

On Redundancy in Linked Geospatial Data

Michael Sioutis¹, Sanjiang Li², and Jean-François Condotta¹

¹ CRIL CNRS UMR 8188, Université d'Artois, Lens, France
{sioutis,condotta}@cril.fr

² QCIS, University of Technology, Sydney, Australia
sanjiang.li@uts.edu.au

Abstract. RCC8 is a constraint language that serves for qualitative spatial representation and reasoning by encoding the topological relations between spatial entities. As such, RCC8 has been recently adopted by GeoSPARQL in an effort to enrich the Semantic Web with qualitative spatial relations. We focus on the redundancy that these data might harbor, which can throttle graph related applications, such as storing, representing, querying, and reasoning. For a RCC8 network \mathcal{N} a constraint is *redundant*, if removing that constraint from \mathcal{N} does not change the solution set of \mathcal{N} . A *prime network* of \mathcal{N} is a network which contains no redundant constraints, but has the same solution set as \mathcal{N} . In this paper, we present a practical approach for obtaining the prime networks of RCC8 networks that originate from the Semantic Web, by exploiting the sparse and loosely connected structure of their constraint graphs, and, consequently, contribute towards offering Linked Geospatial Data of high quality. Experimental evaluation exhibits a vast decrease in the total number of non-redundant constraints that we can obtain from an initial network, while it also suggests that our approach significantly boosts the state-of-the-art approach.

1 Introduction

The Region Connection Calculus (RCC) is the dominant approach in Artificial Intelligence for representing and reasoning about topological relations [10]. RCC can be used to describe regions that are non-empty regular subsets of some topological space by stating their topological relations to each other. RCC8 is the constraint language formed by the following 8 binary topological base relations of RCC: disconnected (*DC*), externally connected (*EC*), equal (*EQ*), partially overlapping (*PO*), tangential proper part (*TPP*), tangential proper part inverse (*TPPi*), non-tangential proper part (*NTPP*), and non-tangential proper part inverse (*NTPPi*). These 8 relations are depicted in [10, Fig. 4].

RCC8 has been recently adopted by GeoSPARQL [9], and there has been an ever increasing interest in coupling qualitative spatial reasoning techniques with Linked Geospatial Data that are constantly being made available [5, 7]. Thus, there is a real need for scalable implementations of constraint network algorithms for qualitative and quantitative spatial constraints, as RDF stores supporting Linked Geospatial Data are expected to scale to billions of triples [5,

7]. In this context, literature has mainly focused on the *satisfiability problem* of a RCC8 network, which is deciding if there exists a solution of the network. Towards efficiently deciding the satisfiability of large real world RCC8 networks that originate from the Semantic Web, there has already been a fair amount of work carried out, presenting promising results, as described in [13, 12, 15]. Lately, the important problem of deriving *redundancy* in a RCC8 network has been considered and well-established in [6]. For a RCC8 network \mathcal{N} a constraint is *redundant*, if removing that constraint from \mathcal{N} does not change the solution set of \mathcal{N} . A *prime network* of \mathcal{N} is a network which contains no redundant constraints, but has the same solution set as \mathcal{N} . Finding a prime network can be useful in many applications, such as computing, storing, and compressing the relationships between spatial objects and, hence, saving space for storage and communication, merging networks [1], aiding querying in spatially-enhanced databases [7, 9], and adjusting geometrical objects to meet topological constraints [17]. Due to space constraints, we refer the reader to [6] for a well-depicted real motivational example and further application possibilities.

In this paper, we propose a practical approach for obtaining the prime networks of RCC8 networks that have been harvested from the Semantic Web. In particular, we exploit the sparse and loosely connected structure of their constraint graphs, by establishing results that allow us to build on the simple decomposition scheme presented in [15]. The paper is organized as follows: in Section 2 we give some preliminaries concerning RCC8, redundant constraints, and the notion of a prime network, in Section 3 we present our practical approach for obtaining the prime networks of RCC8 networks that have been harvested from the Semantic Web, in Section 4 we experimentally show that we can have a vast decrease in the total number of non-redundant constraints that we can obtain from an initial RCC8 network of the considered dataset, with significantly improved performance over the state-of-the-art approach, and, finally, in Section 5 we conclude and make a connection with a relevant late breaking research effort.

2 Preliminaries

A (binary) qualitative constraint language [11] is based on a finite set B of *jointly exhaustive and pairwise disjoint* (JEPD) relations defined on a domain D , called the set of base relations. The base relations of set B of a particular qualitative constraint language can be used to represent definite knowledge between any two entities with respect to the given level of granularity. B contains the identity relation Id , and is closed under the converse operation ($^{-1}$). Indefinite knowledge can be specified by unions of possible base relations, and is represented by the set containing them. Hence, 2^B represents the total set of relations. 2^B is equipped with the usual set-theoretic operations union and intersection, the converse operation, and the weak composition operation denoted by symbol \diamond [11]. In the case of RCC8 [10], as noted in Section 1, B is the set $\{DC, EC, PO, TPP, NTPP, TPPi, NTPPi, EQ\}$, with EQ being relation Id . RCC8 networks can be viewed as qualitative constraint networks (QCNs), defined as follows:

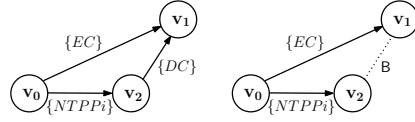


Fig. 1: A RCC8 network (left) and its prime network (right)

Definition 1. A QCN is a pair $\mathcal{N} = (V, C)$ where: V is a non-empty finite set of variables; C is a mapping that associates a relation $C(v, v') \in 2^{\mathbb{B}}$ to each pair (v, v') of $V \times V$. C is such that $C(v, v) = \{\text{Id}\}$ and $C(v, v') = (C(v', v))^{-1}$.

In what follows, given a QCN $\mathcal{N} = (V, C)$ and $v, v' \in V$, $\mathcal{N}[v, v']$ will denote the relation $C(v, v')$. $\mathcal{N}_{[v, v']/r}$, with $r \in 2^{\mathbb{B}}$, is the QCN \mathcal{N}' defined by $\mathcal{N}'[v, v'] = r$, $\mathcal{N}'[v', v] = r^{-1}$, and $\mathcal{N}'[v, v'] = \mathcal{N}[v, v'] \forall (v, v') \in (V \times V) \setminus \{(v, v'), (v', v)\}$. A QCN $\mathcal{N} = (V, C)$ is said to be *trivially inconsistent* iff $\exists v, v' \in V$ with $\mathcal{N}[v, v'] = \emptyset$. A *solution* of \mathcal{N} is a mapping σ defined from V to the domain \mathbb{D} , yielding a valid configuration, such that for every pair (v, v') of variables in V , $(\sigma(v), \sigma(v'))$ can be described by $\mathcal{N}[v, v']$, i.e., there exists a base relation $b \in \mathcal{N}[v, v']$ such that the relation defined by $(\sigma(v), \sigma(v'))$ is b . Two QCNs are *equivalent* iff they admit the same set of solutions. The constraint graph of a QCN $\mathcal{N} = (V, C)$ is the graph (V, E) , denoted by $G(\mathcal{N})$, for which we have that $(v, v') \in E$ iff $\mathcal{N}[v, v'] \neq \mathbb{B}$. $\diamond(\mathcal{N})$ denotes the refined \diamond -consistent QCN of \mathcal{N} , iff $\forall v, v', v'' \in V$ we have that $\diamond(\mathcal{N})[v, v'] \subseteq \diamond(\mathcal{N})[v, v''] \diamond \diamond(\mathcal{N})[v'', v']$. A *sub-QCN* \mathcal{N}' of $\mathcal{N} = (V, C)$, is a QCN (V, C') such that $\mathcal{N}'[v, v'] \subseteq \mathcal{N}[v, v'] \forall v, v' \in V$ where $\mathcal{N}'[v, v'] \neq \mathbb{B}$. Given a QCN $\mathcal{N} = (V, C)$, $\mathcal{N}_{\downarrow V'}$, with $V' \subseteq V$, is QCN \mathcal{N} restricted to V' . If b is a base relation, then $\{b\}$ is a singleton relation. A subclass of relations is a set $\mathcal{A} \subseteq 2^{\mathbb{B}}$ closed under converse, intersection, and weak composition. In what follows, all the considered subclasses will contain the singleton relations of $2^{\mathbb{B}}$. Given three relations r, r' , and r'' , we say that weak composition distributes over intersection if we have that $r \diamond (r' \cap r'') = (r \cap r') \diamond (r \cap r'')$ and $(r' \cap r'') \diamond r = (r' \cap r) \diamond (r'' \cap r)$.

Definition 2. A subclass $\mathcal{A} \subseteq 2^{\mathbb{B}}$ is a distributive subclass if weak composition distributes over non-empty intersections for all relations $r, r', r'' \in \mathcal{A}$. A subclass $\mathcal{A} \subseteq 2^{\mathbb{B}}$ is a maximal distributive subclass if there exists no other distributive subclass \mathcal{B} with $\mathcal{B} \supset \mathcal{A}$.

Notably, RCC8 has two maximal distributive subclasses, namely, \mathcal{D}_8^{41} and \mathcal{D}_8^{64} [6]. Given a QCN $\mathcal{N} = (V, C)$, we say that \mathcal{N} *entails* a constraint $r(v, v') \in 2^{\mathbb{B}}$, with $v, v' \in V$, if for every solution σ of \mathcal{N} , the relation defined by $(\sigma(v), \sigma(v'))$ is a base relation b such that $b \in r(v, v')$. Relation $\mathcal{N}[v, v']$ is *redundant* if network $\mathcal{N}_{[v, v']/\mathbb{B}}$ entails $\mathcal{N}[v, v']$. Note that by definition every universal relation \mathbb{B} in a QCN is redundant. Recalling the fact that the constraint graph of a QCN involves all the non-universal relations, we can obtain the following lemma:

Lemma 1. Given a QCN $\mathcal{N} = (V, C)$ and its constraint graph $G(\mathcal{N}) = (V, E)$, a relation $\mathcal{N}[v, v']$, with $v, v' \in V$, is redundant if $(v, v') \notin E$.

We now recall the following definition of a reducible and a prime QCN:

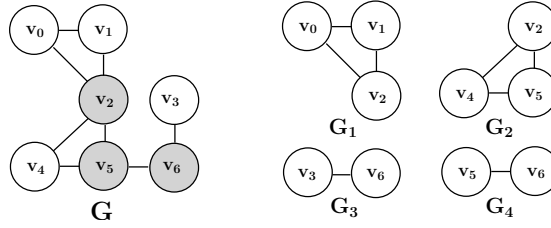


Fig. 2: A graph G (left) with its biconnected components (right)

Definition 3 ([6]). A QCN $\mathcal{N} = (V, C)$ is reducible if it comprises a redundant relation other than relation \mathbf{B} , and irreducible otherwise. An equivalent irreducible sub-QCN of \mathcal{N} , is called a prime QCN of \mathcal{N} . If a prime QCN of \mathcal{N} is also unique, it is denoted by $\mathcal{N}_{\text{prime}}$.

In Figure 1, a QCN \mathcal{N} of RCC8 and its prime QCN are depicted. Relation $\{DC\}$ is redundant as it can be entailed by $\mathcal{N}_{[v_1, v_2]/\mathbf{B}}$ and, thus, can be replaced with relation \mathbf{B} (denoting the lack of a constraint between two entities in a QCN).

Property 1 ([6]). Let $\mathcal{N} = (V, C)$ be a satisfiable QCN of RCC8. Then, \mathcal{N} will be said to satisfy the *uniqueness* property iff $\forall u, v \in V$, with $u \neq v$, we have that \mathcal{N} does not entail a relation $r \subseteq \mathcal{N}[u, v]$ where $r = \{EQ\}$.

The *uniqueness* property specifies that every region in a QCN of RCC8 should be unique and not identical to any other region. This is a necessary property to be able to obtain the unique prime network of a QCN [6] and will hold for all the considered QCNs in what follows. We recall the following important lemma to be used in the sequel:

Lemma 2 ([6]). Let $\mathcal{N} = (V, C)$ be a not trivially inconsistent and \diamond -consistent QCN of RCC8 defined on one of the maximal distributive subclasses \mathcal{D}_8^{41} , or \mathcal{D}_8^{64} , and having the uniqueness property. Then, a relation $\mathcal{N}[v, v']$, with $v, v' \in V$, is non-redundant in \mathcal{N} iff $\mathcal{N}[v, v'] \neq \bigcap \{\mathcal{N}[v, v''] \diamond \mathcal{N}[v'', v'] \mid v'' \in V \setminus \{v, v'\}\}$.

Finally, we have the following result with respect to the unique prime network of a QCN \mathcal{N} of RCC8, namely, $\mathcal{N}_{\text{prime}}$:

Theorem 1 ([6]). Let $\mathcal{N} = (V, C)$ be a satisfiable QCN of RCC8 defined on one of the maximal distributive subclasses \mathcal{D}_8^{41} , or \mathcal{D}_8^{64} , and having the uniqueness property. Further, let χ be the set of non-redundant relations in $\diamond(\mathcal{N})$. Then, $\forall u, v \in V$ we have that $\mathcal{N}_{\text{prime}}[u, v] = (\mathcal{N}[u, v] \text{ if } \diamond(\mathcal{N})[u, v] \in \chi \text{ else } \mathbf{B})$.

3 Towards Efficiently Characterizing Non-Redundant Relations in a Network

In this section we present a practical approach for characterizing non-redundant relations in QCNs that have been harvested from the Semantic Web. In particular, we exploit the sparse and loosely connected structure of their constraint graphs, by establishing results that allow building on the simple decomposition scheme of [15]. We recall the following definition regarding biconnected graphs:

Algorithm 1: Delphys+(\mathcal{N})

in : A satisfiable QCN $\mathcal{N} = (V, C)$ of RCC8 defined on \mathcal{D}_8^{41} or \mathcal{D}_8^{64} .
output : χ , the set of non-redundant relations in $\diamond(\mathcal{N})$.

```
1 begin
2    $\chi \leftarrow \emptyset$ ;
3   foreach  $n \in \text{Decomposer}(\mathcal{N})$  [15] do
4      $\chi \leftarrow \chi \cup \text{Delphys}(n)$  [6];
5   return  $\chi$ ;
```

Definition 4 ([2]). A connected graph $G = (V, E)$ is said to have an articulation vertex u if there exist vertices v and v' such that all paths connecting v and v' pass through u . A graph that has an articulation vertex is called separable, and one that has none is called biconnected. A maximal biconnected subgraph is called a biconnected component.

Intuitively, an articulation vertex is any vertex whose removal increases the number of connected components in a given graph. Figure 2 depicts a graph G , along with its biconnected components. Vertices in grey are the articulation vertices of G . The biconnected components of a graph $G = (V, E)$ can be obtained in $O(|E|)$ time [2]. We recall the following result from [15]:

Proposition 1 ([15]). Let \mathcal{N} be a QCN of RCC8, and $\{G_1, \dots, G_k\}$ the biconnected components of its constraint graph $G(\mathcal{N})$. Then, \mathcal{N} is satisfiable iff \mathcal{N}_i is satisfiable for every $i \in \{1, \dots, k\}$, where \mathcal{N}_i is $\mathcal{N} \downarrow_{V(G_i)}$.

Then, by Proposition 1 and Lemma 1 we can obtain the following result:

Proposition 2. Let \mathcal{N} be a satisfiable QCN of RCC8, and $\{G_1, \dots, G_k\}$ the biconnected components of its constraint graph $G(\mathcal{N})$. Then, a relation $\mathcal{N}[v, v']$, with $v, v' \in V$, is non-redundant in \mathcal{N} iff $(v, v') \in E(G_i)$ and $\mathcal{N}[v, v']$ is non-redundant in \mathcal{N}_i , where \mathcal{N}_i is $\mathcal{N} \downarrow_{V(G_i)}$, for some $i \in \{1, \dots, k\}$.

Proof. By Lemma 1 we know that a relation $\mathcal{N}[v, v']$ is redundant if $(v, v') \notin E(G_i)$. Let us consider a relation $\mathcal{N}[v, v']$ where $(v, v') \in E(G_i)$. Let $\mathcal{N}' = \mathcal{N}_{[v, v']/\mathbb{B}}$ and \mathcal{N}'_i the restriction of \mathcal{N}' to $V(G_i)$. Then, by Proposition 1 we have that a mapping σ is a solution of \mathcal{N}' iff σ is a solution of \mathcal{N}'_i . On the other hand, since G_i is a biconnected component, any solution of \mathcal{N}'_i is the restriction of some solution of \mathcal{N}' to $V(G_i)$. Thus, \mathcal{N}' entails $\mathcal{N}[v, v']$ iff \mathcal{N}'_i entails $\mathcal{N}[v, v']$, and, consequently, $\mathcal{N}[v, v']$ is redundant in \mathcal{N} iff $\mathcal{N}[v, v']$ is redundant in \mathcal{N}_i . \square

An algorithm based on Lemma 2 to obtain the set of non-redundant relations was provided in [6] with a time complexity of $O(|V|^3)$ for a given QCN $\mathcal{N} = (V, C)$ of RCC8, which we here call Delphys. Proposition 2 allows us to establish a time complexity of $O(\frac{|V|}{c} \cdot c^3) = O(|V| \cdot c^2)$, where $c \leq |V|$ is the maximum order among the biconnected components of constraint graph $G(\mathcal{N})$, as it suggests that we can consider the smaller biconnected components of $G(\mathcal{N})$ instead of

Table 1: Biconnected components of real RCC8 networks

network	# of components	max order	median order	min order	avg. order
nuts	1624	52	2	2	2
adm1	27	11 665	2	2	437
gadm1	712	19 864	2	2	61
gadm2	113 097	2 371	2	2	3
adm2	2 893	22 808	579	2	600

the entire constraint graph when characterizing non-redundant relations in \mathcal{N} . This new approach, which we call **Delphys+**, is shown in Algorithm 1.³

It remains to be seen if there is any significant difference between the order of the constraint graph of a QCN and the maximum order among the biconnected components of that graph, that is, if the value of the latter is significantly smaller than the value of the former, so that **Delphys+** can be considered as an advancement over **Delphys**. We will see that this is indeed the case for the considered QCNs of RCC8 that have been harvested from the Semantic Web (originally appeared in [8]), which we introduce as follows.

- **nuts**: a RCC8 network that describes a nomenclature of territorial units and contains 2 235/3 176 nodes/edges.⁴
- **adm1**: a RCC8 network that describes the administrative geography of Great Britain [4] and contains 11 761/44 832 nodes/edges.
- **gadm1**: a RCC8 network that describes the German administrative units and contains 42 749/159 600 nodes/edges.⁴
- **gadm2**: a RCC8 network that describes the world’s administrative areas and contains 276 727/589 573 nodes/edges (<http://gadm.geovocab.org/>).
- **adm2**: a RCC8 network that describes the Greek administrative geography and contains 1 732 999/5 236 270 nodes/edges.⁴

The aforementioned QCNs are satisfiable⁵, comprise relations that are properly contained in any of the two maximal distributive subclasses \mathcal{D}_8^{41} and \mathcal{D}_8^{64} for RCC8, and originate from the Semantic Web, also called the Web of Data, which is argued to be scale-free [16]. Graphs of scale-free structure are relatively sparse [3], as it can be also observed by the $\frac{|E|}{|V|}$ ratio of the constraint graphs of our real world QCNs, thus, we expect these constraint graphs to be loosely connected and yield a high number of biconnected components. We can view information regarding biconnected components of the constraint graphs of our QCNs in Table 1. The findings are quite impressive, in the sense that the maximum order among the biconnected components of a constraint graph is significantly smaller than the order of that graph. For example, the constraint graph of the biggest real RCC8 network, namely, **adm2**, has an order of value

³Check $|V(g)| > 2$ within algorithm **Decomposer** as it appears in [15] must be removed for appropriate use of **Decomposer** in **Delphys+**.

⁴Retrieved from: <http://www.linkedopendata.gr/>

⁵As obtaining the prime network of a QCN requires that the QCN is satisfiable (see Theorem 1), we fixed some inconsistencies with **gadm1** and **gadm2** that were originally unsatisfiable. Also, identical regions were properly amalgamated to satisfy the uniqueness property.

1 733 000, but the maximum order among its biconnected components is only of value 22 808. Note also that, as the other metrics suggest, the largest proportion of the biconnected components of a graph have an order much closer to the minimum order than the maximum order among the components of that graph.

4 Experimental Evaluation

In this section, we compare the performance of Delphys+ with that of Delphys [6] using the dataset presented in Section 3. Experimentation was carried out on a PC with an Intel Core 2 Quad Q9400 processor, 8 GB RAM, and the Precise Pangolin x86_64 OS. Both Delphys and Delphys+ were written in Python and run with with PyPy 2.4.0 (<http://pypy.org/>). Only one CPU core was used.

Table 2: Performance comparison on CPU time

network	Delphys	Delphys+	speedup (%)
nuts	45.98s	0.26s	99.4%
adm1	30 917.23s	29 489.02s	4.6%
gadm1	∞	151 295.62s	$\sim 100\%$
gadm2	∞	12.05s	$\sim 100\%$
adm2	∞	∞	?

The results on the performance of Delphys+ and Delphys are shown in Table 2. Note that symbol ∞ signifies that a reasoner hit the memory limit. The speedup for Delphys+ reaches as high as nearly 100% for the cases where Delphys was actually able to fully reason with the networks (e.g., **nuts**). Regarding **adm1** the speedup was limited and expected as the maximum order among the biconnected components of the constraint graph of **adm1** is very close to the order of the entire graph itself (see Table 1). We also note that despite the overall much better performance of Delphys+, it was unable to fully reason with **adm2**. (We will refer to a late breaking research effort regarding this issue in the closing section.)

Table 3: Effect on obtaining non-redundant relations

network	initial # of relations	non-redundant # of relations	decrease (%)
nuts	3 176	2 249	29.19%
adm1	44 832	44 601	0.52%
gadm1	159 600	158 440	0.73%
gadm2	589 573	292 331	50.42%
adm2	5 236 270	?	?

Regarding redundancy, Table 3 shows the decrease that we can achieve with respect to the total number of non-redundant constraints that we can obtain from an initial network, which allows one to construct sparse constraint graphs that can boost various graph related tasks, such as storing, representing, querying, and reasoning. Notably, for the biggest network that Delphys+ was able to fully reason with, namely, **gadm2**, the decrease is more than 50%, yielding a number of non-redundant constraints which is almost linear to the number of its vertices, confirming a similar observation in [6].

5 Conclusion

We focused on the redundancy that is harbored in RCC8 networks that originate from the Semantic Web, and proposed a practical approach for sanitizing such networks of any redundancy, by obtaining the set of their non-redundant constraints and, consequently, offering Linked Geospatial Data of high quality. Experimental evaluation exhibited a vast decrease in the total number of non-redundant constraints that we can obtain from an initial network, with significantly improved performance over the state-of-the-art approach. A late breaking research effort, presented in [14], builds on our approach and uses a particular partial consistency to significantly boost its performance. Notably, it is able to tackle even the largest of networks considered in our evaluation in this paper.

References

1. Condotta, J., Kaci, S., Marquis, P., Schwind, N.: Merging Qualitative Constraint Networks in a Piecewise Fashion. In: ICTAI (2009)
2. Dechter, R.: Constraint processing. Elsevier Morgan Kaufmann (2003)
3. Del Genio, C.I., Gross, T., Bassler, K.E.: All Scale-Free Networks Are Sparse. *Phys. Rev. Lett.* 107, 178701 (2011)
4. Goodwin, J., Dolbear, C., Hart, G.: Geographical Linked Data: The Administrative Geography of Great Britain on the Semantic Web. *TGIS* 12, 19–30 (2008)
5. Koubarakis, M., et al.: Challenges for Qualitative Spatial Reasoning in Linked Geospatial Data. In: BASR@IJCAI (2011)
6. Li, S., Long, Z., Liu, W., Duckham, M., Both, A.: On redundant topological constraints. *AIJ* 225, 51–76 (2015), in press
7. Nikolaou, C., Koubarakis, M.: Querying Incomplete Geospatial Information in RDF. In: SSTD (2013)
8. Nikolaou, C., Koubarakis, M.: Fast Consistency Checking of Very Large Real-World RCC-8 Constraint Networks Using Graph Partitioning. In: AAAI (2014)
9. Open Geospatial Consortium: OGC GeoSPARQL - A geographic query language for RDF data. OGC[®] Standard (2012)
10. Randell, D.A., Cui, Z., Cohn, A.: A Spatial Logic Based on Regions and Connection. In: KR (1992)
11. Renz, J., Ligozat, G.: Weak Composition for Qualitative Spatial and Temporal Reasoning. In: CP (2005)
12. Sioutis, M.: Triangulation versus Graph Partitioning for Tackling Large Real World Qualitative Spatial Networks. In: ICTAI (2014)
13. Sioutis, M., Condotta, J.F.: Tackling large Qualitative Spatial Networks of scale-free-like structure. In: SETN (2014)
14. Sioutis, M., Li, S., Condotta, J.F.: Efficiently Characterizing Non-Redundant Constraints in Large Real World Qualitative Spatial Networks. In: IJCAI (2015), to appear
15. Sioutis, M., Salhi, Y., Condotta, J.: A Simple Decomposition Scheme For Large Real World Qualitative Constraint Networks. In: FLAIRS (2015), to appear
16. Steyvers, M., Tenenbaum, J.B.: The Large-Scale Structure of Semantic Networks: Statistical Analyses and a Model of Semantic Growth. *Cog. Sci.* 29, 41–78 (2005)
17. Wallgrün, J.O.: Exploiting qualitative spatial reasoning for topological adjustment of spatial data. In: SIGSPATIAL (2012)