Approximate Reasoning with Fuzzy-Syllogistic Systems

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Abstract. The well known Aristotelian syllogistic system consists of 256 moods. We have found earlier that 136 moods are distinct in terms of equal truth ratios that range in τ=[0,1]. The truth ratio of a particular mood is calculated by relating the number of true and false syllogistic cases the mood matches. A mood with truth ratio is a fuzzy-syllogistic mood. The introduction of (n-1) fuzzy existential quantifiers extends the system to fuzzy-syllogistic systems $^{n}\mathbb{S}$, 1<n, of which every fuzzy-syllogistic mood can be interpreted as a vague inference with a generic truth ratio that is determined by its syllogistic structure. We experimentally introduce the logic of a fuzzy-syllogistic ontology reasoner that is based on the fuzzy-syllogistic systems $^{n}\mathbb{S}$. We further introduce a new concept, the relative truth ratio $^{r}\tau$=[0,1] that is calculated based on the cardinalities of the syllogistic cases.

Keywords. Syllogistic reasoning; fuzzy logic; approximate reasoning.

1 INTRODUCTION

Multi-valued logics were initially introduced by Łukasiewicz [10], as an extension to propositional logic. After Zadeh generalised multi-valued logics within fuzzy logic [19], he discussed syllogistic reasoning with fuzzy quantifiers in the context of fuzzy logic [20]. However, this initial fuzzification of syllogistic moods was experimentally applied on only a few true moods and did not systematically cover all moods. The first systematic application of multi-valued logics on syllogisms were intermediate quantifiers and their reflection on the square of opposition [14]. However only set-theoretic representation of moods as syllogistic cases allow 64analysing the fuzzy-syllogistic systems $^{n}\mathbb{S}$ mathematically exactly, such as by calculating truth ratios of moods [6] and their algorithmic usage in fuzzy inferencing [7]. Here we present a sample application of $^{n}\mathbb{S}$ for fuzzy-syllogistic ontology reasoning.

Learning from scratch can be modelled probabilistically, as objects and their relationships need to be first synthesised from a statistically significant number of perceived instances of similar objects. This leads to probabilistic ontologies [4], [15], [11], in which attributes of objects may be synthesised also as objects.

There are more probabilistic ontology reasoners than fuzzy or possibilistic ones and most of them reason with probabilist ontologies [8]. Several ontology reasoners employ possibilistic logic and reason with fuzzy ontologies. The most popular reasoning logic being hyper-tableau, for instance in HermiT [12]. Other experimental reasoning logics are also interesting to analyse, such as fuzzy rough sets and Łukasiewicz logic [3] in FuzzyDL [1], Zadeh and Gödel fuzzy operators in DeLorean [2], Mamdani inference in HyFOM [18] or possibilistic logic in KAON [15]. Fuzzy-syllogistic reasoning (FSR) can be seen as a generalisation of both, fuzzy-logical and possibilistic reasoners.

A fuzzy-syllogistic ontology (FSO) extends the concept of ontology with the quantities that led to the ontological concepts. A FSO is usually generated probabilistically, but does not preserve any probabilities like probabilistic ontologies [11] or probabilistic logic networks [5] do. A FSO can be a fully connected and bidirectional graph.

Several generic reasoning logics are discussed in the literature, like probabilistic, non-monotonic or non-axiomatic reasoning [17]. Fuzzy-syllogistic reasoning in its basic form [21] is possibilistic, monotonic and axiomatic.

Syllogistic reasoning reduced to the proportional inference rules deduction, induction and abduction are employed in the Non-Axiomatic Reasoning System (NARS) [16]. Whereas FSR uses the original syllogistic moods and their fuzzified extensions [22].

There is one implementation mentioned in the literature that is close to the concept of syllogistic cases: Syllogistic Epistemic REAsoner (SEREA) implements poly-syllogisms and generalised quantifiers that are associated with combinations of distinct spaces, which are mapped onto some interval arithmetic. Reasoning is then performed with concrete quantities, determined with the interval arithmetic [13].

First the fuzzy-syllogistic systems ${}^{n}\mathbb{S}$ are discussed, thereafter fuzzy-syllogistic reasoning is introduced, followed by its sample application on a fuzzy-syllogistic ontology and the introduction of relative truth ratios ${}^{r}\tau$.

2 FUZZY-SYLLOGISTIC SYSTEMS

The fuzzy-syllogistic systems ${}^{n}\mathbb{S}$, with $1<n$ fuzzy quantifiers, extend the well known Aristotelian syllogisms with fuzzy-logical concepts, like truth ratio for every mood and fuzzy quantifiers or in general fuzzy sets. We discuss first the systems ${}^{n}\mathbb{S}$ and introduce them further below as the basic reasoning logic of FSR.

2.1 Aristotelian Syllogistic System $\mathbb{S}$

The Aristotelian syllogistic system $\mathbb{S}$ consists of inclusive existential quantifiers ψ, ie I includes A and O includes E as one possible case:

Universal affirmative: All S are P: ψ=A: $\{x \mid x \notin P\text{-}S \wedge x \in P \cap S\}$

Universal negative: All S are not P: ψ=E: $\{x \mid x \in S\text{-}P \wedge x \notin P\text{-}S\}$

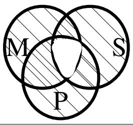
Inclusive existential affirmative: Some S are P: ψ=I: $A \cup \{x \mid (x \notin S\text{-}P \wedge x \notin P\text{-}S \wedge x \in P \cap S) \vee (x \notin S\text{-}P \wedge x \in P \cap S)\}$

Inclusive existential negative: Some S are not P: ψ=O: $E \cup \{x \mid (x \in S\text{-}P \wedge x \notin P\text{-}S \wedge x \notin P \cap S) \vee (x \in S\text{-}P \wedge x \notin P \cap S)\}$

A categorical syllogism $\psi_1\psi_2\psi_3F$ is an inference schema that concludes a quantified proposition $\Phi_3=S\psi_3P$ from the transitive relationship of two given quantified proportions $\Phi_1=\{M\psi_1P, P\psi_1M\}$ and $\Phi_2=\{S\psi_2M, M\psi_2S\}$:

$$\boldsymbol{\psi_1\psi_2\psi_3F = (\Phi_1=\{M\psi_1P,\ P\psi_1M\},\ \Phi_2=\{S\psi_2M,\ M\psi_2S\},\ \Phi_3=S\psi_3P)}$$

Table 1: Binary coding of the 7 possible distinct spaces for three sets.

<table>
<tr><td colspan="3">Syllogistic Case Δ_{95}</td></tr>
<tr><td>Binary code Δ_j= $\delta_1\delta_2\delta_3\delta_4\delta_5\delta_6\delta_7$*</td><td>Venn Diagram</td><td>Space Diagram+</td></tr>
<tr><td>Δ_{96}=1111110#</td><td></td><td>
δ_1 δ_2 δ_3 δ_4 δ_5 δ_6 δ_7</td></tr>
</table>

*Binary coding of all possible distinct space combinations Δ_j, j=[1,96] that can be generated for three sets.
#δ_i=0: space i is empty; δ_i=1: space i is not empty; i=[1,7].
+Every circle of a space diagram represents exactly one distinct sub-set of M∪ P∪ S.

where F={1, 2, 3, 4} identifies the four possible combinations of Φ_1 with Φ_2, namely syllogistic figures. Every figure produces 4^3=64 moods and the whole syllogistic system $\mathbb{S}$ has 4x64=256 moods.

2.2 Syllogistic-Cases

Syllogistic cases are an elementary concept of the fuzzy-syllogistic systems ${}^n\mathbb{S}$, for calculating truth ratios [6] of the moods algorithmically [7].

For three sets, 7 distinct spaces δ_i, i=[1,7] are possible, which can be easily identified in a Venn diagram (Table 1). There are in total j=96 distinct combinations of the spaces $\Delta_j=\delta_1\delta_2\delta_3\delta_4\delta_5\delta_6\delta_7$, j=[1,96] [22], which constitute the universal set of syllogistic moods. Out of this universe, we determine for every mood true and false matching space combinations (Fig 1).

2.3 Fuzzy-Syllogistic Moods

We extend the ancient binary truth classification of moods, to a fuzzy classification with truth values in [0,1]. For this purpose, first the above set-theoretical definitions of the quantifiers of a particular mood are compared against the set of all syllogistic cases Δ_j, j=[1,96], in order to identify true and false matching cases:

$$\textbf{True syllogistic cases: } \boldsymbol{\Lambda^t = {}_{j=1}\cup^{96}\Delta_j \in (\Phi^\Delta_1 \cap \Phi^\Delta_2) \rightarrow \Delta_j \in \Phi^\Delta_3}$$

$$\textbf{False syllogistic cases: } \boldsymbol{\Lambda^f = {}_{j=1}\cup^{96}\Delta_j \in (\Phi^\Delta_1 \cap \Phi^\Delta_2) \rightarrow \Delta_j \notin \Phi^\Delta_3}$$

where Λ^t and Λ^f is the set of all true and false matching cases of a particular mood, respectively (Fig 1) and Φ^Δ is a proposition in terms of syllogistic cases. For instance, the two premisses Φ_1 and Φ_2 of the mood IAI4 of the syllogistic system $\mathbb{S}$, match the 10 syllogistic cases $\Lambda^t = \{\Delta_4, \Delta_{19}, \Delta_{67}, \Delta_{24}, \Delta_{43}, \Delta_{46}, \Delta_{68}, \Delta_{74}, \Delta_{48}, \Delta_{76}\}$, which are all true for the conclusion Φ_3 as well. Thus the mood has no false cases $\Lambda^f = \emptyset$.

The truth ratio of a mood is then calculated by relating the amounts of the two sets Λ^t and Λ^f with each other. Consequently the truth ratio becomes either more true or more false $\tau \in \{\tau^f, \tau^t\}$:

$$\textbf{More true: } \boldsymbol{\tau^t \in \{|\Lambda^f|<|\Lambda^t| \rightarrow 1-|\Lambda^f|/(|\Lambda^t|+|\Lambda^f|)\} = [0.545,1]}$$

$$\textbf{More false: } \boldsymbol{\tau^f \in \{|\Lambda^t|<|\Lambda^f| \rightarrow |\Lambda^t|/(|\Lambda^t|+|\Lambda^f|)\} = [0,0.454]}$$

where $|\Lambda^t|$ and $|\Lambda^f|$ are the numbers of true and false syllogistic cases, respectively. A fuzzy-syllogistic mood is then defined by assigning an Aristotelian mood $\psi_1\psi_2\psi_3F$ the structurally fixed truth ratio τ:

Fuzzy-syllogistic mood: ($\psi_1\psi_2\psi_3$F, τ)

The truth ratio identifies the degree of truth of a particular mood, which we will associate further below in fuzzy-syllogistic reasoning with generic vagueness of inferencing with that mood.

The analysis of the Aristotelian syllogistic system $\mathbb{S}$ with these concepts reveals several interesting properties, like $\mathbb{S}$ has 136 distinct moods, 25 true moods τ=1, of which 11 are distinct, and 25 false moods τ=0, of which 11 are distinct, and that $\mathbb{S}$ is almost point-symmetric on syllogistic cases and truth ratios of the moods [22], [9].

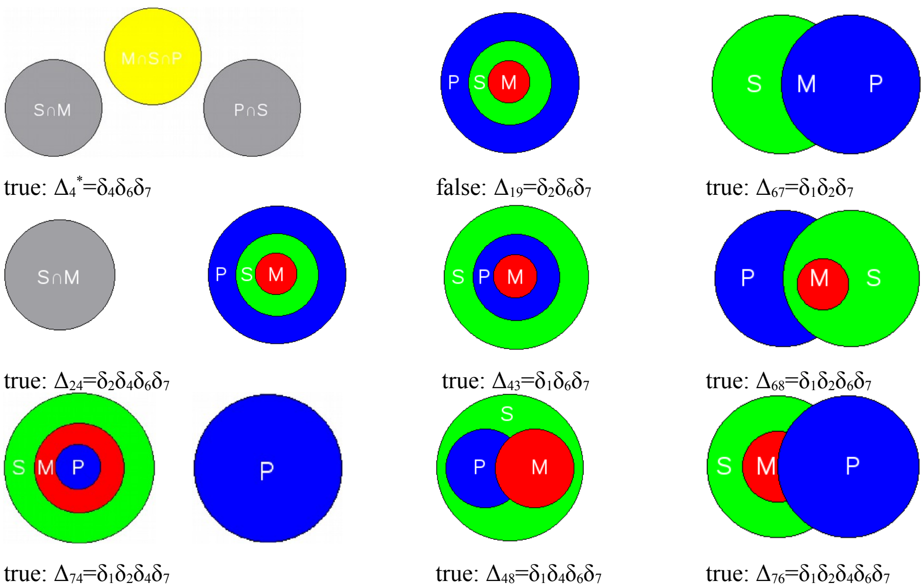
2.4 Fuzzy-Syllogistic System $^2\mathbb{S}$

In the fuzzy-syllogistic system (FSS) $^2\mathbb{S}$, the universal cases A and E are excluded from the existential quantifiers I and O, respectively:

Exclusive existential affirmative: Some S butNotAll are P: ψ=I: {x| (x∉S-P ∧ x∉P-S ∧ x∈P∩S) ∨ (x∉S-P ∧ x∈P∩S)}

Exclusive existential negative: Some S butNotAll are not P: ψ=O: {x| (x∈S-P ∧ x∉P-S ∧ x∉P∩S) ∨ (x∈S-P ∧ x∉P∩S)}

For instance the mood IAI4 of $\mathbb{S}$, becomes $^{2/1}$IA^{1}I4 in $^2\mathbb{S}$. Because of the exclusive existential quantifier $^{2/1}$I, the case Δ_{46} is no more matched by of the first premiss Φ_1 and the conclusion Φ_3 becomes false for the case Δ_{19} (Fig 1).



* A full list of all syllogistic cases Δ_j, j=[1,96], can be found elsewhere [22].

Fig 1. 9 syllogistic cases Δ_j of the mood $^{2/1}$IA^{1}I4 of the fuzzy-syllogistic systems $^2\mathbb{S}$.

Table 2. Value ranges of affirmative quantifiers of various fuzzy-syllogistic systems $^n\mathbb{S}$

Syllogistic System		Quantifier ψ*					
Aristotelian	$\mathbb{S}$	A=all	I=some (including A)				
Fuzzy	$^2\mathbb{S}$	A=all	$^{2/1}$I=some (excluding A)				
	$^3\mathbb{S}$	A=all	$^{3/2}$I=most		$^{3/1}$I=several		
	$^4\mathbb{S}$	A=all	$^{4/3}$I=most	$^{4/2}$I=half	$^{4/1}$I=several		
	$^5\mathbb{S}$	A=all	$^{5/4}$I=many	$^{5/3}$I=most	$^{5/2}$I=several	$^{5/1}$I=few	
	$^6\mathbb{S}$	A=all	$^{6/5}$I=many	$^{6/4}$I=most	$^{6/3}$I=half	$^{6/2}$I=several	$^{6/1}$I=few
	$^n\mathbb{S}$	A=all	$^{n/n-1}$I	...			$^{n/1}$I

* Column breadths are not drawn proportional to the overall value range and to the other quantifiers.

The analysis of the FSS $^2\mathbb{S}$ shows that $^2\mathbb{S}$ has 70 distinct moods, 11 true moods $\tau=1$, of which 5 are distinct, and 40 false moods $\tau=0$, of which 13 are distinct, and that $^2\mathbb{S}$ is not point-symmetric [22], [9].

2.5 Fuzzy-Syllogistic System $^n\mathbb{S}$

By using (n-1) fuzzy-existential quantifiers, the total number of fuzzy-syllogistic moods of the FSS $^n\mathbb{S}$ increases to $(2n)^3$. The sample mood IAI4 of $\mathbb{S}$ can now be generalised to $^{n/k1}$IA^{k2}I4, $1<n$, $0<k_1,k_2<n$ of $^n\mathbb{S}$. $^{n/k1}$IA^{k2}I4 consists of $(n-1)^2$ fuzzy-moods, all having the very same 9 syllogistic cases (Fig 1).

Same linguistic terms used in different FSSs do not necessarily equal each other. For instance, "most" may have different value ranges in the FSSs $^3\mathbb{S}$, $^4\mathbb{S}$, $^5\mathbb{S}$, $^6\mathbb{S}$ and therefore are in general not equal $^{3/2}I \neq {}^{4/3}I \neq {}^{5/3}I \neq {}^{6/4}I$, respectively. Likewise for "half" in $^4\mathbb{S}$ and $^6\mathbb{S}$ the quantifiers may not exactly equal $^{4/2}I \neq {}^{6/3}I$, respectively (Table 2).

3 FUZZY-SYLLOGISTIC ONTOLOGY

A fuzzy-syllogistic ontology (FSO) consists of concepts, their relationships and assertions on them, whereby all quantities are given with fuzzy-quantifications:

Fuzzy-syllogistic ontology: FSO=^{k}(C, R, A)

where C is the set of all concepts, R is the set of all directed relationships between the concepts and A is the set of all assertions. A FSO may be specified top-down or may be transformed from any existing ontology, provided that all quantities are determined systematically, in compliance with one of the FSSs $^k\mathbb{S}$, $1<k\leq n$, (Table 2). In a bottom-up approach, a FSO may be learned from given domain data.

3.1 Learning Fuzzy Quantifiers

Although existing learning approaches generate ontological concepts and their relationships through probabilistic analysis of the data [4], [15], [11], the quantities that actually imply the concepts and relationships, are not preserved in the ontology [8]. Therefore we sketch here briefly how to learn such quantities of a FSO.

For any directly connected triple concept relationship of the FSO, seven distinct relationships are possible (Table 1). The quantity of every such relationship has to be stored with the FSO. Since the relationships may be bi-directional or a concept may be involved in multiple triple relationships (Fig 2), the quantities of all these cases need to be stored too.

The objective of learning a FSO=^{k}(C, R, A) is, to update the FSO against changing domain data and to determined the most appropriate FSS $^{k}\mathbb{S}$ out of $^{n}\mathbb{S}$, $1<k\leq n$.

3.2 Relative truth ratio

Relative truth ratios are calculated from the exact quantities of all syllogistic cases of a particular mood, rather than from just the amount of the cases:

$$\textbf{Relative true: } {}^{r}\tau^{t} = \lambda^{f}<\lambda^{t} \rightarrow \lambda^{f}/(\lambda^{f}+\lambda^{t})$$

$$\textbf{Relative false: } {}^{r}\tau^{f} = \lambda^{t}<\lambda^{f} \rightarrow \lambda^{t}/(\lambda^{t}+\lambda^{f})$$

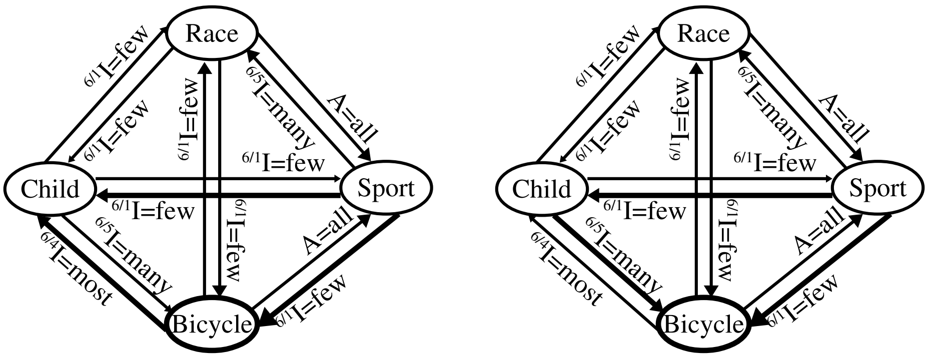
where $\lambda^{t} = {}_{j=1}\sum^{|\Lambda t|} |\Delta^{t}_{j}|$ and $\lambda^{f} = {}_{j=1}\sum^{|\Lambda f|} |\Delta^{f}_{j}|$ is the total number of elements accumulated over all true and false syllogistic cases, respectively. Where $|\Lambda^{t}|$ and $|\Lambda^{f}|$ is the number of true and false cases of the mood, respectively. Accordingly, we can re-define a fuzzy-syllogistic mood with relative truth ratio $^{r}\tau$:

Fuzzy-syllogistic mood with relative truth ratio: ($\psi_1\psi_2\psi_3$F, $^{r}\tau$)

The structural truth ratio τ of a particular mood represents the generic vagueness of the mood and is constant, whereas the relative truth ratio $^{r}\tau$ adjusts τ by weighting every case of the mood with its actual quantity.

4 FUZZY-SYLLOGISTIC REASONING

The fuzzy-syllogistic systems $\mathbb{S}$, $^{2}\mathbb{S}$ and $^{6}\mathbb{S}$ are currently implemented experimentally as the reasoning logic of the fuzzy-syllogistic reasoner (FSR), for reasoning over FSOs [21]. Our objective is to generalise the logic of the reasoner to $^{n}\mathbb{S}$ and to use it as a cognitive primitive for modelling other cognitive concepts within a cognitive architecture. We now sketch the algorithmic design of the FSR.



$^{6}\mathbb{S}$: ($^{6/4}$I^{1}I^{1}I1, 40/47=0.851)

Φ_1: Most **bicycles** are good for children

Φ_2: Few sports are good for **bicycles**

Φ_3: Few **sports** are good for **children**

$^{6}\mathbb{S}$: ($^{6/5}$I^{1}I^{1}I2, 40/48=0.833)

Φ_1: Many children have **bicycles**

Φ_2: Few sports are good for **bicycles**

Φ_3: Few **sports** are good for **children**

Fig 2. Sample fuzzy-syllogistic ontology with affirmative relationships and the best matching fuzzy-syllogistic moods from the syllogistic figures 1 and 2.

4.1 Reasoning Algorithm

FSR is concerned with identifying for any given concept $c \in C$, all possible triple concept relationships $r \in R$, $r=\{M,P,S\}$, of the given $FSO={}^{k}(C, R, A)$ and to reason with the most appropriate fuzzy-syllogistic moods of its FSS ${}^{k}\mathbb{S}$. Whereby associated assertions $a \in A$ may be used for exemplifying a particular reasoning.

For instance, for the concept c=Bicycle, multiple triple relationships r={Bicycle, Child, Sports} exist in the sample $FSO={}^{6}(C, R, A)$ (Fig 2). The reasoner iterates for the FSSs ${}^{k}\mathbb{S}$, $k=[2,n]$, and for their moods, in order to match the moods with the closest fuzzy-syllogistic quantities of relationships r. The reasoner determines the FSS k=6 and the mood ${}^{6/k1}IA^{k2}I4$, $0<k_1,k_2<6$ as best matches for this example.

In the below example with $\mathbb{S}$, I in Φ_3 may include A and therefore is less true. Whereas in ${}^{3}\mathbb{S}$, ${}^{3/1}I$ in Φ_3 is still too general. The best matching quantifiers are found in ${}^{6}\mathbb{S}$ (Fig 3).

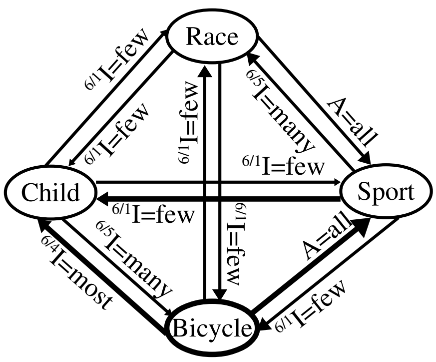
$\mathbb{S}$: (IAI3, 10/10=1.0)
Φ_1: Some **bicycles** are good for children
Φ_2: All **bicycles** are good for sports
Φ_3: Some **sports** are good for **children**

${}^{3}\mathbb{S}$: (${}^{3/2}IA^{1}I3$, 6/6=1.0)
Φ_1: Most **bicycles** are good for children
Φ_2: All **bicycles** are good for sports
Φ_3: Several **sports** are good for **children**

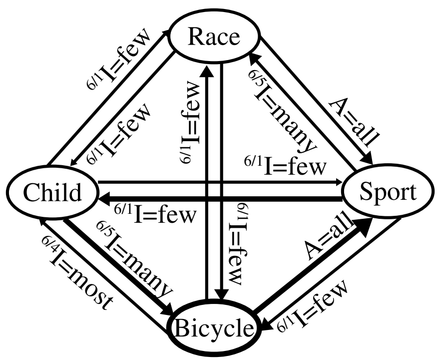
5 CONCLUSION

The FSSs $\mathbb{S}$, ${}^{2}\mathbb{S}$, ${}^{6}\mathbb{S}$ were introduced as the fundamental logic of the fuzzy-syllogistic reasoner (FSR) and its usage was exemplified on a sample fuzzy-syllogistic ontology (FSO). The relative truth ratio ${}^{r}\tau$ of a mood was introduced, which adapts the structural truth ratio τ of the mood to the amount of elements of its syllogistic cases. FSR with FSOs is a generic possibilistic reasoning approach, since the employed reasoning logic ${}^{n}\mathbb{S}$ is generic.

We are currently implementing a sample educational application that extends an existing probabilist ontology learning tool and generates a $FSO={}^{k}(C, R, A)$ for a given



${}^{6}\mathbb{S}$: (${}^{6/4}IA^{1}I3$, 6/6=1.0)
Φ_1: Most **bicycles** are good for children
Φ_2: All **bicycles** are good for sports
Φ_3: Few **sports** are good for **children**



${}^{6}\mathbb{S}$: (${}^{6/5}IA^{1}I4$, 8/9=0.888)
Φ_1: Many children have **bicycles**
Φ_2: All **bicycles** are good for sports
Φ_3: Few **sports** are good for **children**

Fig 3. Sample fuzzy-syllogistic ontology with affirmative relationships and the best matching fuzzy-syllogistic moods from the syllogistic figures 3 and 4.

domain. For a user-chosen concept C from the ontology FSO, FSR is then used to reason with all associated quantities R and present the user all associated scenarios A.

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