Cognitive Control of Pacing During Endurance Exercise: Everyone is a Quitter

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Abstract

Contemporary accounts of control of pacing during endurance exercise focus on physical limitations, generally assuming humans work to that physical limit. Conceptually, control is ceded to the body at the beginning of an exercise bout and is returned to central cognition upon achieving a state of exhaustion. We advance an alternative decision-making model of control of pacing, where the decision whether to persist in the effort is revisited continuously, and cessation of the exercise bout is an explicit, cognitively controlled decision. Our model depends on the following assumptions and features: 1) decisions are made in discrete cycles, 2) repetitive bodily motions depend on a central pattern generator, 3) afferent physiological feedback produces a sense of perceived exertion, 4) central cognition mediates between perceived exertion and the value of persisting (motivation) to perform an ongoing cost-benefit analysis, and 5) cessation of exercise occurs when an explicit decision is made to discontinue the effort.

Keywords: cognitive control of pacing; central pattern generator; perceived exertion; cognitive models of exercise

Cognitive Control of Pacing

Why do we continue in the face of fatigue, and when and why do we give up? This question is so fundamental, an entire body of research in the field of exercise physiology has been dedicated to answering it. Much of the field has focused on physical determinants of eventual performance, whether it be maximal oxygen consumption, hydration, or mechanical factors such as leg length or body composition. Motivational factors are often assumed away, frequently by studying world-class athletes who can be broadly characterized as exceptionally motivated, and cognitive factors are easily neglected. The physical performance is treated much as one would test an internal combustion engine, with maximal performance determined by the physical properties of the engine. That these athletes would persist during testing is a given, and that they produce a maximal effort is one of the further assumptions that defines the implications of the studies. Cognition is given short shrift, both in terms of theory and methodology.

Common protocols consist of exercising to exhaustion at a specified intensity, or completing a set distance in a minimal time. Pacing models have been proposed to describe human behavior; these are predicated on an often hidden assumption that physiology determines pacing (via physical fatigue), rather than that behavior drives pacing (via cognition). This assumption is rarely exposed in constant load exercise because experimenters have worked to remove cognition from the performance and isolate the physical aspect, often through the use of expert athletes who can push themselves to the limits of physiology. A maximal physical effort, if it is purely limited by bodily constraints, has little need to involve cognition. A sufficiently motivated participant is assumed to produce a maximal effort, and cognition is reduced to a simple on/off switch, where it invokes the required effort.

Pacing strategy is a matter of identifying the maximal power output that can be sustained across the expected time interval. For the elite athletes commonly used in pacing studies, the self selection of a maximal pace is done with relative ease, resulting in models that have slight variations from constant power output, but which can be explained through simple physiological explanations such as reserves of anaerobic energy. One commonly used model, known as the Hill model (after A.V. Hill), is based on the idea that exercise produces linear changes in metabolism, until demand exceeds capacity, resulting in fatigue and cessation of exercise. This simple, conventional model of exercise performance also produces some surprising predictions, however, which have been justifiably criticized (Noakes, 2011). Among these suspect predictions are the existence of a single maximal workload (regardless of distance or time), and the inability to lift the pace at the end of a "maximal" session. The ubiquity of a mad sprint for the line in long distance endurance events falsifies the prediction outright. That the Hill model has survived nearly a century of application is a testament to its utility in explaining some important phenomena, and to the lengths to which experimenters have succeeded in removing the brain of the experimental participant from the experiment.

Putting the Brain Back in Charge of Pacing

The trend toward removing cognitive aspects of "exercise to fatigue" has been turned on its head in several studies, however, where deception about pacing has been used to examine the cognitive inputs to sustained physical exertion. For example, (Stone et al., 2011) conducted a study in which participants completed a cycling time trial (a timed solo effort across a fixed distance) in a simulation environment against an avatar that represented their own best prior performance (supposedly a maximal effort). The critical manipulation was a deception condition, where that prior performance was augmented with a 2% increase in power output. Participants, believing that performance represented something they had already done, consistently outperformed the deceptive performance. The researchers concluded that participants all had a metabolic reserve that they strategically conserved, and thus none had completed a truly maximal effort during their initial best efforts. From this perspective, pacing strategy reflects cognitive budgeting of available resources against anticipated demands.

Tucker (2013) argues that pacing is the application of a plan, the entirety of which the participant is not completely aware of, to spend available resources to achieve the goal in a near-optimal fashion (where the difference between optimal and failure is often less than 1%). He defines pacing during exercise as an attempt to optimally meet the following goals and constraints:

1. use available energy at the optimal rate
2. gain heat slowly enough to complete the task, but not so slowly as to reduce intensity
3. accumulate metabolites at a low enough rate to avoid being overwhelmed by them
4. meet oxygen requirements of muscle, brain and other tissues
5. compete with other runners, the clock or whatever other motivational factors impact on performance

The conceptual model of Tucker (2013) depends on a template matching process that occurs continuously, weighing task demands against these templates for performance. Tucker (2013) additionally posits that pacing differences are due to "uncertainty" about the interpretation of templates in the context of the task, which results in the maintenance of a metabolic reserve. What this model lacks, however, is any specificity or concrete definition of what these templates might be, or what the uncertainty is and how it is applied.

It is exactly those theoretical gaps that we intend to address here, by making our model computationally explicit. We assert that, while participants may have a rough goal to do their best, they are engaged in an ongoing comparison between the expected duration of the work bout and their current intensity of effort with reference to their prior experiences. That is, they are retrieving prior events from memory for comparison, where the content of this memory includes aspects such as effort, duration, environmental conditions, and, critically, sustainability of the effort.

The participant need not balance, nor even be aware of, most of the factors that define endurance performance during the majority of work bouts. The surprising concordance of physiological limitations (where body temperature, energy reserves, and cardiac output, for example, simultaneously reach their limits during work to exhaustion) can be explained largely by physiological adaptations: systems that fail often adapt first, until all are roughly on par with each other. For example, the ability to handle heat stress can change dramatically with only a few weeks of training. There is no need for the athlete to attempt to optimize these factors individually, much less be aware of them in many cases. On the other hand, a participant is likely to be acutely aware of any single system (whether body cooling, oxygen delivery, or bodily afferent feedback such as muscular pain) that signals an impending or realized failure. Thus, by reacting to the system that corresponds to the weakest link and matching the current effort level based on personal history, cognitive control can give the appearance of near-optimality without reference to a preconceived plan.

In the remainder of this paper we will 1) formalize this theory within the framework of a cognitive architecture, and 2) demonstrate its utility through a computational model of exercise pacing.

Using a Unified Theory of Cognition to Constrain a Theory of Cognitive Control of Pacing

Unified theories of cognition (UTCs; Newell, 1994) attempt to collect the invariants of human cognitive behavior within a single, computationally realizable framework. One of the primary benefits of depending on UTCs is the requirement to make process models explicit and comprehensive. We turn to one candidate UTC, the ACT-R cognitive architecture (Anderson et. al., 2004), as a source of structural constraints on cognitive processing to inform the development of a theory of cognitive pacing. Critical to this paper, ACT-R has also been mapped onto a variety of brain areas, and can be used to predict and explain brain activity during task performance. A core tenet of ACT-R is that central cognition can be very finely approximated using a discrete decision-making cycle., with a pattern matching system implemented as a production system (that maps onto the basal ganglia) performing a repetitive decision-making inner loop during task performance. The central decision-making process interacts on each cycle with peripheral systems such as memory, visual, auditory, and haptic, perception, and bodily motor functions through a set of low capacity interfaces, or buffers, which allow limited parallelism.

When running or cycling, ~150-200 individual leg movements are typically made per minute. This automaticity requires a helper system capable of regulating repetitive motions without the need to burden central cognition. That is, without such a helper system, one would be unable to do anything other than perform the exercise itself because there would be no free cycles to devote anywhere else. Thus, it is apparent, even without recourse to a UTC-based analysis, that because endurance exercise does not overwhelm central cognition, it follows that it must primarily be handled elsewhere.

Turning back to UTCs, modeling highly interactive real-time tasks often requires helper systems running at higher frequencies than central cognition. For example, Best and Lebiere (2003) were only able to demonstrate smooth targeting, object tracking, and movement behavior in a virtual environment by reducing the cycle time to ~10ms, violating the fundamental of the ACT-R cognitive architecture ("overclocking" central cognition). Salvucci et al. (2001) addressed this limitation in a driving task by modifying the core architecture to interact with a slave system running at a higher frequency, thereby respecting the constraints ACT-R places on central cognition.

The implication of this analysis is that, in the context of endurance exercise, there must be a "helper" that conducts and regulates much of the activity involved in endurance exercise, and central cognition can be expected to primarily interact with that helper.

One candidate slave system that might provide a link between central cognition and endurance exercise is the Central Pattern Generator, a spinal network capable of producing rhythmic limb movements in the absence of cognitive control (Dimitrijevic et al., 1998). The CPG tends toward a natural resonant frequency of just under 3Hz, which corresponds to a natural cadence of ~90, matching up closely with observed freely selected cadences of runners and cyclists. This proposed model of control thus provides a level of indirection between central cognition and the exercising muscles. Fatigue signals from the muscles operate directly on CPG, which then passes this information on to the central executive. An effort signal from the Central Executive causes firing, but fatigue of neural pathways will cause reduced output for the same input firing signal.

Within the central executive, this model of control is based on the retrieval of relevant templates and comparison to ratings of perceived exertion (RPE). Given a target time/distance, memory can be scanned for a relevant effort that was successfully made at that time/distance. Given ongoing RPE feedback, the effort can be increased or reduced relative to the current effort.

Using this model, undershooting and overshooting of pacing efforts are both possible and likely. Overshooting has worse outcomes (failure), while undershooting can result in less than optimal performance. Specifically, undershooting leaves energy to be spent more rapidly at the end (end spurt), but due to task limitations, it may not be possible to spend all of the available energy. In all cases, these experiences are learned and stored, resulting in accumulated knowledge with experience. In the absence of experience, pacing can be expected to fail often, since there are no successes to draw from. This naturally produces learning predictions as well, since lack of experience should result in many more failures in pacing.

Specific Theoretical Predictions The constraints discussed above result in the following theoretical predictions:

- Completely inexperienced athletes (as young athletes often are) are likely to have more failures of over-pacing and under-pacing
- Experienced athletes, after learning to avoid failures, will pace conservatively, and will need to spend more of their energy budget at the end of the effort, producing an end spurt (increase in output intensity near the end of a work bout)
- Attentional manipulations that divide attention will negatively impact pacing (often through a reduction in cadence that reduces work rate)
- Attentional manipulations that focus attention will positively impact pacing (through maintenance of an even pacing strategy)
- Perceived Effort interacts by way of interruption, focuses attention on perception of effort (pain, discomfort, effort)
- Central Executive needs to increase effort signal to CPG as fatigue occurs to maintain the same muscular input
- Pacing should not be natural, but should emerge with learned experience over time
- With experience, cadence will tend toward just under 90, but while learning it will be lower. This is not automaticity or power law speedup, but rather a removal of the cognitive effort of deliberative processing that is replaced with the natural frequency (speedup beyond a cadence of 90 should not happen with greater experience).

We will next examine whether these predictions are sustained or contradicted by the existing literature, and we will explore a computational model that implements this model, providing a proof of the theoretical concept**.**

Constraints of Human Physiology on Endurance

The preceding discussion focuses on the cognitive control of pacing. There are also hard limits on endurance performance imposed directly by human physiology. An individual's capacity to perform endurance exercise is characterized by many features; chief among these are:

- Aerobic capability: the ability to metabolize oxygen to produce work, at lower intensities and long time scales. This ability may be defined in terms of critical power (the maximal work rate that can be indefinitely sustained), and is commonly expressed in units of oxygen consumed per measure of body weight.
- Anaerobic work capacity (AWC): the conversion of stored chemical energy to work without the use of oxygen), at higher intensities and shorter time scales. This anaerobic work creates an oxygen debt that must be repaid through respiration.
- Heat tolerance: the ability to maintain homeostasis in response to heat produced through exercise, primarily through sweat production and diversion of blood to surface skin capillaries to radiate heat.
- Maximal power output: the greatest work rate that can be sustained regardless of the timespan.
- Muscular fatigue: the generation of chemical waste products that inhibit further muscular action.

While there are many other factors that influence the ability to perform endurance exercise, these factors capture many of the main constraints that impact pacing. Aerobic capability and anaerobic work capacity are the two factors used in the critical power model of Monod and Scherrer (1965), which predicts the maximal duration *Tlim* of endurance exercise as a function of work rate *P*, the anaerobic work capacity *AWC*, and the work rate that can be sustained indefinitely *CP*. Their relationship is given by:

$$Tlim=AWC/(P-CP)$$

Graphically, the CP model traces an asymptotic hyperbolic curve, predicting work rates that approach infinity as the time approaches 0, and work rates that never decrease beyond asymptote as time approaches infinity. Despite this limitation, the CP model provides an excellent account of performance from durations of several minutes to several hours. Figure 1 below depicts the CP model in relation to actual historical performances for one individual athlete, showing the ability of this model to predict limitations while taking individual differences into account.

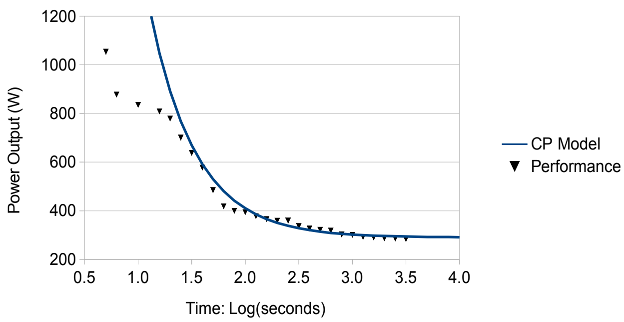

Figure 1: Historical time-intensity endurance curve for an individual athlete. The solid line shows the prediction of the CP model of Monod and Scherrer (1965) across the range of its practical applicability; the triangular markers show the athlete's actual historical performances.

Our modeling goal is to predict these work rate – duration curves through a model of cognitive control, given a set of inputs available to the cognitive system. This requires interaction with a physiology module that incorporates the modeling of heat generation and dissipation, oxygen consumption (as a function of resting metabolism, aerobic exercise, and repayment of oxygen debts), anaerobic energy use and reserves, and the provision of a perceived exertion signal. This last quantity, perceived exertion, is primarily based on cardiac output (the original RPE scale, in fact, was a linear transformation of heart rate), but also includes heat stress, and muscular fatigue components (Borg, 1982). We have implemented such a module, allowing us to derive oxygen consumption, heart rate, heat generation, and anaerobic energy status from a particular work rate given an athlete's individual physiological parameters. We now turn to the model of control.

Cognitive Control of Pacing Behavior

The preceding discussion outlines the effort-duration relationship for endurance exercise. The cornerstone of our model is the use of memory for historical efforts as the basis for establishing and refining a current effort. Those memories, or their absence, are exactly the source of predictions of expert-novice differences. Given a specific duration, an athlete will gauge their effort based on historical experience. Specifically, we suggest that a blended memory retrieval (Anderson et al., 2004) is performed, which combines prior experiences similar to the current context, producing an assessment of whether a particular effort will succeed or fail, weighted toward recent memories (that is, exhibiting recency). Thus, we might expect athletes after a layoff period of reduced or no training to overestimate the appropriate pace, because their last memory corresponded to a higher level of performance. In our model, this matching process is a noisy, inexact match to prior memory, which might be influenced by context, recency, and other similar factors known to influence memorability.

The conventional orient-decide-act cycle, embedded within cognitive architectures, is also a critical component of our model. As we previously pointed out, if endurance exercise required constant attention, it would overwhelm attentional resources. The central pattern generator may largely be responsible for handling the continuance of exercise under non-challenging circumstances. The critical question relates to when and how often attention focuses the individual on reconsidering the decision to either persist, increase, decrease, or entirely cease the effort, during challenging efforts.

Fatigue and pain are both implicated in this refocusing of attention. Anyone who has ever engaged in repetitive physical activity knows that the activity will become more difficult during a work bout, no matter the intensity. One important reason for this is the fatigue of neuromuscular circuits. Specifically, achieving the same muscular output (measured via EMG) requires stronger and stronger neural input, which corresponds to an increasing perception of effort. Voluntary muscle activation, which is measured as the percentage of neural activity achieved under voluntary control when compared to direct electrical stimulation, is approximately 90% prior to fatigue, but drops to less than 75% under conditions of fatigue. This mechanism protects muscle from catastrophic damage, regulating exercise by adjusting output from the brain (Amann, et al, 2008).

Exercising muscles also experience microscopic damage and produce metabolic waste products, leading to a perception of muscular pain and fatigue. This ever-increasing pain signal further attracts attention. The neural fatigue protective mechanism also interacts with regulation of activity via the CPG indirectly. At exactly the time when an athlete needs to focus attention on making the decision to maintain an effort (to increase the neural output signal to the exercising muscles), they may be interrupted to process the urgent sensory input of a pain signal.

Finally, while the CPG might allow the repetition of rhythmic activities, it will not drive effortful performance on its own. In the absence of a decision to continue, CPG-driven behavior will revert to the natural resonant frequency of the CPG, possibly reducing cadence during high effort times, and the same CPG signal will result in a decreasing output effort due to fatigue of neural pathways, despite any conscious decision to continue.

The decision to persist during endurance exercise to exhaustion, thus, must be revisited more and more frequently as the interval continues through the involuntary action of the attentional system. We might choose, if we decide to continue, to either attenuate or increase the effort, or we might choose to continue it, but this decision must be made, at increasingly short time intervals. Distractions, such as loud music, can be helpful since they focus attention away from remaking this decision. Persistence to the point of actual physical failure is exceedingly rare (Tucker, 2013). Eventually, we almost always quit, whether a rank amateur or an Olympic athlete. Stopping is a decision, albeit one that might seem forced upon us.

This appraisal process is depicted in Figure 2 below.

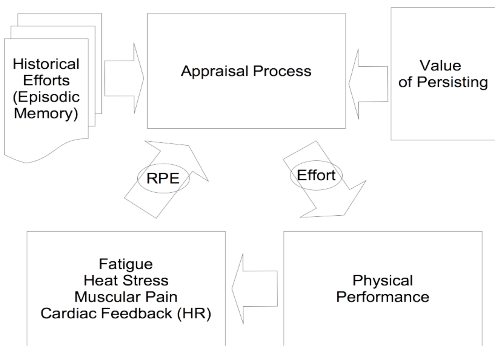

Figure 2: Simulation of 5 participants engaging in a 5-minute time trial at maximum effort.

Model Details The theory discussed here has been explicitly operationalized in the form of a running computer simulation. Given a task to complete a time trial, or volitional effort to exhaustion at the maximum work rate possible, the process model attempts to match historical efforts at the current estimated remaining duration (initially, the entire length of the effort). This model can be conceived of as an expert model, since it has examples of successful efforts that have been completed over a variety of durations.

In quantifying (output) effort, the model depends on an implementation of Stevens' Power Law (Stevens, 1957) as applied to muscular force, anchoring the ends of the scale at the greatest force the athlete can apply (for example, approximated 1400 watts for the athlete depicted in Figure 1) and 0, using an exponent of 1.7 (Zwislocki, 2009).

The simulation uses power output to determine oxygen consumption and heat accumulation. These factors, combined with the individual cardiac and oxygen uptake properties of an athlete, can then be used to predict heart rate, and use and depletion of anaerobic resources, thus providing key information regarding the status of the exercising athlete during the work bout.

These physiological inputs are then used by the model to appraise its efforts., by combining to form a Rating of Perceived Effort (RPE; Borg, 1982). This perceived effort is operationalized as a linear combination of heart rate (we note that the original Borg scale was a linear multiple of heart rate; Borg, 1982), effort, heat stress, and fatigue (following the method of Gonzalez et. al. 2011 for modeling physical fatigue using ACT-R). Under conditions of a constant work rate, fatigue and heat stress (due to accumulated heat) will increase, creating an ever-increasing sensation of perceived effort under constant load.

The model conducts a process of constant reappraisal (Figure 2), adjusting its effort based on current perceived effort, fatigue, and its memory of prior performances that best match the estimated time remaining in the effort. These memories are sourced from historical data such as the data presented in Figure 1. A graph of 5 simulated participants conducting a 5 minute cycling time trial at maximal effort (after 30 seconds of easy exercise) is presented in Figure 3 below.

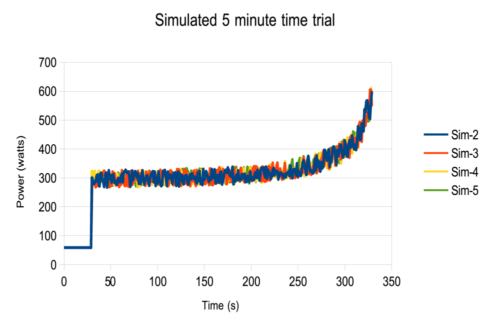

Figure 3: Simulation of 5 participants engaging in a 5-minute time trial at maximum effort.

As time decreases, the memory of having successfully completed efforts at higher and higher intensities are retrieved. Figure 3 shows that this produces the end-spurt phenomenon through application of prior memories, without recourse to uncertainty per-se.

We note that this mechanism differs from that of Tucker (2009), who suggested that the participant was engaging in uncertainty reduction. Our model suggests instead that, due to failures that occur when the effort goes over a sustainable baseline, such extreme efforts are only feasible near the end of an interval. The model has no successful instances of an effort with an early extreme effort to draw from. On the other hand, the model can draw from other historical examples of completing shorter duration efforts at higher intensity. The end-spurt is an emergent phenomenon of this memory-based appraisal process.

The model thus reasons backwards from the time it can quit throughout the effort. While we model uncertainty in that estimation, resulting in the jitter exhibited by individual simulations, the uncertainty does not change over time. The model simply continuously appraises the maximal effort it can maintain until the time when it can quit. We also note that a variety of alternative pacing strategies can also be implemented within this framework – the strategy described here is perhaps the simplest, but many are realizable.

Conclusions and Future Directions

Some widely accepted current theories of human endurance exercise pacing have been criticized as “brainless” (Noakes, 2011). Indeed, these theories treat the exercising individual much like a solid rocket – once ignition is achieved, the rocket burns until the fuel is gone, often at the ideal rate for the event duration. While this may seem silly, the ability of athletes to finely gauge their performances to match the limit of their endurance often masks the role of cognition, allowing this description to capture the rough shape of behavior. It is not until we look at inexperienced athletes that we must account for the import of learning, and thus cognition, in pacing behavior.

Our model, though simple, accomplishes the following:

- Defines a conceptual theoretical model of cognitive control of pacing
- Defines the interaction of central cognition with a central pattern generator as a mechanism for rhythmic exercise.
- Defines the role of memory of prior efforts in determining appropriate efforts.
- Predicts developmental failures in pacing and, in particular, oscillations between excessive and insufficient effort to maximize performance.
- Predicts improvements in pacing performance with accumulation of memories of prior successes and failures of pacing to draw from.
- Predicts a conservative undershoot heuristic to avoid failure of the effort.
- Predicts an end-spurt phenomenon as a means of spending an energy budget after undershooting

To our knowledge, this is the first model of endurance exercise pacing to situate pacing as a series of ongoing decisions within a broader cognitive framework capable of performing other cognitive tasks. As such, it represents a bridge between two disparate research communities. This disparity presents the corresponding challenge of sharing ideas in the absence of a shared common vocabulary to describe the phenomenon. This paper is one step toward establishing some of that shared context within the cognitive science community, connecting cognitive behavior back to the physical world of human physiology.

Our future efforts will also explore modeling incomplete knowledge and attempt to capture the developmental trends more clearly. When presented with few or no relevant instances of prior behavior, the current model easily ends up in a failure state, where it is unable to complete the effort, or unable to settle on a steady state that corresponds to a sustainable effort. While we have not yet attempted to model that learning phenomenon, our initial explorations suggest that this platform will provide an excellent path forwards to modeling the learning of pacing behavior.

Finally, what the model presented here lacks, in its current form, are a complete instantiation within a cognitive architecture, and a thorough validation against rich data sets. Our future efforts will be concentrated in this direction.

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