Contextual Effects on Vagueness and the Sorites Paradox:
A Preliminary Study

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Abstract. Context is a prominent theme in accounts of the semantics of vagueness. Contextual theories of vagueness have been motivated in part by a desire to disarm skeptical arguments that make use of vagueness; another motive is the hope that they might help to resolve the Sorites Paradox. This paper is intended as part of a larger project, which aims at describing and comparing the leading theories that have been presented, to bring out the ways in which contextual theories have informed the semantic phenomenon of vagueness, and to make strategic assesments and suggestions concerning the crucial case of adjectives. It concentrates on Hans Kamp's 1975 theory, and two recent contributions by Haim Gaifman and Joseph Halpern. The specific goal of this paper is to examine the role of context in the semantics of certain adjectives, and assess to what extent context can help to solve what I take to be the most challenging version of the Sorites Paradox.

1   Introduction

Since it began to attract the attention of philosophical logicians in the late 1960s, the Sorites paradox has puzzled many authors, and has inspired a wide spectrum of proposed solutions. After nearly fifty years of work on the topic, it still seems that we lack a satisfactory methodology for attacking this problem. The main positive contribution of this paper is to propose such a methodology; at the same time I provide a survey of some of the proposals for bringing context into the semantics of vagueness and the solution of the Sorites.

2   Kamp's semantic theory of adjectives

Contextual theories of vagueness, formulated using the apparatus of formal semantics, go back at least to (Goguen, Jr., 1969), but perhaps the most influential (and, in my opinion, the most rewarding) early work of this sort is (Kamp, 1975).[1] Kamp's theory is confined to adjectives: adjectives in predicative position, like 'tall' in 'That man is tall', are a notorious source of vagueness. Kamp's

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[1] (Fine, 1975) appeared in the same year as Kamp's paper. Fine's paper is important for the theory of vagueness, but has little or nothing to say about context.

chief goal in the paper seems to have been to give an account of the semantics of adjectives according to which the meaning of the comparative would be determined from the meaning of the absolute form. But in the course of providing such an account he formulates a detailed model theory for vague predicates, a theory which also provides for contextual sharpenings of (reductions in the vagueness of) predicates.

Kamp's theory treats contexts as unanalyzed primitives. A contextual "devaguification"[2] is a function from contexts to (partial) models.[3] Kamp does not have a lot to say about the informational side of contexts—about what factors in practice help us to devaguify adjectives. But he does illustrate how the informational context can influence the interpretation of various sorts of adjectives. He notes that if the adjective modifies a noun, the noun can play an important part in these contextual effects.[4] And he indicates that the interests of the speakers—what issues they are trying to resolve—can also be important.

(Kamp, 1975), then, provides two important ideas: the use of partial models (with a supervaluational interpretation) for interpreting vagueness, and the idea that the appropriate partial model can be a function of context.

3   The Sorites Paradox

The Sorites Paradox (or the Paradox of the Heap), deserves to be counted among the more profound logical paradoxes, and has been known since ancient times. The paradox is well known, and I will present it in a brief and abstract way here—the chief purpose of this section is merely to establish some terminology.

It takes the form either of an induction or of a finite chain of applications of *modus ponens*. Given a vague predicate P, you set up the paradox by finding a series $P(a_1), \ldots, P(a_n)$ of applications of the predicate meeting the following conditions.

(1) For all i, $1 \leq i < n$, $P(a_i)$ and $P(a_{i+1})$ are indistinguishable for the purposes at hand, so that (apparently) the conditionals $P(a_i) \rightarrow P(a_{i+1})$ are all true, for $1 \leq i < n$.

(2) $P(a_1)$ is definitely true.

(3) $P(a_n)$ is definitely false.

In this form of the paradox (which depends on $n-1$ *modus ponens* applications), I'll call (1) the *major premises* of the paradoxical argument, (2) the *base premise*, and (3) the *absurd conclusion*.

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[2] Awkward as it is, I prefer this term to Kamp's 'disambiguation'. Ambiguity is not the same as vagueness and it is good policy to keep the two distinct.

[3] Kamp's paper is rich in technical detail, and actually develops a sophisticated formalism for formalizing vagueness and natural language adjectives. Since what I say will not rely on these technical details, I am suppressing them here.

[4] For instance, 'That author is famous' is (apparently), less vague than 'that is a famous author', since it rules out cases in which the author is famous, but not famous as an author.

In the inductive form, you define a numerical predicate Q with analogous properties. (Indeed, if we express the dependence of a_i on i in the above example as an explicit numerical function, so that $a_i = f(i)$, we can let $Q = \lambda n P(f(n))$.) In particular, Q is chosen so that the following universal generalization appears be plausible.

(1′) $\forall n[Q(n) \rightarrow Q(n+1)]$.

I'll call condition (1′) the *inductive premise* of this form of the paradox.

4 Kamp on the Sorites

Kamp's 1975 paper doesn't mention the Sorites Paradox, but it is impossible to develop a theory of vagueness to the point the paper reaches without being aware of the threat of this paradox.[5] Six years later, in (Kamp, 1981), Kamp addresses the paradox directly—and the results are discouraging.

Kamp had adopted and improved van Fraassen's "supervaluational" theory of satisfaction in partial models,[6] and had originally thought (as I myself had) that this approach provided a plausible or at least defensible account of the Sorites paradox. However, the fact that on this theory

(4) $\bigvee \{P(a_i) \wedge \neg P(a_{i+1}) \ / \ 1 \leq i < n\}$ must be definitely true

(in the conditional version of the paradox), and that

(5) $\exists n[Q(n) \wedge \neg Q(n+1)]$ must be definitely true

(in the inductive version of the paradox), came to seem more and more worrisome to Kamp.

Although you might be able to convince yourself that these consequences of the supervaluational approach (which are closely connected to the conservatism of this approach with respect to classical logic) are harmless in the case of adjectives like 'tall', it really seems to be impossible to reconcile them with the behavior of what Kamp (following Dummett) calls "observational predicates." These are predicates with semantic connections to the outcomes of acts of perception or observation.

The problematic status of observational predicates was pointed out in (Dummett, 1975) and (Wright, 1975). (Kamp indicates that the point was originally due to Dummett.) In this connection, the source of the difficulty consists in the fact that an observational adjective or adjective phrase P ('red', or if you prefer, 'apparently red') will satisfy a condition to the following effect:

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[5] Since Kamp's theory is presented model-theoreticaly, the threat of the paradox doesn't come in the form of a worry that the reasoning endorsed by the theory will in itself lead to a contradiction. The worry is that adding axioms that seem, according to common sense, to be plainly true, will induce a contradiction within the theory.

[6] See, for instance, (Van Fraassen, 1966, van Fraassen, 1969).

(6) If a and b are observationally indistinguishable, then $P(a) \leftrightarrow P(b)$ is definitely true.

In Sorites examples where adjacent instances are observationally indistinguishable, (6) is incompatible with (4) (in the conditional versions of the paradox).[7] So it seems that a supervaluational theory is incompatible with the semantic properties of some adjectives. If difficulties of this sort were restricted to supervaluational approaches, it would simply be evidence against these theories. However, the objections to other logical theories of vagueness are equally powerful.

In (Kamp, 1981), Kamp formalizes a theory of the context for adjectives according to which contexts are sets of sentences that may or may not be coherent; the idea is that Dummett-like color spectrum examples may gradually force an agent to occupy an incoherent context.

I won't go into the details of this theory in this paper (which is intended as a brief version of a longer study). I'll merely indicate why I think that an approach of this sort is unsatisfactory.[8]

Contexts consisting of sets of sentences (or of sets of propositions) are appropriate for representing presuppositions. But, since (Stalnaker, 1975), the idea that such contexts have direct effects on the semantic content of expressions has lost all or most of its attractiveness. Linguistic evidence and considerations such as theoretical simplicity provide a great deal of support for this view.[9]

Presuppositional contexts can have indirect effects on content (for instance, by indicating the appropriateness of a context that does affect content). And, in dynamic theories of semantics, the distinction between presuppositional effects on appropriateness and contextual effects on content can become blurred.

Now, formulations of the Sorites along the lines I sketched in Section 3 do not seem to involve dynamic operators in any essential way. (While they can be formulated, for instance, using dynamic conjunction, the paradoxical aspects seem to remain if they are formulated using only static operators.) Because of this, I assume that it is best to seek for a solution within a static logical framework—dynamic logic will only complicate the issues with which we have to deal.

Secondly, although, as I said, presuppositional contexts can indirectly affect content, they can do so only by interacting with independent contextual effects on content. Since the Sorites (like any paradox that results in an explicit contradiction) is plainly a problem of content, the core of any contextual solution has to lie in the identification of contextual effects on content that could resolve the

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[7] The example that is usually used for this purpose is a colored line, extending a great distance, graduated so that any local area appears to be uniform in color, and with distinctly different colors at the extremes.

[8] From conversations with Kamp at the time that the paper was written, I have the impression that Kamp himself wasn't entirely happy with it, either.

[9] See (Beaver, 1996) for a survey of work on presupposition up to 1996, and an assessment of the status of semantic theories of presupposition as of that time.

paradox. This is my reason for abandoning solutions along the lines that Kamp explored in his 1975 paper.

5   Fast forward

This section will draw attention to the leap I now take over many years and pages. It marks a large gap in my account of the literature.

In the 1980s, vagueness and the Sorites paradox came to be perceived as a central problem in metaphysics, and received growing attention in the philosophical literature. See (Keefe and Smith, 1997) for a survey and collection from the late 1990s, and (Keefe, 2000) for more recent material relating to these developments. Delia Graff's work, beginning with her 1997 dissertation, (Graff, 1997), and continued in (Graff, 2000, Graff, 2001, Graff, 2003), deserves particular mention in this regard.

In this paper, I will settle for a discontinuous history, and will pass over this period to two more recent contributions by non-metaphysicians. Actually, although this leaves out interesting and important discussion of the philosophical issues, it omits much less if the purpose is to focus on the incorporation of context into a logical formalization that addresses the Sorites Paradox.

6   Gaifman's proposal

In (Gaifman, 2002), available online as of the date of this paper but as yet (as far as I know) unpublished, Haim Gaifman presents a fully formalized theory of vagueness that presents the role of context in a different light. The paper also presents an extended discussion of the phenomena of vagueness, containing several new insights.

Gaifman proposes that a class of predicates, the "tolerant predicates," including Dummett's observational predicates, satisfy a version of (6), which he formalizes as follows.

(7) $N_P(x, y) \rightarrow [P(x) \leftrightarrow P(y)]$

Here, N_P is an appropriate "sufficiently near" relation for the predicate P: if P is the predicate of being red, for instance, N_P relates two things if and only if they are observationally indistinguishable with respect to redness.

Citing (Kofka, 1922), who in turn refers to experiments performed by others late in the 19th century, as well as (Raffman, 1994), Gaifman suggests that applications of some vague predicates, at least, are relative to a comparison class. In the psychological experiments that Gaifman cites, a subject is presented with several stimuli, and asked to compare some of them. The judgments depend not only on the items compared, but on the comparison class.

Here, it is easy to see what the context is: it will consist of the examples that are presented (and, presumably, attended to) as the comparison is made. In other cases (for instance, in evaluating sentences like 'If the car is red, we'll have

to repaint it' or the sentence 'Tomatoes are red'), it may be harder to reconstruct a comparison class.

Gaifman formalizes the context as a list of compared items (he feels that in some cases the order in which the items are presented can be important) and proposes a contextual logic in which context is formalized using modal operators `[C]`, where C is a context. Thus, the formula `[C]`A means that A holds in context C.[10]

Gaifman reformalizes the Tolerance Condition, (6), in the following way to take context into account.

$\quad$ (8) $N_P(x,y) \rightarrow$ `[C]`$[P(x) \leftrightarrow P(y)]$

This idea is well motivated by the comparison experiments, and by the interpretation that psychologists like Kofka made of these results. And it makes good logical sense to say that whether or not two instances are distinguishable depends on the comparison class. Dummett's account of indistinguishability may overlook a hidden variable—whether two presented objects are perceptually distinguishable does not in general depend only on the two objects. Even though, compared just with each other, a and b may appear equally red, introducing a third object c that (compared just with b) appears to be just as red as b but that perceptably differs from a in redness seems to create a context in which a is now (indirectly) perceptually different from c in redness.[11]

As Gaifman points out, this idea disarms the color-spectrum versions of the Sorites that Kamp and others found so disturbing. The disarmament strategy depends on formalizing the conditional version of paradoxical situation so that the major premise conditionals $P(x) \rightarrow P(y)$ give way to `[`x,y`]`$[P(x) \rightarrow P(y)]$, whereas a conjunction like $P(a_1) \wedge P(a_n)$ gives way to `[C]`$[P(a_1) \wedge P(a_n)]$, where C is a list containing at least a_1 and a_n. It's plausible enough that in this example all these formulas are true.

Gaifman's idea raises a number of issues, some of them calling for extended reflection. (1) There is the question of motivating this use of context, and of how in general to motivate a hypothesized contextual effect on content. Since in this case there is some independent plausibility to the psychological motivation, I pass this over here. (2) We may ask whether Gaifman's idea actually solves the Sorites paradox. Of course, Gaifman, like Kamp, is presenting his theory model-theoretically, so consistency is not in doubt. But the question is whether Sorites-like examples could be presented that force you, using Gaifman's formalization policies, to represent sentences that intuitively are true in a way that makes them false. I will return to this question later in this paper; the main issue is how to make a plausible case that a logic and formalization strategy does in fact

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[10] Gaifman seems to be unaware of the AI literature on the logic of context; he thinks that this formal proposal is new.

[11] However, if one takes this point seriously, it seems as if Gaifman's (8) should now give way to

$\quad$ (8$'$) `[C]`$[N_P(x,y) \rightarrow [P(x) \leftrightarrow P(y)]]$.

solve a paradox. (3) Kamp began his investigation with a project having to do with the semantics of adjectives. That project tends to be forgotten in the later work on vagueness, but it is worthwhile to restore it to a proper place in the inquiry. For instance, it provides us with additional constraints that may help us to evaluate proposed solution: a solution that fails to produce a way of defining the comparative form in terms of the absolute form is not as successful, all other things equal, as one that does provide such a definition.

7 Halpern's proposal

Joseph Halpern proposes a formalization that develops and formalizes the intuition that subjectivity plays an important role in the semantics of vagueness. The formalization proposed in (Halpern, 2004) uses a multimodal logic of the sort described in (Fagin *et al.*, 1995). The semantics of multimodal logic treats possible worlds as global states, which are combinations of the (local) states of agents and another state, representing the objective facts in the world: $w = \langle o, s_1, \ldots, s_n, \rangle$, where s_i ($1 \leq i \leq n$) is the state of agent i and s_o is the objective component. In multimodal epistemic logic, the truth of a modal-free sentence in w will depend only on s_o, and an agent state is determined by a two-place relation over worlds. The truth of a sentence involving a modal operator $[a]$ (tracking, say, what agent a knows) may depend on a's local state, which is modeled using a 2-place relation over worlds. To understand the application to vagueness, we can resrict attention to the case where $n=1$; here a world w is a pair $\langle o, s \rangle$.

Suppose that we include all the subjective aspects affecting vagueness (the agent's ability to perceptually discriminate objective states, the choice of a reference class, the agent's possibly idiosyncratic interpretation of an adjective, and any other such factors) in the subjective state s. And suppose that all the objective factors affecting vagueness (if there are any) are included in the objective state o, and that these things are built into a system of possible worlds. This will yield a model for a logic of vagueness.

Halpern's logic has two modal operators:[12] $[R]$ (tracking what the agent reports) and $[D]$ (tracking what is objectively definite). Both are interpreted as usual, using relations $\sim_s$ and $\sim_o$ over worlds. $\langle o, s \rangle \sim_o \langle o', s' \rangle$ iff $o = o'$ and $\langle o, s \rangle \sim_s \langle o', s' \rangle$ iff $s = s'$. The operators are interpreted in the standard way; $\langle M, w \rangle \models [D]\phi$ iff $\langle M, w' \rangle \models \phi$ for all w' such that $w \sim_s w'$, and $\langle M, w' \rangle \models [R]\phi$ iff $\langle M, w' \rangle \models \phi$ for all w' such that $w \sim_o w'$ and $w \in P$, where P is a designated set of "initially plausible worlds."

Take the color spectrum sorites, with 3 propositions. P_i says that the ith sample is red. Say that the objective state records the physical light-reflecting properties of the samples. The samples differ in objective ways that make higher-numbered ones closer to orange and further from red than the lower-numbered ones. The interpretation of 'red' depends on both the subjective and subjective components of a world. Suppose that the agent can perceive that the first sample

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[12] Two operators for each agent, but I am considering the single-agent case here.

is red and that the third is not; in all w' such that $w \sim_s w'$, P_1 is true and P_3 is false. But $w \sim_s w_1$ and P_2 is true in w_1; $w \sim_s w_2$ and P_2 is false in w_2. In these models, the conditional $[R]P_1 \rightarrow [R]P_2$ is simply false at the world w. So on this way of formalizing the Sorites (which is close to the one Halpern uses), the major premisses of the argument are false.

However, Halpern's logic is very flexible and context can be absorbed into his models in various ways. For instance, Gaifman's idea could be incorporated into the formalization of the color spectrum sorites (say, by making the samples to which the agent is attending part of the objective state). Attempts of this kind show that Halpern's logical language may need to be extended to provide an adequate formalization—to represent Gaifman's effect, the modal operators need to contain a representation of the attention list—but Halpern seems to be willing to entertain extensions of this kind.

For this reason, Halpern's proposal is probably best viewed as a general framework for formalizing solutions to the Sorites that are conservative with respect to classical logic, and the main contribution is the distinction between the subjective and objective aspects of possible worlds. I am not sure myself how useful this distinction is. Sometimes it seems to be helpful, in other cases it seems to be arbitrary and perhaps even unhelpful.

8   A possible resolution of the color-spectrum Sorites

As I indicated, Dummett's version of the Sorites has struck many people as especially troublesome, because of the very strong intuition in this case (in the conditional version) that the major premisses are fully true. So a solution to this version of the paradox might be a good place to begin attacking the Sorites. We can think of this case as a problem concerning the semantics of a color adjective, say, 'red'.

Even though it will not deal with the generality problem, a plausible solution to the color spectrum version would certainly be encouraging, since this is generally thought to be the hardest known case. In this section, I'll (briefly) examine a solution that incorporates Gaifman's contextual resolution.

There are many ways in which adjectives can depend on context. The most obvious of these, and probably the most frequently cited, is dependence on a standard of comparison, perhaps given by a comparison class. When something is said to be tall, for instance, or heavy, the content of what is said will vary according to what sorts of things it is compared with. A 10-story building could be truly said to be tall if (implicitly) it is compared with buildings in Missoula, but not at all tall if (implicitly) compared with buildings in New York.

Color adjectives may depend on this factor as well, though it seems to be relatively unimportant for them. However, there does seem to a dependence of color adjectives on what Graff calls *interest.* In (Graff, 2000) she illustrates this with an example in which the purpose is to identify a book on that is gray (but grayish blue) that is on a shelf with other books that are grayish red. In this

case, it seems appropriate to say "bring me the blue book." If this is a matter of truth and not appropriateness, we have another sort of contextual dependence.

Suppose that this sort of contextual effect is determined by a set of alternatives (in this case, the alternatives would be a partition of the set of books on the shelf into two sets, one of which is a unit set). Representations of this sort are also useful in the interpretation of focus; see, for instance, (Rooth, 1996), and I think they may provide a better way of coming to grips with the phenomena than ascribing them to purposes or interests.

In that case (discounting other contextual effects on color adjectives) we could treat a context as a pair consisting (as Gaifman proposes) of a comparison list and a set of alternatives. Rather than formalizing this, with a logical language and a model theory, I will discuss the idea briefly. The role of the alternative set is to locate the boundary of a color adjective in a region where it can achieve one of the partitions (if there is such a place for the adjective in question). In general, this will shift the interpretation of the adjective (in much the same way that an intensifier like 'very' does) without, however, fixing it precisely. We can assume for the purposes of this example that the effects of the attention lists are as Gaifman describes them. In that case (with, of course, an appropriate formalized language and model theory) we can inherit Gaifman's solution to the Sorites in this version of the Sorites.

9   Other adjectives

Other adjectives, however, may not exhibit dependence on a context list: 'intelligent' and 'expensive' may be cases of this sort. Here, we will need to look for other resolutions of the Sorites. If, for instance, we can show in these cases that it is independently plausible to deny the major premises, we do not even need to look for appropriate contextual effects.

The point is that, with an approach that treats the context appropriate for interpreting an adjective as a complex structure that can contain many factors affecting the boundary of the adjective, we can develop a general approach that can examine various adjectives on a case-by-case basis, while at the same time a general logical framework is maintained. Any formalized solution in the literature that respects classical logic could, I think, be incorporated into this framework. This is methodologically useful, for the reasons I mention in the next section.

10   Responding to paradoxical challenges

A paradox confronts us with a plausible argument that has plausible premises and an implausible conclusion. The profound paradoxes (including at least the semantic paradoxes and the Sorites) are *generalizable*, appearing in a variety of forms. This feature makes them difficult to solve. A resolution of one form of a paradox may not generalize to other forms. For instance, a solution to the version of the Liar Paradox involving 'This sentence is false' that simply invokes truth-value gaps will not solve versions that invoke 'This sentence is not true'.

And formalisms that seem to avoid the Russell paradox by eliminating negation can run afoul of the Curry paradox.

A formally adequate solution to a class of paradoxes will involve a formalized language in which the paradoxes can be formalized in a plausible way. It must be semantically sensible, and it must be consistent. The best way of showing this is to produce models of the formalism, and to show that these models are plausible and match the formalism's informal motivation. This part of the project is more or less mathematical. As such, it can be carried through to completion and evaluated using the criteria generally applicable to logical formalizations.

But in the case of generalizable paradoxes, there has to be a more open-ended part of the solution project, in which various generalizations of the paradox are considered, and solved. The main components of the solution—assuming that the formalism doesn't have to be extended to deal with generalizations of the paradox—will consist in exhibiting plausible formalizations of the generalized paradox and showing that the solution strategy extends to these cases. What makes this part of the solution open-ended is the lack of a complete list of all the conceivable variations of a generalizable paradox. In the absense of such a list, one has to fall back on the methods that are used to support claims like Church's thesis and the adequacy of first-order logic as a vehicle for formalizing mathematical logic: (1) an extended suite of case studies illustrating the generality of the formalism, (2) the absence of any plausible counterexamples, once the claim has achieved a certain amount of publicity.

11   Generality and the Sorites

The known Sorites paradoxes do not vary much in the actual reasoning. As we said, they come in two forms; and any solution that deals with the conditional form is pretty certain to deal with the inductive form. But they do occur with predicates that differ widely in their linguistic realizations and semantic properties. Generality poses a threat to solutions of the Sorites—and especially to contextual solutions—mainly through the variety of vague predicates.

A solution that—like Gaifman's, for instance—depends for its plausibility on the way in which an adjective like 'red' depends on context, may be unable to deal with a Sorites paradox involving a different adjective that does not depend on context in this particular way. Not all adjectives are related to perception in a way that makes contextual dependence on a comparison list plausible: 'honest' doesn't seem to enter into any such contextual dependence, for instance. Analysis of other examples, and case-by-case consideration of whether there is a plausible way of disarming the paradox, will be needed to make the generality of the solution plausible. Fortunately, adjectives, like other items in the lexicon, group together into semantic classes, so that the number of cases to examine will be much smaller than the number of adjectives in the lexicon.

Beyond vague adjectives, there are vague adjective phrases. This is uncharted territory; but hopefully, an extended examination of adjectives might yield gen-

eral techniques for dealing with conjunctions and modifications of adjective phrases, and other complex cases.

And, of course, vagueness is not confined to adjectives and adjective phrases. After all, 'sorites' ('heap') is a noun; and a complete account will have to systematically consider the whole list of syntactic categories.

The program I have proposed here attempts to resolve the Sorites by cases. The list of cases may prove to be so open-ended that a completed argument may not emerge from the program; but, with the help of some linguistics, there is some hope that this might happen.

An alternative way to structure the cases would look at the meanings of adjectives and other words; such an attempt, for instance might proceed by distinguishing different types of vague predicates, and showing that each type is tractable. This is more or less what Delia Graff undertakes in her solution strategy. Setting aside the arguments she gives in specific cases, I think that this semantic way of structuring the argument is unsuccessful. Graff has one sort of solution for "phenomenal continua" Sorites arguments and another sort for the rest. The phenomenal continua arguments involve *observational* predicates, which are those predicates whose "applicability to an object (given a fixed context of evaluation) depends only on the way that object appears" (Graff, 2001)[p. 907]. Approaching the problem in this way, by dividing the paradoxical cases into a positively defined class (observational predicates) and a residue class (the other predicates) simply transfers the generality problem to the residue class. Graff deals with some arguments in the residue class elsewhere (Graff, 2000), but even so the generality of her solution strategy (which deploys only two solution methods) is left in doubt. There is no reason to think that the strategy covers all versions of the paradoxical arguments. In fact, "semiobservational" predicates such as redness, which (apparently) depend partly on appearances, may pose a problem.

To address the generality problem, it seems that a semantical case structure would have to provide an exhaustive partition of the predicates without appealing to a residue category—and this strikes me as difficult, if not impossible.

12   Conclusion

The purpose of this paper has been to bring out some useful ideas in the recent literature on the logic of vagueness and to develop a methodology that does justice to the very deep problems raised by the Sorites, rather than to propose a solution. The hope is that, using this framework, incremental progress can be made. If that turns out to be the case (and I can't guarantee at all that it would), this would show that the Sorites paradox, despite its apparent intractibility, is different from the semantic paradoxes. There, I see no hope at present for an incremental program of this kind.

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