Accuracy Analysis of Estimation of 2–D Flow Profile in Conduits by Results of Multipath Flow Measurements

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Abstract. The paper presents a model of distorted velocity distribution of a flow in conduit, which passes through its bent section. The proposed model is properly analyzed. An algorithm for recovering the pipe profile by multipath ultrasonic measurements in the presence of *a priori* information is proposed. Numerical simulations with the proposed algorithm are performed as well.

Keywords: flow measurements, flow profile, multipath flowmeter

1 Introduction

In practice, it is important to know the set of values of the stream head of a flow in each section of the pipeline. The flow head is one of the main characteristics of the conduit. When the flow passes through a bent section (such as elbow or valve, etc.), its head gets lost. In practice, losses of the head are determined by experiments [1]. The loss of head is connected with the flow profile distortion. There is a symmetric flow profile in a long straight section of pipeline (Fig. 1a,b). However, when flow passes a bent section (such as elbow, valve, etc.), the flow profile, *i.e.*, the distribution of velocity in the cross section of the pipe, becomes distorted (Fig. 1 c,d). Thus, the profile of flow characterizes a bent section of the pipe [1].

One way to determine the flow profile is to recover it by ultrasonic multipath flow measurements. The ultrasonic technology allows measurement of the velocity distribution in a number of planes between two transducers/receivers of acoustic waves as it is shown in Fig. 2a. The distortion of a flow profile can be recovered by approximation of such measured values in a sufficient number of planes [2].

The task of ultrasonic measurement of the flow rate is the radar task of measurement of propagation time in an investigated media. The emitted wave accelerates when moving in the direction of a flow and slows down in the opposite direction (Fig. 2b). The velocity of a flow is calculated as a difference of measured times of propagation:

$$t_{12} = \frac{L}{c + v_l \cos\alpha}, \quad t_{21} = \frac{L}{c - v_l \cos\alpha}, \tag{1}$$

$$v_l = \frac{\Delta t c^2}{2L\cos\alpha}, \quad v_z = \frac{\Delta t c^2}{2L}\tan\alpha, \tag{2}$$

where t_{12} is the time of propagation in the flow direction; t_{21} is the time of propagation in the opposite direction; c is the speed of acoustic wave; α is the angle between direction of the flow and direction of the wave propagation; L is the length of the wave path in the investigated media; v_l is the projection of the flow velocity on the wave path; v_z is the velocity in z–direction of the flow [3].

The velocity profile is always distributed not uniformly through the cross–section area of a pipe. This means that velocity in the central region is usually higher than near the wall. The measured value of velocity v_l is the average of this distribution in the direction of the path. Measured value usually differs from average velocity through the whole cross section v_c:

$$v_l = \int_0^L v(l)dl, \; Q = \int_s v(x,y)dS, \tag{3}$$

$$k = \frac{Q}{\pi R^2 v_l}, \tag{4}$$

where $v(l)$ is the distribution of velocity in the direction of the wave propagation; Q is the flow rate; $v(x, y)$ is the distribution of velocity through the whole cross–section of the conduit; R is the radius of the pipe; S is the cross–section area of the pipe; k is the meter factor, which is connected with the measured and actual flow rate [3].

Equation (4) is valid for any symmetric flow. However, in the case of asymmetric propagation of the flow wave patch, the flow rate would be calculated with error. The type of function $v(x, y)$ for asymmetric flow does not have general analytical expression. The function of flow distribution strongly depends on the conduit configuration, and its characteristics (such as material, roughness, temperature of the controlled media, etc.). The error of the calculated flow rate for a single path flow meter can achieve a 10% and be even larger [3].

The meter factor can be calibrated in laboratory conditions. Such calibration is generally performed on a long straight section of a conduit where a symmetry flow profile takes place [3]. In the case of multipath flow meter, all asymmetric features are assumed to be present for calibration of a symmetric flow. In - practice, it is necessary to have more than 4 paths to consider the meter factor as 1 with error less than 0.1% [4, 5]. However, the solution of the task of recovering a flow profile with such accurate flow measurements is an open issue.

For some types of flow profiles, the analytical expression has been obtained by Salami [6]. Based on his works, some authors proposed analysis of configuration of the multipath measurements and suggested some interpretations of analytical profiles [4–8]. The solution of the profile recovering problem by ultrasonic flow measurements was only proposed on the basis of ultrasonic tomography or by Abel's integration transform, which doesn't take into account asymmetry features of flow [2, 9].

In this work, a technique is suggested for recovering the flow profiles using the multipath measurements and *a priori* information about discretionary function, which can be determined for each type of a bent section of the conduit.

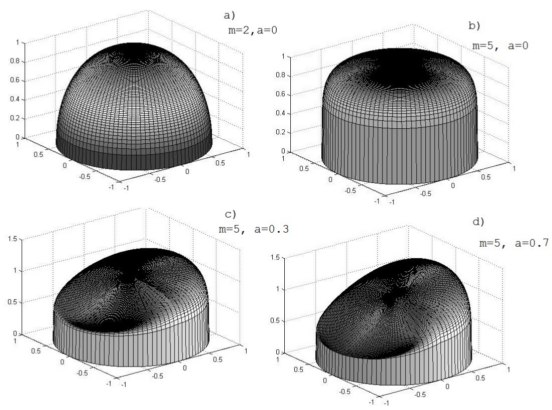

Fig. 1. Flow profiles for a) m=2, a=0; b) m=5, a=0;) m=5, a=0.3; d) m=5, a=0.7

2 Model of flow profile

The model of flow profile, which have been proposed by authors [6, 7] can be considered as a particular case of a flow profile that has passed through a conduit section, whose specified characteristic determines the distortion of the velocity distribution. The proposed model and generalized approach can be analyzed regarding to the accuracy of the profile recovering.

In general, the flow rate could be calculated as an integral of velocity distribution over the cross-section area as

$$Q = \int_{s(z)} v_z(x, y) dS = \int_{x_1=-R}^{x_2=R} \left[\int_{y_1=-\sqrt{(R^2-x^2)}}^{y_2=\sqrt{(R^2-x^2)}} v_z(x, y) dy \right] dx, \tag{5}$$

Thus

$$Q = \int_{-R}^{R} v_z(x) dx = \frac{c^2 \tan(\alpha)}{2} \int_{-R}^{R} \Delta t(x) dx, \tag{6}$$

where $\Delta t(x)$ is the distribution of difference values of measured propagation time in the flow direction and in the opposite one versus the coordinate. In theory of numerical integration, it is proposed to substitute the integral by sum with specified weight, which may be chosen, for instance, by criteria of optimal distribution of nodes locations and corresponding coefficients (the Gauss quadrature

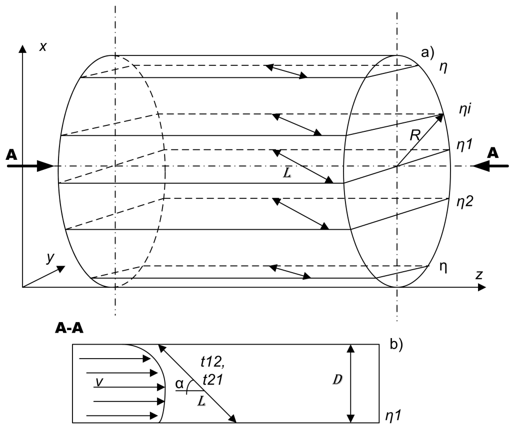

Fig. 2. Schematic draw picture a) multipath flowmeter and b) the center plane cut, η_i is the ith plane

method). The solution is presented in Table 1.

$$\xi = \frac{x}{R}\int_{-R}^{R} \Delta(x)dx \;\rightarrow\; R\int_{-1}^{1} \Delta(\xi)d\xi = R\sum_{i}^{n} \lambda_i \Delta t(\xi), \tag{7}$$

where ξ is the normalized coordinate of the measurement plane, λ_i is the coefficient of weight function (determined by solution of the Gauss quadrature task for number of measurements); $\Delta t(\xi_i), i = 1, 2 \ldots n$ are differences of propagation times in planes with coordinate ξ_i; n is the number of planes. In the case of a

Table 1. Solution of Gauss quadrature task [10]

	n=2	n=3		n=4		n=5		
λ_i	± 0,5773	0	± 0,7746	± 0.3400	± 0.8611	0	± 0.5385	± 0.9062
ξ_i	1	0.8889	0.5556	0, 6521	0,3479	0.5689	0.2369	0.4786

multipath flow meter, the flow rate can be calculated in accordance with (6) as

$$Q = k\frac{c^2 R \; tan(\alpha)}{2}\sum_{i}^{n} \lambda_i \Delta t(\xi_i) = k\sum_{i=1}^{n} \lambda_i v_z(\xi_i), \tag{8}$$

where the meter factor k can be considered as 1 if there are more than 4 measured planes [4]. There are many profiles proposed by Salami [6], which have been verified by experiments. For instance, authors [2] noted that profile of flow that passes through a single elbow could be expressed as follows:

$$\frac{v_z(r,\varphi)}{v_0} = \sin\left(\frac{\pi}{2}(1-r)^{1/m}\right) + \alpha\sin\left(\pi(1-r)^{1/2}\right)\exp(-0.2\varphi)\sin(\varphi), \tag{9}$$

where $v_z(r, \varphi)$ is the velocity distribution in cylindrical coordinates; r is the radius; φ is the angle; v_0 is the velocity of flow at the center of cross-section ($v_z(0,0) = v_0$); α is the velocity of flow at the center of cross-section; m is the coefficient of symmetric flow profile, which characterizes the flow profile depending on velocity if asymmetric coefficient is equal to zero. In the original work [6], the author proposed values $m = 5$, $\alpha = 0.3$. The first part of the right side of equation (4) corresponds to the symmetric part of the flow, and the second part corresponds to asymmetry distortion with coefficient α [6]. Without asymmetry distortion, the flow rate is calculated according to the equations (8)-(9) as

$$Q = \int_S \sin\left(\frac{\pi}{2}(1-r)^{\frac{1}{m}}\right) dS = \int_0^{2\pi} \int_{-R}^{R} \sin\left(\frac{\pi}{2}(1-r)^{\frac{1}{m}}\right) r dr d\varphi, \tag{10}$$

$$Q = kR \sum_{i=1}^{n} \lambda_i v_z(\xi_i). \tag{11}$$

For symmetric model of flow (10, 11) calibration $m = f(k) = f(Q/v_0)$ could be performed. Normalized profiles are shown in Fig. 1 a,b. It can be noted that profile with *m=2* corresponds to the laminar flow mode, and cannot be considered in model. Profile with *m=5* corresponds to the turbulent mode [2]. It is well known that the profile of a flow changes toward symmetry distribution with distance from a section of the conduit, which caused distortion [3]. So, it may be supposed that the influence of the second part of equation (9) decreases as asymmetric coefficient tends to zero with increasing distance from the distorted section of the conduit. Based on the condition of constant flow ($Q(z) = const$) it should be noted that m is also changing with distance from the section of the conduit that caused the distortion.

It may be proposed that type of the second part of the right side of equation (9) depends on the type of section of the conduit, which caused distortion. Such expressions can be defined theoretically or from experiments. In this work, type of the profile (9), which corresponds to signal elbow [2], is considered. Profiles of flow for $m = 5, \alpha = 0.3\ and\ \alpha = 0.7$ are shown in Fig. 2c,d. It has been mentioned above that in each measurement plane, the value of velocity is the integral over the direction of acoustic wave propagation. The normalized measured velocity in the plane with coordinate x_i corresponds to equation (9) can be expressed as

$$\frac{v_z(x_i)}{v_0} =$$

$$= \int_{-A}^{+A} \left[\sin\left(\frac{\pi}{2}(1-(x_i^2+y^2)^{1/2}\right)^{1/m}\right] + \left[\alpha \sin\left(\pi\left(1-(x^2+y^2)^{1/2}\right)^{1/2}\right)\right] \times$$

$$\times \exp(-0.2 \arctan\left(\frac{y}{x_i}\right) \sin\left(\arctan\left(\frac{y}{x_i}\right)\right) dy, \tag{12}$$

where $A = \sqrt{R^2 - x_i^2}$. Denote:

$$F_1(x_i, m) = \int_{-A}^{+A} \left[\sin\left(\frac{\pi}{2}(1 - (x_i^2 + y^2)^{1/2}\right)^{1/m} \right] dy, \tag{13}$$

$$F_2(x_i) = \left[\alpha \sin\left(\pi \left(1 - (x^2 + y^2)^{1/2}\right)^{1/2}\right)\right] \times$$
$$\times \exp(-0.2 \arctan\left(\frac{y}{x_i}\right) \sin\left(\arctan\left(\frac{y}{x_i}\right)\right) dy. \tag{14}$$

Consequently,

$$\frac{v_z(x_i)}{v_0} = F_1(x_i, m) + \alpha F_2(x_i), \tag{15}$$

where $F_1(x_i, m)$ is the function, which characterizes symmetric part of flow, $F_2(x_i)$ is the function, which characterizes an asymmetric part of flow when it passes through a bent section. Expression of the flow rate corresponding to (15) is

$$Q = \int_{-R}^{R} v_z(x) dx = v_0 \int_{-R}^{R} \left[F_1(x_i, m) + \alpha F_2(x_i)\right] dx \approx kR \sum_{i=1}^{n} \lambda_i v_z(\xi_i). \tag{16}$$

In accordance with (15), the flow rate can be expressed as

$$Q \approx v_0 R \sum_{i}^{n} \lambda_i \left[F_1(\xi_i, m) + \alpha F_2(\xi_i)\right]. \tag{17}$$

In equation (17), the meter factor is not used because since it is included in the calibration dependence for $F_1(x_i, m)$. From equation (17) in accordance with (11) and (16), the symmetric flow rate normalized on v_0 can be expressed as

$$\frac{Q_c}{v_0} = R \left[k \sum_{i=1}^{n} \lambda_i v_z(\xi_i) - \alpha v_0 \sum_{i=1}^{n} \lambda_i F_2(\xi_i 2) \right] = f^{-1}(m), \tag{18}$$

where $f^{-1}(m) = Q_c/v_0$ denotes the calibration relation in coordinates $f(Q_c/v_0)$. Such calibration can be implemented because of symmetry of function $F_1(x_i, m)$ and symmetry of profile Q_c. Hence, if we know the flow rate under symmetric conditions and radius of the pipe, we can deduce v_0.

There are three unknown parameters in equation (17): v_0 is the value of velocity at the center of the pipe cross-section; m is the coefficient of symmetric part of flow; α is the asymmetric coefficient. Here, it is assumed that type of function $F_2(x_i)$ is known as a function of distorted section of the conduit. Relation $m = f(F_1(x, m)) = f(Q/v_c)$ assumed to be known from preliminary calibrations on a long straight pipeline. The determination of parameters named above requires solution of the system of equations for each parameter. The position of planes (x_i) is supposed to be chosen here in accordance with Table 1 and from symmetry of $F_1(x_i, m)$. The solution of the system mentioned above has the following expression:

$$\begin{cases} \alpha = [(v_z(x_3) - v_z(x_1))\, F_1(x_1, m)]/[F_2(x_2)v_z(x_1) - v_z(x_3)F_2(x_1)], \\ v_0 = [F_2(x_3)v_z(x_1) - v_z(x_3)F_2(x_1)]/[F_1(x_1, m)\,(F_2(x_1) - F_2(x_3))], \\ \sum\limits_{i=1}^{n} \lambda_i F_1(x_i, m) = \Big[k \sum\limits_{i=1}^{n} \lambda_i v_z(x_i) - \alpha v_0 \sum\limits_{i=1}^{n} \lambda_i F_2(x_i)\Big]/v_0 = f^{-1}(m). \end{cases} \tag{19}$$

Here, the first and the second equations are calculated depending on F_1 and hence on m. However, the multiplier αv_0 may be calculated without the third equation:

$$\alpha v_0 = \frac{v_z(x_3) - v_z(x_1)}{F_2(x_1) - F_2(x_3)}. \tag{20}$$

In the third equation, the meter factor can be considered as 1 if more than 4 measurement planes are used. Consequently, in the third equation m is determined from the calibration relation. The solution of system (19) allows one to calculate velocity distribution in accordance with (15) for distorted flow profile in cross-section of the conduit.

3 Results of Modeling

In the Matlab software, simulation of the algorithm described above is carried out for function of distorted profile (9). The first stage of modeling requires the number of measurement planes and their positions. In our study, *4* planes ($k \approx 1$ in this case) have been chosen according to the Table 1. For the chosen model of distortion relation, functions $m = f(Q/v_0)$ have been calibrated, shown on Fig. 3.

$$x_1 = 0.34R,\ x_2 = 0.86R,\ x_3 = -0.34R,\ x_4 = -0.86R,$$

$$\lambda_1 = 0.652,\ \lambda_2 = 0.347,\ \lambda_3 = 0.652,\ \lambda_4 = 0.347.$$

In Figure 4 function $F_2(x)$ for the chosen type of distortion section of the conduit is shown. In the model of profile (9), coefficients were selected as $m = 5$, $\alpha = 0.3$, $v_0 = 1$, along with the measured values of velocities:

$$v_z(x_1) = 1.9;\ v_z(x2) = 1;\ v_z(x3) = 1.4;\ v_z(x_4) = 0.5.$$

The calculated value αv_0 accordingly to (20) $\alpha v_0 = 0.3$. The calculated value of the flow rate (considering that $k = 1$) is $Q = 2.7307$.

The relation for m (Fig.3) has been found from the third equation of system (9). For calibration, the value $m = 2.83$ has been determined. The difference of the measured and the preliminary chosen m value may be explained by not sufficient accuracy of the assumption that $k \approx 1$ or with influence of F_1 on F_2, which is not taken into account in the proposed model.

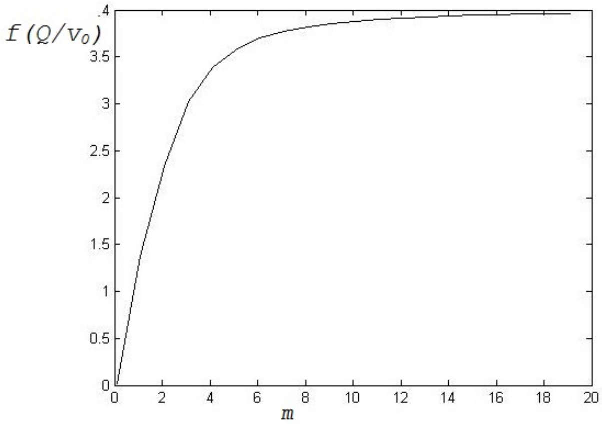

Fig. 3. - Relation $m = f(Q/v_0)$

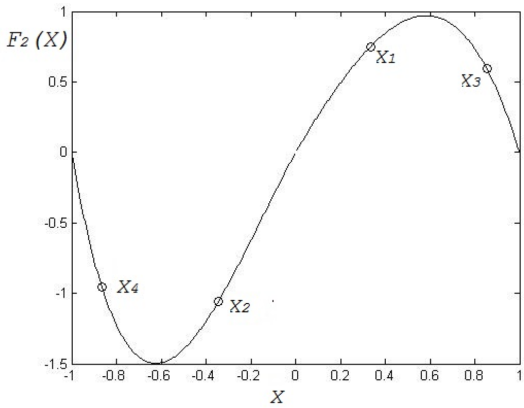

Fig. 4. Function $F_2(x)$ for the chosen type of distortion section of the conduit

The calculated values of $F_1(m, x_i)$, in accordance with (13) and the determined m, are

$$F_1(m, x_1) = F_1(m, x_3) = 1.51;\ F_1(m, x_2) = F_1(m, x_4) = 1.1.$$

From the estimated value of m $F_1(m, x_i)$ and by calculation of equations (20) $\alpha = 0.26, v_0 = 1.16$. The error of determination α, v_0 obviously connected with accuracy of estimation m.

The error of recovering the profile of a flow (9) with the calculated values of parameters and set values estimated as

$$\left(1-\frac{Q_{teor}}{Q_{calc}}\right) 100\% = 6.7\%, \tag{21}$$

where Q_{teor} is the flow rate calculated form the selected values m, α, v_0, and Q_{calc} is the flow rate calculated from estimated values of parameters. The relation of error versus the asymmetry coefficient and $m = 5,\ 7,\ 10$ shown in Fig. 5. The

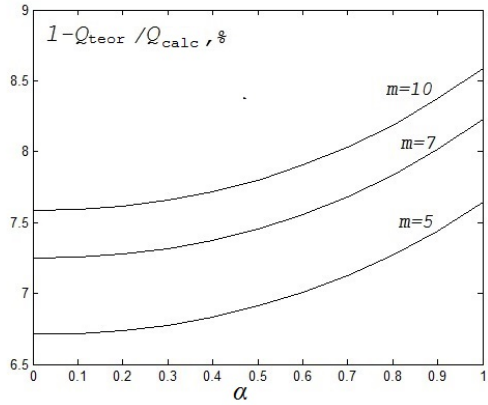

Fig. 5. Relation of error depending on asymmetry coefficient with $m = 5,\ 7,\ 10$ and $v_0 = 1$

accuracy analysis shows that the error dependence has a constant part that corresponds to symmetry flow distribution $m = f(Q/v_0)$. Such constant value of the error can be decreased by correction of calibration relationship $m = f(Q/v_0)$. The result with such calibration for each value m is shown in Fig. 6. In the whole, such behavior of the error can be caused by the insufficient accuracy of the model assumptions. Particularly, it may be supposed that the asymmetry part of $F_2(x)$ in (13) is influenced by the symmetry part of $F_1(x, m)$.

The general algorithm of flow profile recovering on the basis of multipath ultrasonic flow rate measurements and *a priori* information about distorting function of the conduit section has the following stages:

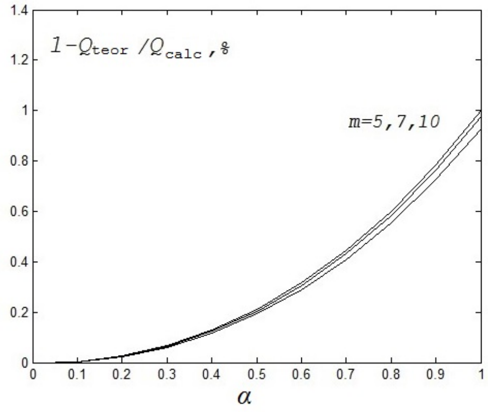

Fig. 6. Relation of error with corrected $m = f(Q/v_0)$ depending on asymmetry coefficient with $m = 5,\ 7,\ 10$ *and* $v_0 = 1$.

1. selection of the function $F_2\,(x, m)$ type;
2. carrying out calibration for symmetric flow $m = F(Q)$;
3. measurements of flow velocities in planes for each x_i and determination the flow rate by equation (16);
4. calculation value αv_0 from (20);
5. determination of m in agree with (18) and calibration of the relationship;
6. evaluation of correction relation $m = F(Q)$ for decreasing symmetric flow error ($\alpha = 0$);
7. calculation of values $F_1(m, x_i)$, α and v_0 by equations (19);
8. calculation of velocity distribution and flow profile.

4 Conclusion

The new model of behavior of velocity distribution in conduit and the algorithm for flow profile recovering based on it are proposed in the paper. The model generalizes analytical expressions for distorted flow profiles in the conduit, which were proposed by Salami. In contrast to [6], here, some coefficients of the model has been generalized and their physical interpretation provided.

The algorithm for profile recovering on the basis of multipath ultrasonic measurements and *a priori* information about distorting section of conduit is proposed. The model can be used for any type of profile with theoretical or experimental description.

The numerical simulation for generalized expression of the function for profile after single elbow has been carried out [2]. The results of simulation have been presented in the paper. The flow profile after single elbow, measured by ultrasonic in 4 planes is considered. The error is estimated as the relation between the flow rate calculated with theoretical set of parameters and parameters that were determined by the proposed algorithm. The obtained accuracy gives less than 1% error.

The reached that the model can be considered as reliable. In further developments of the model, the influence of symmetry part of flow on asymmetry part will be investigated. Moreover, investigation of number of measurements and orientation of planes influence on accuracy should be done.

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