$$[M^{\alpha,\iota}]_k := \bigvee_{s \in \iota} I_s(\mathbf{w}_{0,0}) \wedge \bigwedge_{j=1}^{f_k(\alpha)} \bigvee_{l=0}^{k} \mathcal{N}_l^{=}(\mathbf{u}_j) \wedge \bigwedge_{j=1}^{f_k(\alpha)} \bigwedge_{i=0}^{k-1} \mathcal{T}(\mathbf{w}_{i,j}, (\mathbf{a}_{i,j}, \mathbf{d}_{i,j}), \mathbf{w}_{i+1,j})$$

where $\mathbf{w}_{i,j}$, $\mathbf{a}_{i,j}$, $\mathbf{d}_{i,j}$, and $\mathbf{u}_j$ are, respectively, symbolic states, symbolic actions, symbolic weights, and symbolic numbers, for $0 \leqslant i \leqslant k$ and $1 \leqslant j \leqslant f_k(\alpha)$.

Then, the next step is a translation of a WECTL*KD state formula α to a propositional formula $[\alpha]_{M,k}$. In order to define $[\alpha]_{M,k}$, we have to know how to divide the set A of k-paths such that $|A| = f_k(\alpha)$ into subsets needed for translating the subformulae of α. To accomplish this goal we use some auxiliary functions that were defined in [24]. We recall their definitions now. First, the relation $\prec$ is defined on the power set of $\mathbb{N}$ as follows: $A \prec B$ iff for all natural numbers x and y, if $x \in A$ and $y \in B$, then $x < y$. Further, let $A \subset \mathbb{N}$ be a finite non-empty set, and $n, m \in \mathbb{N}$, where $m \leqslant |A|$. Then, $g_l(A,m)$ denotes the subset B of A such that $|B| = m$ and $B \prec A \setminus B$, $g_r(A,m)$ denotes the subset C of A such that $|C| = m$ and $A \setminus C \prec C$, $g_s(A)$ denotes the set $A \setminus \{min(A)\}$, and if n divides $|A| - m$, then $hp(A,m,n)$ denotes the sequence $(B_0, \ldots, B_n)$ of subsets of A such that $\bigcup_{j=0}^{n} B_j = A$, $|B_0| = \ldots = |B_{n-1}|$, $|B_n| = m$, and $B_i \prec B_j$ for every $0 \leqslant i < j \leqslant n$. Now let $h_k^{\mathrm{U}}(A,m) := hp(A,m,k)$ and $h_k^{\mathrm{R}}(A,m) := hp(A,m,k+1)$. Note that if $h_k^{\mathrm{U}}(A,m) = (B_0, \ldots, B_k)$, then $h_k^{\mathrm{U}}(A,m)(j)$ denotes the set B_j, for every $0 \leqslant j \leqslant k$. Similarly, if $h_k^{\mathrm{R}}(A,m) = (B_0, \ldots, B_{k+1})$, then $h_k^{\mathrm{R}}(A,m)(j)$ denotes the set B_j, for every $0 \leqslant j \leqslant k+1$.

The functions g_l and g_r are used in the translation of the formulae with the main connective being either conjunction or disjunction: for a given WECTL*KD formula $\varphi \wedge \psi$, if the set A is used to translate this formula, then the set $g_l(A, f_k(\varphi))$ is used to translate the subformula φ and the set $g_r(A, f_k(\psi))$ is used to translate the subformula ψ; for a given WECTL*KD formula $\varphi \vee \psi$, if the set A is used to translate this formula, then the set $g_l(A, f_k(\varphi))$ is used to translate the subformula φ and the set $g_l(A, f_k(\psi))$ is used to translate the subformula ψ.

The function g_s is used in the translation of the formulae with the main connective $\mathrm{Q} \in \{\mathrm{E}, \overline{\mathrm{K}}_{\mathbf{c}}, \overline{\mathrm{D}}_{\Gamma}, \overline{\mathrm{E}}_{\Gamma}, \overline{\mathcal{O}}_{\mathbf{c}}, \underline{\widehat{\mathrm{K}}}_{\mathbf{c_1}}^{\mathbf{c_2}}\}$: for a given WECTL*KD formula $\mathrm{Q}\varphi$, if the set A is used to translate this formula, then the path of the number $min(A)$ is used to translate the operator Q and the set $g_s(A)$ is used to translate the subformula φ.

The function h_k^{U} is used in the translation of subformulae of the form $\varphi \mathrm{U}_I \psi$: if the set A is used to translate the subformula $\varphi \mathrm{U}_I \psi$ at the symbolic k-path $\boldsymbol{\pi}_n$ (with the starting point m), then for every j such that $m \leqslant j \leqslant k$, the set $h_k^{\mathrm{U}}(A, f_k(\psi))(k)$ is used to translate the formula ψ along the symbolic path $\boldsymbol{\pi}_n$ with starting point j; moreover, for every i such that $m \leqslant i < j$, the set $h_k^{\mathrm{U}}(A, f_k(\psi))(i)$ is used to translate the formula φ along the symbolic path $\boldsymbol{\pi}_n$ with starting point i. Notice that if k does not divide $|A| - d$, then $h_k^{\mathrm{U}}(A, d)$ is undefined. However, for every set A such that $|A| = f_k(\varphi \mathrm{U}_I \psi)$, it is clear from the definition of f_k that k divides $|A| - f_k(\psi)$.

The function h_k^{R} is used in the translation of subformulae of the form $\varphi \mathrm{R}_I \psi$: if the set A is used to translate the subformula $\varphi \mathrm{R}_I \psi$ along a symbolic k-path $\boldsymbol{\pi}_n$ (with the starting point m), then for every j such that $m \leqslant j \leqslant k$, the set $h_k^{\mathrm{R}}(A, f_k(\varphi))(k+1)$ is used to translate the formula φ along the symbolic paths $\boldsymbol{\pi}_n$ with starting point j; moreover, for every i such that $m \leqslant i \leqslant j$, the set $h_k^{\mathrm{R}}(A, f_k(\varphi))(i)$ is used to translate the formula ψ along the symbolic path $\boldsymbol{\pi}_n$ with starting point i. Notice that if $k+1$ does not divide $|A| - 1$, then $h_k^{\mathrm{R}}(A, p)$ is undefined. However, for every set A such that $|A| = f_k(\varphi \mathrm{R}_I \psi)$, it is clear from the definition of f_k that $k+1$ divides $|A| - f_k(\varphi)$.

Let α be a WECTL*KD state formula and $A \subset \mathbb{N}_+$ be a set of numbers of symbolic k-paths such that $|A| = f_k(\alpha)$. If $n \in \mathbb{N} \setminus A$ and $0 \leqslant m \leqslant k$, then by $\langle \alpha \rangle_k^{[m,n,A]}$ we denote the translation of a WECTL*KD state formula α at the symbolic state $\mathbf{w}_{m,n}$ by using the set A. Let φ be a WECTL*KD path formula and $B \subset \mathbb{N}_+$ be a set of numbers of symbolic k-paths such that $|B| = f_k(\varphi)$. If $n \in \mathbb{N}_+ \setminus A$ and $0 \leqslant m \leqslant k$, then by $[\varphi]_k^{[m,n,A]}$ we denote the translation of a WECTL*KD path formula φ along the symbolic k-path $\boldsymbol{\pi}_n$ with starting point m by using the set A. Furthermore, we define $[\alpha]_{M,k}$ as $\langle \alpha \rangle_k^{[0,0,F_k(\alpha)]}$, where $F_k(\alpha) = \{j \in \mathbb{N} \mid 1 \leqslant j \leqslant f_k(\alpha)\}$, and:

$\langle \mathrm{true} \rangle_k^{[m,n,A]} := \mathrm{true}$, $\langle \mathrm{false} \rangle_k^{[m,n,A]} := \mathrm{false}$,

$\langle p \rangle_k^{[m,n,A]} := p(\mathbf{w}_{m,n})$, $\quad \langle \neg p \rangle_k^{[m,n,A]} := \neg p(\mathbf{w}_{m,n})$,

$$\begin{aligned}
\langle \alpha \wedge \beta \rangle_k^{[m,n,A]} &:= \langle \alpha \rangle_k^{[m,n,g_l(A,f_k(\alpha))]} \wedge \langle \beta \rangle_k^{[m,n,g_r(A,f_k(\beta))]},\\
\langle \alpha \vee \beta \rangle_k^{[m,n,A]} &:= \langle \alpha \rangle_k^{[m,n,g_l(A,f_k(\alpha))]} \vee \langle \beta \rangle_k^{[m,n,g_r(A,f_k(\beta))]},\\
\langle \mathrm{E}\varphi \rangle_k^{[m,n,A]} &:= H(\mathbf{w}_{m,n}, \mathbf{w}_{0,\min(A)}) \wedge [\varphi]_k^{[0,\min(A),g_s(A)]},\\
[\alpha]_k^{[m,n,A]} &:= \langle \alpha \rangle_k^{[m,n,A]},\\
[\varphi \wedge \psi]_k^{[m,n,A]} &:= [\varphi]_k^{[m,n,g_l(A,f_k(\varphi))]} \wedge [\psi]_k^{[m,n,g_r(A,f_k(\psi))]},\\
[\varphi \vee \psi]_k^{[m,n,A]} &:= [\varphi]_k^{[m,n,g_l(A,f_k(\varphi))]} \vee [\psi]_k^{[m,n,g_r(A,f_k(\psi))]},
\end{aligned}$$

$$
\begin{aligned}
[\mathrm{X}_I\alpha]_k^{[m,n,A]} &:= \begin{cases} \mathcal{D}^I_{m,m+1}(\boldsymbol{\pi}_n) \wedge [\alpha]_k^{[m+1,n,A]}, & \text{if } m<k \\ \bigvee_{l=0}^{k-1}(\mathcal{D}^I_{l,l+1}(\boldsymbol{\pi}_n) \wedge \mathcal{L}^l_k(\boldsymbol{\pi}_n) \wedge [\alpha]_k^{[l+1,n,A]}), & \text{if } m=k \end{cases} \\
[\alpha\mathrm{U}_I\beta]_k^{[m,n,A]} &:= \bigvee_{j=m}^{k}(\mathcal{D}^I_{m,j}(\boldsymbol{\pi}_n) \wedge [\beta]_k^{[j,n,h_k^{\mathrm{U}}(k)]} \wedge \bigwedge_{i=m}^{j-1}[\alpha]_k^{[i,n,h_k^{\mathrm{U}}(i)]})\vee \\
&\quad (\bigvee_{l=0}^{m-1}(\mathcal{L}^l_k(\boldsymbol{\pi}_n)) \wedge \bigvee_{j=0}^{m-1}(\mathcal{N}_j^{>}(\mathbf{u}_n) \wedge [\beta]_k^{[j,n,h_k^{\mathrm{U}}(k)]}\wedge \\
&\quad \bigvee_{l=0}^{m-1}(\mathcal{N}_l^{=}(\mathbf{u}_n) \wedge \mathcal{D}^I_{m,k;l,j}(\boldsymbol{\pi}_n))\wedge \\
&\quad \bigwedge_{i=0}^{j-1}(\mathcal{N}_i^{>}(\mathbf{u}_n) \to [\alpha]_k^{[i,n,h_k^{\mathrm{U}}(i)]})) \wedge \bigwedge_{i=m}^{k}[\alpha]_k^{[i,n,h_k^{\mathrm{U}}(i)]}), \\
[\alpha\mathrm{R}_I\beta]_k^{[m,n,A]} &:= \bigvee_{j=m}^{k}(\mathcal{D}^I_{m,j}(\boldsymbol{\pi}_n) \wedge [\alpha]_k^{[j,n,h_k^{\mathrm{R}}(k)]} \wedge \bigwedge_{i=m}^{j}[\beta]_k^{[i,n,h_k^{\mathrm{R}}(i)]})\vee \\
&\quad (\bigvee_{l=0}^{m-1}(\mathcal{L}^l_k(\boldsymbol{\pi}_n)) \wedge \bigvee_{j=0}^{m-1}(\mathcal{N}_j^{>}(\mathbf{u}_n) \wedge [\alpha]_k^{[j,n,h_k^{\mathrm{R}}(k)]}\wedge \\
&\quad \bigvee_{l=0}^{m-1}(\mathcal{N}_l^{=}(\mathbf{u}_n) \wedge \mathcal{D}^I_{m,k;l,j}(\boldsymbol{\pi}_n))\wedge \\
&\quad \bigwedge_{i=0}^{j}(\mathcal{N}_i^{>}(\mathbf{u}_n) \to [\beta]_k^{[i,n,h_k^{\mathrm{R}}(i)]}) \wedge \bigwedge_{i=m}^{k}[\beta]_k^{[i,n,h_k^{\mathrm{R}}(i)]})\vee \\
&\quad (\neg\mathcal{B}^I_k(\boldsymbol{\pi}_n) \wedge \bigwedge_{j=m}^{k}(\mathcal{D}^I_{m,j}(\boldsymbol{\pi}_n) \to [\beta]_k^{[j,n,h_k^{\mathrm{R}}(k)]}))\vee \\
&\quad (\mathcal{B}^I_k(\boldsymbol{\pi}_n) \wedge \bigwedge_{j=m}^{k}(\mathcal{D}^I_{m,j}(\boldsymbol{\pi}_n) \to [\beta]_k^{[j,n,h_k^{\mathrm{R}}(k)]})\wedge \\
&\quad \bigvee_{l=0}^{k-1}[\mathcal{L}^l_k(\boldsymbol{\pi}_n) \wedge \bigwedge_{j=l}^{k}(\mathcal{D}^I_{m,k;l,j}(\boldsymbol{\pi}_n) \to [\beta]_k^{[j,n,h_k^{\mathrm{R}}(k)]})]), \\
\langle \overline{\mathrm{K}}_{\mathbf{c}}\alpha \rangle_k^{[m,n,A]} &:= \bigvee_{s\in\iota} I_s(\mathbf{w}_{0,min(A)}) \wedge \bigvee_{j=0}^{k}([\alpha]_k^{[j,min(A),g_s(A)]} \\
&\quad \wedge H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,min(A)})), \\
\langle \overline{\mathrm{D}}_{\Gamma}\alpha \rangle_k^{[m,n,A]} &:= \bigvee_{s\in\iota} I_s(\mathbf{w}_{0,min(A)}) \wedge \bigvee_{j=0}^{k}([\alpha]_k^{[j,min(A),g_s(A)]}\wedge \\
&\quad \bigwedge_{\mathbf{c}\in\Gamma} H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,min(A)})), \\
\langle \overline{\mathrm{E}}_{\Gamma}\alpha \rangle_k^{[m,n,A]} &:= \bigvee_{s\in\iota} I_s(\mathbf{w}_{0,min(A)}) \wedge \bigvee_{j=0}^{k}([\alpha]_k^{[j,min(A),g_s(A)]}\wedge \\
&\quad \bigvee_{\mathbf{c}\in\Gamma} H_{\mathbf{c}}(\mathbf{w}_{m,n}, \mathbf{w}_{j,min(A)})), \\
\langle \overline{\mathrm{C}}_{\Gamma}\alpha \rangle_k^{[m,n,A]} &:= \left\langle \bigvee_{j=1}^{k}(\overline{\mathrm{E}}_{\Gamma})^j\alpha \right\rangle_k^{[m,n,A]}.
\end{aligned}
$$

Theorem 2. *Let M be a model and α be a WECTL*KD state formula. Then for some $s \in \iota$ and for every $k \in \mathbb{N}$, $M, s \models_k \alpha$ if, and only if, the propositional formula $[M,\alpha]_k$ is satisfiable.*

5 Conclusions

We have proposed the SAT-based BMC for WECTL*KD and for WDIS. The BMC of the WDIS may also be performed by means of Ordered Binary Diagrams (OBDD). This will be explored in the future. Moreover, our future work include an implementation of the algorithm presented here, a careful evaluation of experimental results to be obtained, and a comparison of the OBDD- and SAT-based BMC method for WDIS.

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