# Calculation of a mode set in weakly guiding fibers

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**Abstract.** The aim of the paper is to calculate the mode set in weakly guiding fibers using a calculation of the eigenmodes of the optical fiber with a step index of refraction. The superposition of modes with various properties of self-reproduction for a given set of physical characteristics was determined. The propagation of light signals in the non-ideal optical waveguides has been studied by computer simulation using the commercial software BeamProp.

Keywords: optical waveguides, weakly guiding fibers, calculation of the eigenmodes

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## Introduction

The optical fiber is considered now to be the perfect physical environment for information transfer as well as the most preferable environment for significant data flows over considerable distances. Optical fibers have wider applications in computer networking and telecommunications thanks to a number of the features inherent in optical waveguides.

Success achieved in the production of optical fibers allows information to be transferred at high speeds over hundreds of kilometres without regeneration of a signal. High-noise immunity, safety of the transmitted data and electromagnetic compatibility of communication channels are serious arguments in favour of fiber-optical systems.

There are two types of optical fibers: single-mode and multi-mode. Fibers with various refractive index profiles (step-index profile or gradient-index profile) are used, depending on the field of application. For step-index optical fiber, a refractive index profile characterized by a uniform refractive index within the core and a sharp decrease in the refractive index at the core-cladding interface is used so that the cladding is of a lower refractive index. Gradient-index fiber is an fiber whose core has a refractive index that decreases with increasing radial distance from the optical axis of the fiber. In this investigation, we have looked at fibers with a step profile index of refraction because of their wide extension.

The term "mode division multiplexing" (MDM) is used for multimodal optical fibers when describing methods for data transmission channel multiplexing, with each

spatial fiber mode being treated as a separate channel that carries its own signal [1,2]. The essence of mode division multiplexing is as follows: as a linear superposition of fiber modes, laser beams can be used to generate signals that will effectively transmit data in a physical carrier - a multimodal fiber; the data transmitted can be contained both in the modal composition and in the energy portion associated with each laser mode [3-13]. In addition, the division of the vortex basis connected with orbital angular momentum is especially perspective [7-14].

Recent years have witnessed a number of research activities in the field of singular optics [15, 16, 25-27]. In terms of quantum theory vortex modes they are characterized as spin-orbital states that the current speed of transfer on one fiber allows to increase repeatedly without additional polarizing multiplexing.

#### 1. Mode set in weakly guiding stepped-index fibers

Use of the diffraction optical elements is the most popular method for devices of generation and selection of vortex modes [8, 10, 17-21]. For most popular commercial fibers, the core-cladding index contrast,  $\Delta n = n_1 - n_2$ , is less than 1%. For such fibers, termed weakly guiding fibers, assuming  $n_1 \approx n_2$ , in place of the hybrid modes of the propagating electromagnetic field we can consider their linearly polarized superpositions. Considering that for the LP-mode the transverse field is essentially linearly polarized, a complete set of modes takes place when only one electric and one magnetic component are predominant.

The aim of this study is to simulate an ideal optical fiber with a superposition of linearly polarized modes determined by the given physical characteristics of a possible set of modes and their superpositions, with various properties of selfreproduction.

Moreover, the intention is to study the propagation of light signals in non-ideal optical waveguides with a Beam PROP simulation tool (RSoft Design, USA), which implements the well-known Beam Propagation Method (BPM).

To research a set of modes, we considered the weakly guiding cylindrical optical fiber with a step profile of index of refraction. The core radius is a, the cladding radius is b and the respective refractive indices of the core and cladding are  $n_1$  and  $n_2$  (Fig.1). The electromagnetic field extending in such a waveguide is conveniently described using the Bessel functions [22-24].



Fig. 1. - The structure of a typical single-mode fiber

Approximation LP-modes are applicable for weakly guiding fibers:

$$LP_{pq}(r,\phi) = \begin{cases} \cos(p\phi) \\ \sin(p\phi) \end{cases} \begin{cases} \frac{J_p(u_{pq}r/a)}{J_p(u_{pq})}, & 0 \le r \le a \\ \frac{K_p(w_{pq}r/a)}{K_p(w_{pq})}, & a \le r \le b \end{cases}$$
(1)

In Eq. (1), the first-kind Bessel functions  $J_p(x)$  describe the field in the fiber core, whereas the modified Bessel functions  $K_p(x)$  are used for the cladding.

$$\frac{uJ_m(u)}{J_{m+1}(u)} + \frac{wK_m(w)}{K_{m+1}(w)} = 0,$$
(2)

where the parameters  $u^2 + w^2 = V^2$ ,  $V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 + n_2^2}$  form the cut-off number and  $\lambda$  is the wavelength of laser light in air. The cut-off number V, which includes the main

The value of tasks region of tasks region in an interaction number v, which includes the main parameters of fibers and laser radiation, is the number of modes propagating in the fiber. The numerical simulation parameters are as follows: the core radius is  $a=5\mu$ m, the cladding radius is  $b=62.5 \mu$ m, and the respective refractive indices of the core and cladding are  $n_1=1.45$  and  $n_2=1.44$ . For example, when  $\lambda=0.633 \mu$ m and  $V\approx8,4398$ , fiber with the above parameters in addition to the fundamental mode LP01 will behave as LP02, LP03, LP11, LP12, LP21, LP41. Figure 2 shows cross-section distributions for some modes for a stepped-index fiber with the cut-off number V=8.4398.



**Fig. 2.** – Superposition of the (p,q) modes: (1,1)+(-1,1) with different complex coefficients: (a) transverse amplitude distribution, (b) transverse intensity distribution, and (c) phase in the plane *z*=0, and (d) phase distribution at distance *z*=200 µm

We consider the propagation of a linear superposition of LP-modes in an ideal stepped index optical fiber:

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$$U_0(r,\phi) = \sum_{p,q\in\Omega} C_{pq} \Psi_{pq}(r,\phi)$$
(3)

where  $C_{pq}$  are the complex coefficients and  $\Psi_{pq}(r,\phi)$  are the modes at z=0, whose angular component is represented in a different way without a loss of generality:  $\Psi_{-}(r,\phi,z) = \exp(-i\beta_{-}z)T_{-}(\phi)R_{-}(r) =$ 

$$= \exp(-i\beta_{pq}z) \exp(ip\phi) \begin{cases} \frac{J_{p}(u_{pq}r/a)}{J_{p}(u_{pq})}, & 0 \le r \le a \\ \frac{K_{p}(w_{pq}r/a)}{K_{p}(w_{pq})}, & a \le r \le b \end{cases}$$

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**Fig. 3.** – The (p,q) modes: (0,3), (1,2), (2,1), (4,1): (a) transverse amplitude distribution (negative), (b) transverse intensity distribution (negative) in the plane *z*=0; transverse phase distribution (white: zero phase, black:  $2\pi$ ) in the planes (c) *z*=0 and (d) *z*=100 µm

Models of optical fibers with the different parameters, structures and shapes have been established in the software BeamProp in order to research, analyse and compare non-ideal optical waveguides (Fig. 4).



Fig. 4. – Various models of waveguides

The main characteristic of optical fiber is the set of modes extending within it (Fig. 5).

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Fig. 5. – Some of the modes supported by a simple 3D fiber structure

Figure 6 shows the distribution of radiation *S*-shaped optical fiber and power of the propagating radiation.



Fig. 6. – The completed s-bend circuit in the CAD window; the simulation results found using the arc waveguides

Figure 7 shows the distribution of radiation of *X*-shaped coupler and the power of the propagating radiation zero and first modes.

By modelling the propagation of radiation in optical fibers, it is possible to determine their output values and make sure the elements have the given parameters, and to predict their behaviour depending on external influences.

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**Fig. 7.** – X-shaped coupler

## Conclusions

Using the BeamProp program it is possible to investigate various samples of nonideal optical fibers with a step profile of index of refraction, according to following characteristics:

- profile and index of refraction;
- difference of indices of refraction;
- waveguide length;
- a type of function on which the index of refraction changes;
- etc.

Studying the resistance of vortex modes to fiber bends under various characteristics of a core and cover of optical fiber is of great interest for the researchers.

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