Study of the chromatic properties of harmonic diffractive lens

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Abstract. This work is devoted to the study of the axial image point formed by a layered diffractive lens illuminated by laser light of different wavelengths. The influence of chromaticity can be identified when there is a change of quantisation levels and harmonic interval (bringing the phase to the level $2\pi n$). The influence of aberrations introduced in the form of converging or diverging conical and cubic wavefronts will also be considered in this study.

Keywords: diffractive optical elements, aberrations, chromatic properties of lens


Introduction
Diffractive optical elements (DOE) are used for the miniaturisation and weight saving of optical systems. Furthermore, they enable the formation of light beams with properties that can not otherwise be obtained by classical optical refractive elements. However, DOE are characterised by significant chromatic aberration, which can have both negative and positive impacts upon the imaging system [1-10].

Traditional lenses and mirrors are designed on the basis of geometrical optics. Due to their large size there are not suitable for use in optical and optoelectronic microsystems. Furthermore, the formation of difficult complex distributions of the laser fields can not be performed by classical refractive elements. However, this problem is solved effectively by means of diffractive optics. DOE are able to take into account the wave nature of light, so as to successfully carry out the conversion of laser radiation in practically any amplitude-phase distribution [11, 12].

The main property of DOE is the use of the diffraction phenomenon to change the propagation direction of light beams. The diffraction elements split the light beam into multiple beams, each of which is redirected from different angles. Diffraction angles are different for different wavelengths: longer wavelengths are diffracted at higher angles than shorter wavelengths. Thus, a chromatic dispersion effect manifests itself in this case [13, 14]. The negative impact of this case affects both the imaging [1, 6, 9], and focusing [15, 16] systems containing DOE. To compensate for the
chromaticity DOE, the use of a hybrid system is often suggested – i.e., a combination of refractive and diffractive elements having opposite chromatism [13, 14, 17, 18].

Several DOE can combine the properties of both diffractive and refractive lenses. An example of this is the so-called harmonic diffractive elements, whereby the diffractive and refractive properties of the lens depends on the phase of bringing the different intervals [19-21]. Harmonic intervals, as they are known, bring the phase to the level $2\pi n$. Obviously, the larger the $n$, the closer it is to the properties of harmonic DOE refractive element. However, the manufacture of such elements faces technological difficulties, since in this case deep etching is required. It is therefore desirable to find a compromise between the need to increase the harmonic interval to overcome the chromaticity and opportunities for the process of manufacturing the optical element.

1. Theoretical bases and study chromatic properties

In this work the optical system is shown in Figure 1, where $z_1$ is the distance from the radiation source to the DOE, and $z_2$ is the distance from the DOE to the image plane. Diverging spherical waves are assumed, for which a high $z_1$ obtains parallel beams of light coming from the source of radiation.

Fig. 1. – Optical system with DOE

The main stages of the formation of the DOE are as follows:
1. Continuous phase transmission function $\varphi(r)$ is reduced to the interval $[0, 2\pi n)$, where $n$ - number of periodicity;
2. Resulting $n$ – periodic phase $\varphi_j(r) = \text{mod}_{2\pi n} \varphi(r)$ are quantised with a similar step height, forming $m$ levels.

The greater the number of quantisation levels in phase $m$, the more accurate the resulting phase repeats will be for the relief of the continuous case. An increase in the periodicity $n$ allows relief to be brought to the refractive diffraction.

In this work the focus of study was the intensity of light passing through the multi-level DOE when changes were made to the quantisation levels of the element. Figures 2, 3 and 4 are graphs of the intensities obtained for the refractive, binary and sixteen-level lens respectively.

For multi-level DOE we see the difference in foci for different wavelengths and a reduction in intensity with decreasing quantisation levels in comparison with the refractive lens. This reduction in intensity is due to the fact that part of the energy for the quantised DOE is redistributed along the optical axis, closer to the optical element.
In addition, a study was conducted of the intensity obtained by introducing a phase transmission function of various kinds of aberrations – namely, the introduction of a wavefront with a radial phase function of the first or third degree. It is known that such changes of wavefront lead to a deepening focus of the imaging system [22, 23], which may also help to compensate for chromatic effects. A conical wavefront can be implemented using an additional axicon, while a cubic wavefront can be implemented using the appropriate DOE.

In general, the phase transmission function of the DOE is:

$$\tau(r) = \exp \left[ ik \left( c_0 + c_1 r + c_2 r^2 + ... + c_j r^j + \frac{c_1}{r} + \frac{c_2}{r^2} \right) \right]$$

If the lens is parabolic: \(\tau(r) = \exp \left( -ik \frac{r^2}{2f} \right)\), then \(c_2 = -\frac{1}{2f}\), and the other factors set the aberration changing point, thus spreading the function of the imaging.
system. Tables 1, 2 and 3 are graphs of the intensities depending on changes in factors which are responsible for conical and cubic wavefronts.

**Table 1.** Graphs of the intensity at the optical axis caused by the addition of a divergent conical wavefront.

<table>
<thead>
<tr>
<th>$c_1$</th>
<th>Graph of intensity</th>
<th>$c_1$</th>
<th>Graph of intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00001</td>
<td><img src="image1" alt="Graph" /></td>
<td>0.0001</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Table 2.** Graphs of the intensity at the optical axis caused by the addition of a convergent conical wavefront.

<table>
<thead>
<tr>
<th>$c_1$</th>
<th>Graph of intensity</th>
<th>$c_1$</th>
<th>Graph of intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.00001</td>
<td><img src="image3" alt="Graph" /></td>
<td>-0.0001</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Table 3.** Graphs of the intensity at the optical axis caused by the addition of a cubic diverging wavefront.

<table>
<thead>
<tr>
<th>$c_3$</th>
<th>Graph of intensity</th>
<th>$c_3$</th>
<th>Graph of intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00001</td>
<td><img src="image5" alt="Graph" /></td>
<td>0.000015</td>
<td><img src="image6" alt="Graph" /></td>
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Tables 1, 2 and 3 show that when a conical or cubic aberration occurs, there is a drop in intensity at the focal point and an increased depth of focus. The point of maximum intensity is displaced along the optical axis. Note that the introduction of such wave aberrations may help to compensate for chromatic aberrations associated with different focal lengths for different wavelengths.

In addition, a study was carried out of the formation of an image depending on the change of quantisation levels of the element, bringing the phase to the interval $[0, 2\pi n]$, where $n = 1, 2, ..., 10$. The study was conducted for the elements with different radii and illustrates the dependence of $M$ and $N$ on the radius of the lens.

A number of factors were found to affect the intensity of light obtained by passing through the radiation element; not only the number of quantisation levels and relief depth, but also the number of zones of the element. Figure 5 shows the DOE with...
radius $R = 1$ mm, the number of quantisation levels of $M = 16$, the periodicity $N = 1$, for which the number of zones is 10 (the peaks in the image). The intensity on the optical axis to such an element is shown in Figure 6.

![Figure 5](image1)

**Fig. 5.** – 16-level DOE with periodicity $N = 1$

![Figure 6](image2)

**Fig. 6.** – The intensity on the optical axis for 16-level DOE with periodicity $N = 1$

In Figure 6 we see the divergence of foci for different wavelengths. The convergence of points of focus is necessary to reduce the number of zones to one element by increasing the periodicity of interval (increasing the depth of the relief elements). Thus, DOE will tend to refract when the number of zones is reduced, but will never reach this point.

Figures 7 and 8 demonstrate the above effect. Elements with one zone of periodicity $N = 9$, and the intensity obtained when the radiation passes through the DOE are shown respectively.

![Figure 7](image3)

**Fig. 7.** – 16-level DOE with periodicity $N = 9$

Graphs of the intensity of light at changing radii of the lens and the quantisation levels $M$ are shown in Table 4. The periodicity $N$ for each element was chosen so that the number of zones of the lens is equal to one.
From the table above it is clear that increasing the radius of the lens to shift the focus is necessary to increase the number of quantisation levels (a lens radius of 1 mm is sufficient at 32 level quantisation for a lens with a radius of 10 mm - 2048 levels). Thus, the lenses of one zone with a larger radius can be achieved through relief depth increases (lens radius 1 mm $N = 9$ to 10 mm radius $N = 910$).

**Conclusions**

The study results are as follows:

- If you reduce the level of quantisation of elements for different wavelengths, different focal lengths can be observed, along with a drop in intensity and the redistribution of available energy along the optical axis;
- In the case of entering divergent or convergent conical and cubic wavefronts, a drop in intensity at the focal point while increasing the depth of focus could be observed. The point of maximum intensity is displaced along the optical axis;
- By increasing the radius of the DOE to achieve the shift in focus for different wavelengths at one point necessary to increase the number of quantisation levels of the lens, and choose a value of the periodicity $N$, the number of zones are required to be equal to one element.

The problems associated with chromatic aberration of DOE for some applications can be solved by entering different aberrations into the wavefront and increasing the depth of focus.

**Acknowledgements**

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Table 4. Graphs showing intensities, obtained by varying the quantisation levels $M$ for elements with different radii $R$

<table>
<thead>
<tr>
<th>$R = 1$ mm, $N = 9$</th>
<th>$R = 2$ mm, $N = 82$</th>
<th>$R = 5$ mm, $N = 228$</th>
<th>$R = 10$ mm, $N = 910$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 4$</td>
<td>$M = 4$</td>
<td>$M = 4$</td>
<td>$M = 4$</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
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<td>$M = 32$</td>
<td>$M = 256$</td>
<td>$M = 512$</td>
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<tr>
<td></td>
<td>$M = 512$</td>
<td>$M = 2048$</td>
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References
23. Khonina SN. Phase apodization of imaging system to increase the focal depth in coherent and incoherent cases. Computer Optics, 2012; 36(3): 357-364. [in Russian]