Fault Diagnosis of P-Time Labeled Petri Net Systems

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This paper focuses on the fault diagnosis problem of systems modeled with P-time labeled Petri nets with partial information. Indeed, the set of transitions is partitioned into those labeled with the empty string ϵ called silent (as their firin cannot be detected) including the faulty transitions and the observable ones. The proposed approach is based on the synthesis of a function called diagnoser allowing to determine the diagnosis state of the system based on the current observation. The novelty of the developed approach resides in the fact that, although the time factor is considered as intervals, the diagnoser is computed thanks to the underlying untimed Petri net structure of the P-time labeled model considered. Furthermore, the method relies on the schedulability analysis of particular firin sequences exhibited by the analysis of the obtained diagnoser and does not require the building of the state class graph.

Discrete event systems. Petri nets. Time labeled systems. Observability. State estimation. Fault diagnosis.

1. INTRODUCTION

The correct behavior of a real-world application is the ultimate requirement, particularly for systems such as communication protocols, manufacturing and real-time systems. Indeed, a drift from an expected behavior can be of crucial importance and can even lead, in extreme cases, to severe consequences including human losses. So, knowing the current state of a system in order to take the appropriate decisions and determining the malfunction of a system component are nowadays fundamental issues.

From a practical point of view, associating a dedicated sensor to each variable of interest in order to monitor its internal state is inconceivable. This restriction, due to economical or physical accessibility reasons leads to a system analysis in presence of uncertainties as the state information cannot be directly obtained. This particularity has gave rise to the introduction of the observers paradigm in the classical system theory. Indeed, an observer can be viewed as a mechanism allowing to estimate or reconstruct the internal state of a system based on some measurements. From a discrete event dynamic systems point of view and more precisely from a Petri net (PN on short) perspective this issue corresponds to the estimation of a PN marking based on some event observations.

Thus, being given a sequence of observed events (called word or trace) the challenge consists in determining if a fault has occurred, eventually or for sure!

It can be noticed that the problems of fault diagnosis has receive extensive attention these recent years and particularly in the framework of automata models and regular languages (Sampath et al. (1995), Cassandras and Lafortune (2008), Lin (1994), Cassez and Tripakis (2008)) but there are few studies in the time discrete event systems context.

A preliminary version of this paper was presented in (Bonhomme (2014)) where an approach allowing to estimate the marking of a P-time labeled Petri net (P-TLPN) system based on the observation of particular labels was presented. The plant observation is given by a set of labels whose occurrence can be detected/observed by an external agent (called observer or estimator) - these particular labels are associated to observable transitions. The other transitions, the unobservable ones (called silent transitions) are labeled with the empty string ϵ.

In this extended and enriched version, a fault diagnosis problem is solved thanks to the introduction of a function called diagnoser which associates to each observation a diagnosis state. In the proposed technique the set of unobservable transitions is further partitioned into the set of faulty transitions and the set of regular ones. The regular transitions are unobservable and non faulty.

The proposed approach does not require the state class graph construction and consequently it is designed to alleviate the state space explosion problem. Indeed, the construction of the considered state observer is based on the analysis of the underlying untimed PN structure of the P-time labeled PN considered.

In particular, the following four assumptions are made:

1. the net structure and the initial marking are known,
2. the fault model is known,
3. the underlying untimed PN, of the P-TLPN considered is bounded,
4. the Petri net induced by the set of unobservable transitions does not contain circuit of null length.

Note that this latter assumption is adopted to exclude the situation where an infinit of actions may take place in a finit amount of time: it prevents the net induced by the set of unobservable transitions from being Zeno (Hadjidj et al. (2007)) which is in contradiction with a diagnosability scheme. In addition, there is no assumption on the backward conflic freeness of the subnet induced by the set of unobservable transitions as in (Giua et al. (2007)).

The paper is organized as follows: after an overview of the relevant literature in the next section, a brief reminder of the basics of untimed Petri nets followed by a formal definitio of P-time labeled Petri nets is realized in the third section. Section four covers the procedure of estimation and the construction of the state observer. The schedulability analysis of the occurrence sequence highlighted by the state observer and its application to the estimation problem are studied in the fift section. In the sixth section the fault diagnosis problem is solved. Section seven presents an illustration of the developed method and the last section concludes the paper and gives suggestions for future research.

2. LITERATURE REVIEW

For discrete event system (DES) state estimation has been addressed by several researchers. For instance, in (Giua et al. (2007)) the authors deal with the marking estimation of a labeled Petri net system. Thanks to structural assumptions on the subnet induced by the set of unobservable transitions, they propose an algebraic characterization of the set of consistent markings once a sequence is observed.

In the framework of fault detection or fault diagnosis several approaches can also be found in the literature - fault diagnosis is closed to the state estimation problem. Note that a complete survey of fault diagnosis methods for DES can be found in (Zaytoon and Lafortune (2013)). In (Cabasino et al. (2010)) the authors proposed a diagnosis approach based on the concept of basis marking and justificatio under the acyclicity assumption of the unobservable subnet of the system considered. Intuitively, for an observed sequence (word) ω, a justificatio can be thought as the set of minimal (in terms of firin vector) unobservable transitions interleaved with ω necessary to complete ω into a fireabl sequence on the net considered, from the initial marking. They extended their work in (Cabasino et al. (2014)) to provide a diagnosability approach for bounded labeled PN by introducing two graphs, namely the modifie basis reachability graph (MBRG) and the basis reachability diagnoser (obtained from the MBRG). Necessary and sufficien conditions for diagnosability are given but the construction of the two graphs is of exponential complexity with respect to the structure of the PN considered and its initial marking.

There are relatively few works in this topic in the time discrete event systems scheme where the time factor is modeled as intervals, so, numerous problems are still open. Concerning the time Petri net model of Merlin (Merlin and Faber (1976)), the authors in (Basile et al. (2013)) proposed a procedure for estimating the marking of the model in presence of unobservable transitions. They introduced a modifie state class graph which captures the required information on the possible evolution of the system starting from a given initial marking. Thanks to this graph, being given a timed sequence and a time instant, the set of markings consistent with the current observation is determined via integer linear programming techniques. The approach is restricted to bounded time Petri nets.

In a recent paper, the authors in (Basile et al. (2015)) extend the previously mentioned approach developed in (Basile et al. (2013)) to deal with the state estimation and the fault diagnosis problem for systems modeled by time PN augmented with labels.

The authors in (Wang et al. (2013)), thanks to a fault diagnosis graph (FDG) which is a truncation of the conventional state class graph (SCG) (Berthomieu and Diaz (1991)), developed an online technique for the fault diagnosis of systems modeled by unlabeled time Petri nets. The FDG is constructed incrementally with respect to the current observation and its number of states can be, in the worst case, the same as the one of the traditional state class graph. Indeed, the FDG is obtained from the SCG by only keeping the information required for the evaluation of the fault states and the authors concentrate on the sequence information and remove the irrelevant state classes (i.e., which are not used in the fault diagnosis). Intuitively, the state classes which are obtained after the firin of an unobservable transition are discarded as the diagnosis state is updated after an observation.

The acyclicity assumption of the subnet induced by the unobservable transitions is also considered. The authors further extend the method in (Wang et al. (2014)) by using reduction rules and model checking techniques.

3. PETRI NETS

3.1. Untimed Petri Nets

The reader unfamiliar with Petri nets can refer to (Murata (1989)), in the following only the basic notions are recalled.

A Place/Transition net (P/T net) is a structure $N = (P, T, Pre, Post)$, where P is a set of m places; T is a set of n transitions. $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$ are the pre and post incidence functions that specify the arcs; $C = Post - Pre$ is the incidence matrix. The preset and postset of a node $X \in P \cup T$ are denoted $^\circ X$ and X°. A marking is a vector $M : P \to \mathbb{N}$ that assigns to each place of a P/T net a non-negative integer number of tokens, represented by black dots. $M(p)$ is the marking of place p.

A net system $\langle N; M_0 \rangle$ is a net N with an initial marking M_0. A transition t is marking enabled at M if $M \geq Pre(\cdot, t)$. A transition t enabled at M may fire yielding the marking $M' = M + C(\cdot, t)$. We write $M[\sigma >$ to denote that the sequence of transitions σ is enabled at M, and we write $M[\sigma > M'$ to denote that the firin of σ yields M'. A marking M is reachable in $\langle N; M_0 \rangle$ iff there exists a firin sequence σ such that $M_0[\sigma > M$.

The set of all sequences that are enabled at the initial marking M_0 is denoted $L(N, M_0)$ i.e., $L(N, M_0) = \{\sigma \in T^\star | M_0[\sigma >\}$ with $T^\star$ the Kleene closure of set T i.e. the set of all firin sequences of elements of T of arbitrary length, including the empty sequence λ. The notation $\sigma'\sigma$ will correspond to the firin sequence σ' followed by firin sequence σ, i.e., the concatenation operation ; σ' is the prefi of firin sequence $\sigma'\sigma$.

The set of all markings reachable from M_0 define the reachability set of $\langle N; M_0 \rangle$ and is denoted $R(N, M_0)$.

Given a net $N = (P, T, Pre, Post)$ and a subset $T_s \subseteq T$, the T_s-induced subnet of N is the net $N_s = (P, T_s, Pre_s, Post_s)$ where Pre_s and $Post_s$ are the restrictions of Pre and $Post$ to T_s. So, the net N_s is obtained from N by removing all transitions in $T \setminus T_s$, it is denoted also by $N_s \angle_{T_s} N$.

3.2. Labels mapping

A labels mapping $\mathcal{LM}$ is associated to each transition of the net considered as follows

$$\mathcal{LM} : T \to \Omega \bigcup \{\epsilon\},$$

with Ω a finit alphabet and ϵ the empty string.

In the proposed approach, the set of transitions is partitioned into two sets: observable transitions whose firin can be detected by an external observer, denoted as T_o and unobservable transitions whose firin cannot be detected, denoted as T_u with $T = T_o \cup T_u$ and $T_o \cap T_u = \emptyset$.

More precisely, the following stands:

- $T_u = \{t \in T | \mathcal{LM}(t) = \epsilon\}$, transitions in T_u are also called silent,
- $T_o = \{t \in T | \mathcal{LM}(t) \neq \epsilon\}$ (i.e., T_o is the set of transitions labeled with a symbol in Ω).

In the proposed approach, the same label $\zeta \in \Omega$ can be shared by several transitions, i.e., two transitions t_i, t_j with $t_i \neq t_j$ will be called indistinguishable if:

$$\mathcal{LM}(t_i) = \mathcal{LM}(t_j) = \zeta.$$

The extension of the label mapping can be realized over sequences, $\mathcal{LM} : T^\star \to \Omega^\star$, recursively as follows:

1. $\mathcal{LM}(t_i) = \zeta \in \Omega$ if $t_i \in T_o$,
2. $\mathcal{LM}(t_i) = \epsilon$ if $t_i \in T_u$,
3. let $\sigma \in T^\star$ and $t_i \in T$ then $\mathcal{LM}(\sigma t_i) = \mathcal{LM}(\sigma)\mathcal{LM}(t_i)$,
4. $\mathcal{LM}(\lambda) = \epsilon$ where λ is the empty sequence.

3.3. P-time Petri Nets

Definitio 1 *The formal definitio of a P-TPN (Khansa et al. (1996)) is given by a pair $\langle N; I \rangle$ where:*

- *N is a marked Place/Transition net (a P/T net system augmented with a marking)*
- $P \to (\mathbb{Q}^+ \cup \{0\}) \times (\mathbb{Q}^+ \cup \{\infty\})$,
- $p_i \to I(p_i) = [a_i, b_i]$ *with* $0 \leq a_i \leq b_i$

With:

- P: the set of places of the net N,
- $\mathbb{Q}^+$: the set of positive rational numbers,
- I_i define the static interval of the operation duration of a token in a place p_i.

A token in place p_i will be considered in the enabledness of the output transitions of this place if it has stayed for a_i time units at least and b_i at the most. Consequently, the token must leave p_i, at the latest, when its operation duration becomes b_i. After this duration b_i, the token will be "dead" and will no longer be considered in the enabledness of the transitions. According to the strong firin mode, a transition in a P-TPN, is forced to fir unless it is disabled by the firin of another conflictin transition.

Let consider α_i the clock associated with the token denoted $i \in TK$ of the P-TPN (TK being the set of tokens of the P-TPN considered). υ is a valuation of the system, i.e., a mapping associating to each token i of the P-TPN, an element of $(\mathbb{R}_{\geq 0})$, υ_i, representing the time elapsed since the token i has been created (i.e., the valuation of the clock α_i). So, $\upsilon \in (\mathbb{R}_{\geq 0})^{TK}$ with the notation A^X representing the set of mappings from X to A. $\overline{0}$ is the initial valuation with $\forall i, \overline{0}_i = 0$

The semantics of a P-TPN can be define as a Timed Transition System (TTS). A state of the TTS is a couple $s = (M, \upsilon)$ where M is a marking and υ a valuation of the system.

Definitio 2 *The semantics of a P-TPN $\langle N; I \rangle$ is define by the Timed Transition System $\mathcal{S}_N = (\mathcal{Q}, \{q_0\}, \Sigma, \longrightarrow)$:*

1. $\mathcal{Q} = \mathbb{N}^P \times (\mathbb{Q}_{\geq 0})^{TK}$
2. $q_0 = (M_0, \overline{0})$
3. $\Sigma = T$
4. $\longrightarrow \in \mathcal{Q} \times (\Sigma \cup \mathbb{Q}_{\geq 0}) \times \mathcal{Q}$

- *The continuous transition is define $\forall d \in \mathbb{R}_{\geq 0}$ by:*

$$(M, \upsilon) \xrightarrow{d} (M, \upsilon') \text{ iff} \begin{cases} \upsilon' = \upsilon + d. \\ \forall \text{ token } k \text{ in } p_s \Rightarrow \upsilon'_k \leq b_s. \end{cases}$$

- *The discrete transition is define $\forall t_i \in T$ by:*

$(M, \upsilon) \xrightarrow{t_i} (M', \upsilon')$ *iff:*

$$\begin{cases} M \geq^\circ t_i. \\ \forall \text{ token } k \text{ in } p_l, \upsilon_k \leq b_l. \\ \forall p_s \in^\circ t_i, \forall \text{ token } k \text{ in } p_s \text{ involved in } t_i\text{'s firin :} \\ \bigcap_k [max(0, a_s - \upsilon_k), (b_s - \upsilon_k)] \neq \emptyset. \\ M' = M -^\circ t_i + t_i^\circ. \\ \forall \text{ token } r, \upsilon'_r = \begin{cases} 0 \text{ if created by } t_i. \\ \upsilon_r \text{ otherwise.} \end{cases} \end{cases}$$

The dynamic evolution of a P-TPN depends on the timing situation of each token. Indeed, each token will be associated with a potential firin interval (or dynamic interval) which can be different from its static one. For instance, consider place p_i with static interval $[a_i, b_i]$, let a token arrive in place p_i at absolute time τ. At τ its potential firin interval will correspond to $[a_i, b_i]$. At time $\tau + c$ with $c \leq b_i$ the dynamic interval of the considered token will become $[max(a_i - c, 0), b_i - c]$. It can be noticed that a token is considered as dead when its dynamic interval becomes $[0, 0]$.

Definitio 3 *A P-time labeled Petri net (P-TLPN on short) over an alphabet Ω is a triple $\langle N, I, \mathcal{LM} \rangle$ where $\langle N, I \rangle$ is a P-TPN and $\mathcal{LM} : T \rightarrow \Omega \bigcup \{\epsilon\}$ is a labeling function.*

Finally, given a sequence of labels (a word) $\omega \in \Omega^\star$, it is denoted by ω^k the k^{th} element in ω and the number of elements of ω is denoted by $|\omega|$. For $a \in \Omega$, we write $a \in \omega$ if there exists $k \geq 1$ such that $\omega^k = a$ (i.e., a is an element of the word ω).

Furthermore, let $\omega_1, \omega_2, \ldots, \omega_n$ be n sequences of labels (i.e., $w_i \in \Omega^\star, 1 \leq i \leq n$), the notation $\omega = \omega_1\omega_2 \ldots \omega_n$ will be the concatenation of $\omega_1, \omega_2, \ldots, \omega_n$.

The next section recalls the procedure (Bonhomme (2015)) to construct the state observer.

4. ESTIMATION PROCEDURE

The goal of the observer is to give the current state estimate of the system based on the information of the observed traces. The state of the observer will consist in a set of states the model can be in after a label observation.

The following set will be associated to any observed word ω (i.e., the observed labels sequence):

- $\mathcal{L}(\omega)$ is the set containing all sequences of transitions that are consistent with ω, i.e., the set of all possible firin sequences that produce observation ω.

In general, if ω is an observed word, the associated firin sequence $\sigma \in \mathcal{LM}^{-1}(\omega)$ is not necessarily fireabl on the net as some unobservable transitions should be interleaved to obtain a fireabl sequence that produce ω.

Definitio 4 *Let N be a P-TLPN with $T = T_o \cup T_u$. The following operator is defined*

- *The projection over T_o is $P_o : T^\star \rightarrow T_o^\star$ define as:*
  - $P_o(\lambda) = \lambda$,
  - *for all $\sigma \in T^\star$ and $t \in T$, $P_o(\sigma t) = P_o(\sigma)t$ if $t \in T_o$ and $P_o(\sigma t) = P_o(\sigma)$ otherwise (with λ representing the empty sequence).*

Given a sequence $\sigma \in L(N, M_0)$, $\omega = \mathcal{LM}(P_o(\sigma))$ denotes the corresponding observed word.

Definitio 5 *Let N be a P-TLPN with $T = T_o \cup T_u$ and $\omega \in \Omega^\star$ be an observed word. $\mathcal{L}(\omega)$ is define as:*

$$\mathcal{L}(\omega) = P_o^{-1}(\mathcal{LM}^{-1}(\omega)) \cap L(N, M_0) = \{\sigma \in L(N, M_0) | \mathcal{LM}(P_o(\sigma)) = \omega\},$$

i.e., the set of firin sequences consistent with $\omega \in \Omega^\star$.

Definitio 6 *Let N be a P-TLPN with $T = T_o \cup T_u$ and $\omega \in \Omega^\star$ be an observed word. $\mathcal{C}(\omega)$ is define as:*

$$\mathcal{C}(\omega) = \{M \in R(N, M_0) | \exists \sigma \in \mathcal{L}(\omega) : M_0[\sigma > M\},$$

i.e., the set of markings consistent with ω.

So, being given an observed word ω, $\mathcal{L}(\omega)$ is the set of sequences that may have fire while $\mathcal{C}(\omega)$ is the set of markings in which the system may actually be.

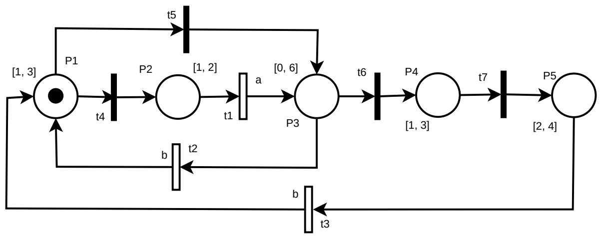

Figure 1: *P-TLPN model.*

Let consider the P-TLPN of Figure 1 with $T_u = \{t_4, t_5, t_6, t_7\}$, $T_o = \{t_1, t_2, t_3\}$, $\Omega = \{a, b\}$. It holds $\mathcal{LM}(t_1) = a, \mathcal{LM}(t_2) = \mathcal{LM}(t_3) = b$ (transitions t_2 and t_3 are indistinguishable) and $\mathcal{LM}(t_i) = \epsilon, \forall t_i \in T_u$.

If the observed word is $\omega = ab$ then $\mathcal{LM}^{-1}(\omega) = \{t_1t_2, t_1t_3\}$ and $\mathcal{L}(\omega) = \{t_4t_1t_2, t_4t_1t_6t_7t_3\}$ and $\mathcal{C}(\omega) =$ [10000].

Definitio 7 *Let N be a P-TLPN with $T = T_o \cup T_u$, the unobservable reachability mapping $\mathcal{UR}$, which enables to fin the markings reachable from a given marking M_i, following the firin of all unobservable sequences is define as:*

$\mathcal{UR} : \mathbb{N}^m \to 2^{\mathbb{N}^m}$,

$M_i \to \mathcal{UR}(M_i) = \{M_j \in \mathbb{N}^m | \exists \sigma_u \in T_u^\star, M_i[\sigma_u > M_j\}$,

with $2^{\mathbb{N}^m}$ the power set of the markings of the PN considered.

4.1. State observer

Let N_i and N_j be two nodes of the graphical representation of the state observer (associated respectively to the states y_i and y_j of the observer) such that it exists a directed arc linking N_i to N_j ($N_i \to N_j$, i.e., N_i is a predecessor of N_j) labeled with a_k with $a_k \in \Omega$ as illustrated on Figure2.

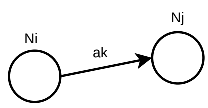

Figure 2: *nodes of the state observer.*

Definitio 8 *The state observer for the partially observable P-TLPN N with initial marking M_0 and $T = T_o \cup T_u$ is define by the 5-tuple $(Y_{so}, E_{so}, f_{so}, y_0, \varsigma_{so})$ where:*

- *Y_{so} is the set of states of the state observer,*
- *$E_{so} = \Omega$ is the set of labels (associated to the observable events),*
- *$\varsigma_{so} : Y_{so} \to 2^{R(N, M_0)}$ is a function associating to each state $y_{so} \in Y_{so}$ a set of reachable markings,*
- *y_0 is the initial state of the state observer and $\varsigma_{so}(y_0) = SEM(N_0) \cup SSM(N_0)$,*
- *$f_{so} : Y_{so} \times E_{so}^\star \to Y_{so}$ is the transition function define as :*
  for $y_l \in Y_{so}$ a state of the observer and $\omega \in E_{so}^\star$ a string of observable labels $f_{so}(y_0, \omega) = y_l$ if $\varsigma_{so}(y_l) \notin \emptyset$ where $\varsigma_{so}(y_l) = \left\{M_l : M_0 \xrightarrow{\tau} M_l \wedge \mathcal{LM}(P_o(\tau)) = \omega\right\} = SEM(N_l) \cup SSM(N_l)$.

With the two sets SSM and SEM define as follows:

Definitio 9 *Sets SSM and SEM*

- *$SEM(N_j)$, the Set of Entry Markings of N_j,*

$$SEM(N_j) = \{M_s \in N_j | \exists M_u \in N_i, t_k \in T_o, \\ a_k \in \Omega, \mathcal{LM}(t_k) = a_k : M_u[t_k > M_s\}$$

- *$SSM(N_j)$, the Set of Shadow Markings of N_j,*

$$SSM(N_j) = \{M_s \in N_j | \exists M_u \in SEM(N_j), \\ \sigma_u \in T_u^\star : M_u[\sigma_u > M_s\}$$

or equivalently, $SSM(N_j) = \mathcal{UR}(SEM(N_j))$.

Intuitively, for a given node N_s of the state observer, after the observation of the word ω, the set $SEM(N_s) \cup SSM(N_s)$ represents the set of markings that are consistent with the current observed word (i.e., $\mathcal{C}(\omega)$). The other nodes can be computed recursively as explained in the following.

1. The state observer starts in the initial state y_0 and its associated initial node N_0 is composed of $SEM(N_0) = \{M_0\}$ and $SSM(N_0) = \mathcal{UR}(M_0)$.
2. as soon as a label a_k (associated with an observable transition $t_k \in T_o$) is observed a new state y_l of the observer is calculated yielding a new node N_l:
   - the set of entry markings of node N_l is obtained by investigating the set of markings resulting from the firin of transition t_k starting from any marking ($SEM \cup SSM$) of N_0,
   - the set of shadow markings of N_l corresponds to the set of markings obtained by the firin of all unobservable sequences of transitions starting from any entry marking of N_l,

3. return to 2 with the newly calculated state as the initial state.

Definitio 10 *Let N_i and N_j be two nodes of the state observer, N_i and N_j are said to be equivalent ($N_i \Leftrightarrow N_j$) if and only if:*

$SEM(N_i) = SEM(N_j)$ *and* $SSM(N_i) = SSM(N_j)$.

Proposition 1 *Two nodes N_i and N_j of the state observer will be equivalent if and only if, the following holds:*

$SEM(N_i) = SEM(N_j)$.

Definitio 11 *Given a marking $M_i \in R(N, M_0)$ and a transition $t_f \in T_o$ (associated with a label $l_f \in \Omega$, i.e., $\mathcal{LM}(t_f) = l_f$), the set of candidate sequences denoted $CS(M_i, t_f)$ is the set of firin sequences, composed of the unique fina observable transition t_f, which can occur from M_i, i.e.:*

$CS(M_i, t_f) = \{s.t_f | s \in T_u^{\star} \cup \lambda, t_f \in T_o : M_i[s.t_f >\}$.

With respect to the timing constraints to be satisfied candidate sequences can be in the state possible or impossible.

As $N_u \angle_{T_u} N$ (i.e., the Petri net induced by the set of unobservable transitions) is not Zeno by assumption, it is ensured that the time is diverging with regard to the length of the firin sequences, thus, the set of candidate sequences from a marking is necessarily finit (at the instant of observation) and it can be investigated. The following section addresses the schedulability analysis (Bonhomme (2013b)) of an occurrence sequence (i.e., a procedure verifying if the considered firin sequence can occur without any violation of timing constraints) and its application to the estimation problem.

5. SCHEDULABILITY ANALYSIS AND ESTIMATION

Let $\sigma = t_a t_b t_c \ldots t_q$ be a firin sequence of length s (denoted $|\sigma| = s$). The j^{th} fire transition of σ will be associated with the j^{th} firin instant (Bonhomme (2013a)). A variable x_i will represent the elapsed time between the $(i-1)^{th}$ firin instant and the i^{th} one (with $x_0 = 0$).

For instance on Figure 3, ($x_2 + x_3$) is the time elapsed between the firs firin instant (associated with transition t_a) and the third one (transition t_c).

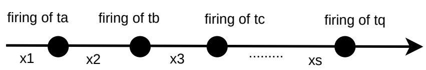

Figure 3: *Firing instants.*

In a P-TPN, the sojourn time (i.e., the amount of time that a token has been waiting in a place) is counted up as soon as the token has been dropped in the place as seen previously. To compute the firin instants, this approach requires that a token is identifie by three parameters: the place that contains it, the information of its creation instant and of its consumption one.

Function TOK is define with this purpose assuming that a FIFO queuing policy in the net is used in the sequel:

TOK:$\mathbb{N} \times (\mathbb{N} \setminus \{0\}) \times T^{\star} \rightarrow \wp(P)$),

$TOK(j, n, \sigma) = \{p \in P | p$ contains a token created by the j^{th} firin instant and consumed by the n^{th} one in firin sequence $\sigma\}$.

With $\wp(P)$ the set of subsets of P (also noted 2^P).

When it is clear from the context σ will be omitted in the notation of $TOK(.)$.

When the weight of the P-TPN arcs is element of $\mathbb{N}$, $TOK(j, n)$ is a multi-set. For the sake of simplicity, only ordinary P-TPN are considered (the arcs weight are element of $\{0, 1\}$).

Tokens, with the same creation instant, located in different places and involved in the same transition firin may mutually constrained their sojourn time, the following quantities, $Dsmin$ and $Dsmax$, are introduced in order to evaluate the contribution of these tokens. So, $Dsmin$ represents their availability in order to participate to this firin and similarly, $Dsmax$ expresses the fact that they all must be prevented from dying (with $[a_i, b_i]$ the static interval associated with the place p_i).

$$Dsmin(j, n) = \begin{cases} \max(a_i), & i \mid p_i \in TOK(j, n) \\ \text{else } 0 \text{ if } TOK(j,n) = \emptyset \end{cases},$$

$$Dsmax(j, n) = \begin{cases} \min(b_i), & i \mid p_i \in TOK(j, n) \\ \text{else } +\infty \text{ if } TOK(j,n) = \emptyset \end{cases}.$$

The definitio of the following set $SEN(q)$, allowing to determine the creation instants of tokens involved in the q^{th} firin instant, is also necessary:

$$SEN(q) = \{u | TOK(u, q) \subset ({}^{\circ}t_q)\}$$

To express more simply the obtained results, the definitio of the following coefficient is required:

$$c_{uq} = \begin{cases} Dsmin(u, q) & \text{if } u \in SEN(q) \\ 0 & \text{else} \end{cases},$$

$$d_{jk} = \begin{cases} Dsmax(j, k) & \text{if } TOK(j, k) \neq \emptyset \\ +\infty & \text{else} \end{cases}$$

With, $\forall (j, k) \in [0, q-1] \times [1, q], j \notin SEN(q)$ and $k \neq q$, then $c_{jk} = 0$, and $\forall k \in [0, q], x_k \geq 0$.

The following proposition is finall obtained:

Proposition 2 *A sequence of transitions $\sigma = t_1t_2....t_q$ is schedulable (i.e., it may be fi ed respectively at firin instants $1, 2, \ldots, q$) if and only if there exist $x_1 \geq 0$, $x_2 \geq 0$,..., $x_q \geq 0$ such that:*

$$\begin{cases} c_{0k} \leq x_1 \leq d_{0k}, k = 1, ..., n \\ \max\limits_{k=2,...,n} (c_{0k}, c_{1k} + x_1) \leq x_1 + x_2 \leq \min\limits_{k=2,...,n} (d_{0k}, d_{1k} + x_1) \\ \ldots \\ \max\limits_{\substack{j=0,...,q-1 \\ k=q,...,n}} (c_{jk} + \sum\limits_{s=0}^{j} x_s) \leq \sum\limits_{s=0}^{q} x_s \leq \min\limits_{\substack{j=0,...,q-1 \\ k=q,...,n}} (d_{jk} + \sum\limits_{s=0}^{j} x_s) \end{cases}$$

In the sequel this system will be denoted as $\mathcal{S}_\sigma(q)$ or simply $\mathcal{S}_\sigma$ when it is clear from the context.

Definitio 12 *The firin space at the q^{th} firin instant, associated with a firin sequence σ, denoted by $\mathcal{FS}_\sigma(q)$ is the set of non negative vectors $(x_1, ..., x_q)$ such that the fi st, the second, ...and the q^{th} firin conditions are satisfied Thus, a firin sequence $\sigma = t_1t_2....t_q$ is schedulable if and only if its associated firin space $\mathcal{FS}_\sigma(q)$ is non-empty.*

Thanks to this characterization of a firin sequence, the Zenoness property can be checked by evaluating the minimal duration of the circuit of unobservable transitions under consideration (for instance, by minimizing the sum of the x_i associated with the considered transitions).

Definitio 13 *A P-TLPN N_r firin schedule, will be a sequence of ordered pairs ($t_i, \sum\limits_{k=0}^{i} x_k$) ; transition t_i fi able at time ($\sum\limits_{k=0}^{i} x_k$), obtained from the state reached by starting from N_r initial state and firin the transitions t_j, $1 \leq j < i$, in the schedule at the given times.*

Finally, as in (Basile et al. (2015)), let denote:

$\omega_t = ((a_1, \tau_1), (a_2, \tau_2) \ldots (a_n, \tau_n)) \in (\Omega \times \mathbb{Q}^+)^\star$,

a time-label sequence (TLS), i.e., a sequence of pairs (observed label-time instant).

Indeed, in the considered sequence, label a_i is observed at absolute time τ_i ($i \geq 1$) and $\tau_1 \leq \tau_2 \ldots \leq \tau_n$.

Now all the required material for the proposed method is given, the principle is presented as follows:

- starting from the initial state, once a label a_f will be observed at the absolute time τ_f,
- the set of associated observable event $T_{a_f} = \{t \in T_o | \mathcal{LM}(t) = a_f\}$ will be evaluated,
- then, $\forall t_f \in T_{a_f}$ the set of feasible candidate sequences $CS(M_0, t_f)$ will be computed,
- a switch from node N_0 to node N_f (created by the observation of label a_f) is realized in the state observer,
- for each $\sigma_f \in CS(M_0, t_f)$ (with $P_o(\sigma_f) = t_f$) the associated linear system $\mathcal{S}_{\sigma_f}$ will be constructed,
- and each σ_f will be checked for schedulability with the following additional constraint:

$$\textstyle\sum_{i=0}^{|\sigma_f|} x_i = \tau_f.$$

Thanks to these considerations it is ensured that sequence σ_f is schedulable and the firin of t_f occurs at τ_f. Once a firin sequence is proved to be possible the set of markings the system can be in is then determined.

Let denote by $FEAS(N_0, t_f)$ the set of schedulable firin sequences from node N_0 ending with the unique observable transition t_f (it is a subset of the set of candidate sequences).

$$FEAS(N_0, t_f) = \{\sigma \in CS(M_0, t_f) | \mathcal{FS}_\sigma(|\sigma|)$$
$$\text{augmented with } \textstyle\sum_{i=0}^{|\sigma|} x_i = \tau_f \text{ is non-empty}\Big\}.$$

Furthermore, based on the knowledge of the schedulable candidate firin sequences only a subset of the set of entry markings of node N_f (resulting from the firin of transition t_f), denoted $SEM'(N_f)$, will be considered for the next step.
It holds:

$$SEM'(N_f) = \{M \in SEM(N_f) | M_0[\sigma > M,$$
$$\sigma \in FEAS(N_0, t_f)\}.$$

With $SEM'(N_f) \subseteq SEM(N_f)$.

Afterwards, if another label a_x is observed at absolute time τ_x then:

- The set of associated observable event $T_{a_x} = \{t \in T_o | \mathcal{LM}(t) = a_x\}$ will be evaluated,
- then, $\forall t_x \in T_{a_x}$ the set of feasible candidate sequences $CS(M_i, t_x)$ will be computed with $M_i \in SEM'(N_f)$,
- a switch from node N_f to node N_x is realized in the state observer,
- for each feasible firin sequence (on the underlying untimed PN) $\sigma'_f\sigma_x$ (i.e., $M_0[\sigma'_f\sigma_x >$) with $\sigma_x \in CS(M_i, t_x)$ and $\sigma'_f \in FEAS(N_0, t_f)$ the associated linear system $\mathcal{S}_{\sigma'_f\sigma_x}$ will be constructed.

  It is recalled that σ'_f is a schedulable firin sequence determined in the previous step with label a_f observed at τ_f and $P_o(\sigma'_f\sigma_x) = t_ft_x$.
- each previously determined $\sigma'_f\sigma_x$ will be checked for schedulability with the following additional constraint:

$$\textstyle\sum_{i=0}^{|\sigma'_f|+|\sigma_x|} x_i = \tau_x.$$

ensuring that the firin of t_x occurs at τ_x.

And so on, the same method is iteratively applied with respect to the current observation.

So, more formally the following principle is obtained: let ω_{obs} be an observed word (i.e., a sequence of labels $\omega_{obs} = a_1a_2a_3 \dots a_ia_{i+1} \dots \in \Omega^\star$) and let N_i $(i \geq 1)$ be the node of the associated state observer obtained after the observation of label $a_i \in \omega_{obs}$ detected at absolute time τ_i, as illustrated on the following figur (Figure 4).

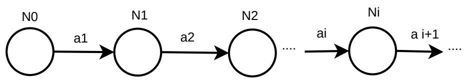

Figure 4: *Observable sequence.*

The associated sets $FEAS$ and SEM' are computed as follows:

Let $t_1 \in T_{a_1} = \{t \in T_o | \mathcal{LM}(t) = a_1\}$,

$FEAS(N_0, t_1) = \{\sigma \in CS(M_0, t_1) | \mathcal{FS}_\sigma(|\sigma|)$ augmented with $\sum_{k=0}^{|\sigma|} x_k = \tau_1$ is non-empty$\Big\}$.

$SEM'(N_0) = SEM(N_0) = \{M_0\}$ and

$$SEM'(N_1) = \{M \in SEM(N_1) | M_0[\sigma > M, \sigma \in FEAS(N_0, t_1)\}.$$

$\forall i > 0$,

Let $\mathcal{LM}(t_{i+1}) = a_{i+1}$,

$FEAS(N_i, t_{i+1}) = \{\sigma \in CS(M_b, t_{i+1}) | M_b \in SEM'(N_i),$ $M_0[\varpi >, \mathcal{FS}_\varpi(|\varpi|)$ augmented with $\sum_{k=0}^{|\varpi|} x_k = \tau_{i+1}$ is non-empty$\Big\}$.

With firin sequence $\varpi = \sigma_1\sigma_2 \dots \sigma_i\sigma$ where $\sigma_s \in FEAS(N_{s-1}, t_s), s \in \{1, \dots, i\}$ and $P_o(\varpi) = t_1t_2t_3 \dots t_it_{i+1}$.

More precisely:

$P_o(\sigma_j) = t_j, j \in \{1, \dots, i\}$ with $\mathcal{LM}(t_j) = a_j$.

$$SEM'(N_{i+1}) = \{M \in SEM(N_{i+1}) | M_k[\sigma > M, \sigma \in FEAS(N_i, t_{i+1}), M_k \in SEM(N_i)\}.$$

$SEM'(N_i)$ is the set of entry markings of node N_i resulting from the firin of schedulable firin sequences with respect to the current observation.

Roughly speaking, $FEAS(N_i, t_k)$ is the set of candidate sequences of node N_i ending with t_k and which can be completed by schedulable sub-sequences into a schedulable firin sequence starting from the initial marking of the P-TLPN considered.

So, by this way it is ensured that the feasible firin sequences associated with the observed time-label sequence $((a_1, \tau_1), (a_2, \tau_2) \dots (a_{i+1}, \tau_{i+1}))$ are effectively computed.

In the next section, addressing the fault diagnosis problem of a P-TLPN system, this set will be used to evaluate the state diagnosis associated with an observed TLS.

6. FAULT DIAGNOSIS

The set of unobservable transitions is partitioned into two subsets, $T_u = T_f \cup T_{reg}$ where the set T_f includes all the fault transitions (modeling anomalous or faulty behavior) while T_{reg} includes all unobservable transitions which correspond to regular events. Furthermore, the set T_f is partitioned into r different subsets T_f^i, where $i = 1, \dots, r$, that models the different fault classes.

Definitio 14 *Let $\langle N; M_0 \rangle$ be a net system with labeling function $\mathcal{LM} : T \rightarrow \Omega \bigcup \{\epsilon\}$, where $N = (P, T, Pre, Post)$ and $T = T_o \cup T_u$. Let consider the TLS $\omega_t = ((a_1, \tau_1), (a_2, \tau_2) \dots (a_n, \tau_n))$ associated with the state observer of Figure 4.*

Let define

$\sum(M_0, \omega_t) = \{\sigma \in T^\star | M_0[\sigma >, \sigma = \sigma_1\sigma_2 \dots \sigma_n :$

$\mathcal{LM}(\sigma_i) = a_i, i = 1, \dots, n, \sigma_s \in FEAS(N_{s-1}, t_s),$

$\mathcal{LM}(t_s) = a_s, s = 1, \dots, n\}$

Indeed, σ can be viewed as a concatenation of subsequences, namely $\sigma_i, i \geq 1$. Each subsequence σ_i is of the form $s.t_i$ with $s \in T_u^\star$, $\mathcal{LM}(t_i) = a_i$ and absolute firin instant of t_i is τ_i.

So, it holds:

$\sigma_i \in CS(M_b, t_i)$ with $M_b \in SEM'(N_{i-1})$.

Definitio 15 *A diagnoser is a function*

$$\Gamma : [\Omega \times \mathbb{Q}^+]^\star \times \{T_f^1, T_f^2, \dots, T_f^r\} \rightarrow \{N, U, F\}$$

that associates with each observed time-label sequence ω_t and each fault class T_f^i, where $i = 1, \dots, r$, a diagnosis state.

- $\Gamma(\omega_t, T_f^i) = N$ if $\forall \sigma \in \sum(M_0, \omega_t)$ and $\forall t_f \in T_f^i$, it is $t_f \notin \sigma$.

  In such a case the i^{th} fault cannot have occurred, because none of the firin sequences consistent with the considered observation contains a fault transition of class i.

- $\Gamma(\omega_t, T_f^i) = U$ if:
  1. $\exists \sigma \in \sum(M_0, \omega_t)$ and $t_f \in T_f^i$ such that $t_f \in \sigma$,
  2. $\exists \sigma' \in \sum(M_0, \omega_t)$ such that $\forall t_f \in T_f^i$, it is $t_f \notin \sigma'$.

  In such a case a fault transition of class i may have occurred or not, the diagnosis is in this case, uncertain.

- $\Gamma(\omega_t, T_f^i) = F$ if $\forall \sigma \in \sum(M_0, \omega_t), \exists t_f \in T_f^i$ such that $t_f \in \sigma$.

  In such a case the fault of class i must have occurred, because all firabl sequences consistent with the considered observation contains at least one fault transition of class i.

Let consider the P-TLPN of Figure1 with $T_u = \{t_4, t_5, t_6, t_7\}$, $T_o = \{t_1, t_2, t_3\}$, $\Omega = \{a, b\}$. It holds $\mathcal{LM}(t_1) = a, \mathcal{LM}(t_2) = \mathcal{LM}(t_3) = b$ (transitions t_2 and t_3 are indistinguishable). Furthermore, $T_f^1 = \{t_5\}$ and $T_f^2 = \{t_7\}$, i.e., there are two fault classes.

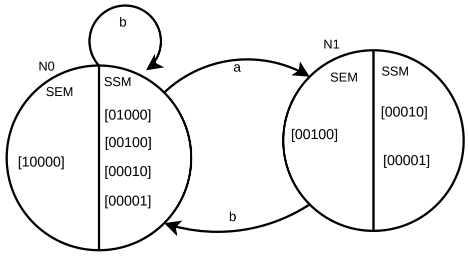

Figure 5: *State observer.*

The corresponding state observer with two nodes is depicted on Figure 5.

Let consider the following observed TLS $\omega_t = ((a, 2), (b, 5))$ then:

$\sum(M_0, \omega_t) = \{\omega_1, \omega_2\}$ with $\omega_1 = t_4 t_1 t_2$ and $\omega_2 = t_4 t_1 t_6 \underline{t_7} t_3$.

We have (according to the notations of definitio 14):

- $\omega_1 = \sigma_1 \sigma_2$ with $\sigma_1 = t_4 t_1$ and $\sigma_2 = t_2$,
- $\omega_2 = \sigma_1 \sigma_2$ with $\sigma_1 = t_4 t_1$ and $\sigma_2 = t_6 \underline{t_7} t_3$.

The two obtained candidate sequences are feasible with regard to the timing constraints. Indeed, the two associated firin schedules can be, for instance, considered respectively for ω_1 and ω_2:

- $((t_4, 1), (t_1, 2), (t_2, 5))$,
- $((t_4, 1), (t_1, 2), (t_6, 2), (t_7, 3), (t_3, 5))$.

It holds $t_7 \in T_f^2$ and $t_7 \in \omega_2$ ($t_7 \notin \omega_1$), and $t_5 \in T_f^1$, $t_5 \notin \omega_1, t_5 \notin \omega_2$.

So, $\Gamma(\omega_t, T_f^1) = N$ and $\Gamma(\omega_t, T_f^2) = U$.

It means, that according to the previous observed time label sequence ω_t, it is known for sure that the fault of class 1 (corresponding to fault transition t_5) cannot have occurred while fault transition $t_7 \in T_f^2$ may have occurred (via ω_2).

If the observed TLS corresponds to $\omega_t = (b, 1)$, it is easy to verify that $\sum(M_0, \omega_t) = \{\omega_3\}$ with $\omega_3 = t_5 t_2$ (the associated firin schedule is $((t_5, 1), (t_2, 1))$) and consequently, $\Gamma(\omega_t, T_f^1) = F$ and $\Gamma(\omega_t, T_f^2) = N$ (i.e., a fault of class T_f^1 occurs for sure and a fault of the second class cannot have occurred).

In the next section an illustrative example is presented where the T_u-induced subnet is cyclic.

7. ILLUSTRATIVE EXAMPLE

Let consider the P-TLPN of Figure 6 with $T_o = \{t_2, t_5\}$, $T_u = \{t_1, t_3, t_4, t_6, t_7\}$, $T_f = \{t_6\}$ and $\mathcal{LM}(t_2) = a, \mathcal{LM}(t_5) = b$. The T_u-induced subnet contains the cycle $(p_3 - t_4 - p_4 - t_6 - p_3)$.

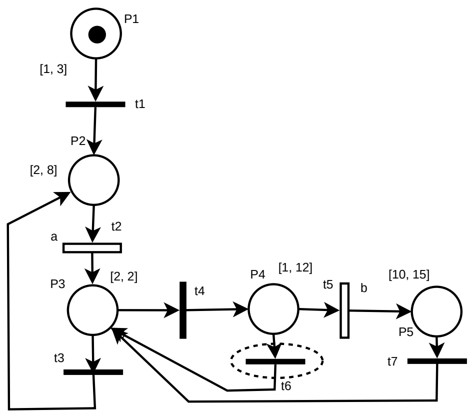

Figure 6: *P-TLPN with a cyclic T_u-induced subnet.*

The state observer is depicted on Figure 7, it consists of three nodes X_0, X_1 and X_2.

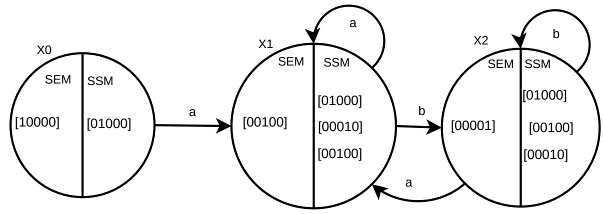

Figure 7: *State observer of the P-TLPN of Figure 6.*

If the observed word is $\omega = (a, b)$ then the set of possible associated firin sequences is of the form $t_1 t_2 t_4 (t_6 t_4)^{\star} t_5$ with the $\star$ after the subsequence $(t_6 t_4)$ (derived from the

Kleene star operator) indicating that it is allowed to occur from zero time to infinitel . Thanks to the time instant of occurrence of each label the set of feasible associated firin sequences is necessarily finite

For instance if the TLS considered is:

$\omega_t = ((a,3),(b,6))$ then $\sum(M_0,\omega_t) = \{\omega_1\}$ with $\omega_1 = t_1t_2t_4t_5$. The associated firin space $\mathcal{FS}_{\omega_1}(|\omega_1|)$ augmented with the following constraints:

- $x_1+x_2 = 3$ (absolute firin instant of transition t_2),
- $x_1+x_2+x_3+x_4 = 6$ (absolute firin instant of transition t_5),

is non-empty.

It holds:

$\omega_1 = \sigma_1\sigma_2$ with $\sigma_1 = t_1t_2$ and $\sigma_2 = t_4t_5$ and an example of firin schedule is:

$\varpi = ((t_1,1),(t_2,3),(t_4,5),(t_5,6))$,

and it is unique with respect to the static intervals of the P-TLPN places. So, it is easy to see that $\Gamma(\omega_t, T_f) = N$ and the faulty transition t_6 cannot have occurred.

If the TLS considered is now: $\omega_t = ((a,3),(b,9))$ then $\Gamma(\omega_t,T_f) = U$, as the computation of the set $\sum(M_0,\omega_t)$ leads to the following possible firin schedules (with the same observable projection), one containing the faulty transition and the other one not:

- $\varpi_1 = ((t_1,1),(t_2,3),(t_4,5),(t_5,9))$,
- $\varpi_2 = ((t_1,1),(t_2,3),(t_4,5),(\underline{t_6},6),(t_4,8),(t_5,9))$.

If the TLS considered is now: $\omega_t = ((a,3),(a,14))$ then $\Gamma(\omega_t,T_f) = F$. Indeed, the computation of the set $\sum(M_0,\omega_t)$ leads to the following possible firin schedule containing the faulty transition:

- $\varpi_2 = ((t_1,1),(t_2,3),(t_4,5),(\underline{t_6},10),(t_3,12),(t_2,14))$.

In this case the faulty transition occurs with certainty thanks to the timing structure of the P-TLPN considered and the occurrence date of the observed labels.

8. CONCLUSION AND PERSPECTIVES

In this paper, a new methodology allowing to analyze the fault diagnosis of systems modeled by P-time labeled Petri nets is developed. It is based on the construction of a function called diagnoser which associates with each observation and each fault class a diagnosis state. This diagnoser is obtained thanks to the synthesis of a state observer which is an automaton allowing to estimate the set of markings in which the system may be, being given a sequence of observed labels.

Furthermore, the considered state observer is computed on the basis of the untimed underlying Petri net of the P-time labeled PN considered. This particularity allows to avoid the combinatorial state space explosion problem usually associated with the consideration of the time factor modeled as time intervals.

Thanks to a schedulability analysis technique, the feasibility of the candidate firin sequences associated with the observed time-label sequence is evaluated via linear programming techniques.

An issue currently being investigated is the extension of the method to test the diagnosability property of P-TLPN systems, i.e., is the fault can be detected within a finit number of steps after its occurrence ?

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