A Syllogistic Reasoning Theory and Three Examples

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Abstract. A recent meta-study shows that the conclusions driven by human reasoners in psychological experiments about syllogistic reasoning are not the conclusions predicted by classical first-order logic. Moreover, current cognitive theories deviate significantly from the empirical data. In the following, three important cognitive approaches are presented and compared to predictions made by a new approach to model human reasoning tasks, viz. the weak completion semantics. Open questions and implications are discussed.

1 Introduction

The way of how humans ought to reason correctly about syllogisms has already been investigated by Aristotle. A *syllogism* consists of two quantified statements using some of the four quantifiers *all* (A), *no* (E), *some* (I), and *some are not* (O)[1] about entities like '*some a are b*' and '*no b are c*', and is questioning about the logical consequences of these statements. E.g., '*some a are not c*' is a logical consequence of the given two statements in classical first-order logic (FOL). The four quantifiers and their formalization in FOL are given in Table 1. The entities can only appear in four different orders called *figures* as shown in Table 2. Hence, a problem can be completely specified by the quantifiers of the first and second premise and the figure. E.g., the example discussed so far is IE1.

Altogether, there are 64 syllogisms and, if formalized in FOL, we can compute their logical consequences in classical logic. However, a meta-study [24] based on six experiments has shown that humans do not only systematically deviate from the predictions of FOL but from any other of at least 12 cognitive theories. In the case of IE1, besides the above mentioned logical consequence, a significant number of humans answered *no a are c* which does not follow from IE1 in FOL.

In recent years, a new cognitive theory based on the weak completion semantics (WCS) has been developed. It has its roots in the ideas first expressed by Stenning and van Lambalgen [32], but is mathematically sound [17], and has been successfully applied – among others – to the suppression task [8], the

* The authors are mentioned in alphabetical order.

1 We are using the classical abbreviations.

Mood	Natural Language	FOL	Short
Affirmative universal (A)	*all a are b*	$\forall X(a(X) \rightarrow b(X))$	Aab
Affirmative existential (I)	*some a are b*	$\exists X(a(X) \wedge b(X))$	Iab
Negative universal (E)	*no a are b*	$\forall X(a(X) \rightarrow \neg b(X))$	Eab
Negative existential (O)	*some a are not b*	$\exists X(a(X) \wedge \neg b(X))$	Oab

Table 1. The four syllogistic moods together with their logical formalization.

	Figure 1	Figure 2	Figure 3	Figure 4
Premise 1	a-b	b-a	a-b	b-a
Premise 2	b-c	c-b	c-b	b-c

Table 2. The four figures used in syllogistic reasoning.

selection task [9], the belief bias effect [28,29], to reasoning about conditionals [5,7] and to spatial reasoning [6]. Hence, it was natural to ask whether WCS is competitive in syllogistic reasoning and how it performs with respect to the cognitive theories considered in [24]. This paper gives some preliminary results by considering FOL, the syntactic rule based theory PSYCOP [31], and two model-based theories that performed well in the meta-study: the verbal model theory [30] and the mental model theory[2] [19].

2 Predictions of Cognitive Theories

Due to space limitations we will refer for the assumed operations and underlying cognitive processes of the other theories to [24]. The predictions of the theories FOL, PSYCOP, verbal, and mental models for the syllogisms IE1, EA3, and AA4 and those of the participants are depicted in Table 3. For the statistical analysis, the reader is refered to [24].

FOL and the other three cognitive theories make different predictions. Additionally, each theory provides at least one prediction which is correct with respect to classical FOL and provides an additional prediction which is false with respect to classical FOL.

3 Syllogisms

Various theories have tried to explain this phenomenon. Some conclusions can be explained by converting the premises [2] or by assuming that the atmosphere of the premises influences the acceptance for the conclusion [34]. Johnson-Laird and Byrne [21] proposed the mental model theory [20], which additionally supposes the search for counterexamples when validating the conclusion. Evans et

[2] http://mentalmodels.princeton.edu/models/mreasoner/

	participants[3]	FOL	PSYCOP	verbal models	mental models
IE1	Eac, Oac	Oac	Oac, Iac, Ica	Oac	Eac, Eca, Oac, Oca, NVC
EA3	Eac, Eca	Eac, Eca	Eac, Eca, Oac, Oca	NVC, Eca	Eac, Eca
AA4	Aac, NVC	Iac, Ica	Iac, Ica	NVC, Aca	Aca, Aac, Iac, Ica

Table 3. The conclusions drawn by the participants are highlighted in gray and compared to the predictions of the theories FOL, PSYCOP, verbal, and mental models for the syllogisms IE1, EA3, and AA4. NVC denotes that there are no valid conclusions.

al. [12,11] proposed a theory which is sometimes referred to as the selective scrutiny model [14,1]. First, humans heuristically accept any syllogism having a believable conclusion, and only check on the logic if the conclusion contradicts their belief. Adler and Rips [1] claim that this behavior is rational because it efficiently maintains our beliefs, except in case if there is any evidence to change them. It results in an adaptive process, for which we only make an effort towards a logical evaluation when the conclusion is unbelievable. It would take a lot of effort if we would constantly verify them even though there is no reason to question them. As people intend to keep their beliefs consistent, they invest more effort in examining statements that contradict them, than the ones that comply with them. However, this theory cannot fully explain all classical logical errors in the reasoning process. Yet another approach, the selective processing model [13], accounts only for a single preferred model. If the conclusion is neutral or believable, humans attempt to construct a model that supports it. Otherwise, they attempt to construct a model, which rejects it.
As summarized in [14], there are several stages in which a belief bias can take place. First, beliefs can influence our interpretation of the premises. Second, in case a statement contradicts our belief, we might search for alternative models and check whether the conclusion is plausible.

4 Weak Completion Semantics

The general notation, which we will use in the paper, is based on [26,16].

4.1 Logic Programs

We assume the reader to be familiar with logic and logic programming, but recall basic notions and notations. A *(logic) program* is a finite set of (program) clauses of the form $A \leftarrow \top$, $A \leftarrow \bot$ or $A \leftarrow B_1 \wedge \ldots \wedge B_n$, $n > 0$, where A is an atom,

[3] 156 participants have been asked where the population ranges from highschool to university students.

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table 4. $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

B_i, $1 \leq i \leq n$, are literals and $\top$ and $\bot$ denote truth and falsehood, resp. A is called *head* and $\top$, $\bot$ as well as $B_1 \wedge \ldots \wedge B_n$ are called *body* of the corresponding clause. Clauses of the form $A \leftarrow \top$ and $A \leftarrow \bot$[4] are called *positive* and *negative facts*, respectively. We restrict terms to be constants and variables only, i.e. we consider *data logic programs*. Throughout this paper, $\mathcal{P}$ denotes a program. We assume for each $\mathcal{P}$ that the alphabet consists precisely of the symbols occurring in $\mathcal{P}$ and that non-propositional programs contain at least one constant. When writing sets of literals we will omit curly brackets if the set has only one element.

$\mathsf{g}\mathcal{P}$ denotes the set of all ground instances of clauses occurring in $\mathcal{P}$. A ground atom A is *defined* in $\mathsf{g}\mathcal{P}$ iff $\mathsf{g}\mathcal{P}$ contains a clause whose head is A; otherwise A is said to be *undefined*. $\mathit{def}(\mathcal{S}, \mathcal{P}) = \{A \leftarrow \mathit{Body} \in \mathsf{g}\mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$ is called *definition* of $\mathcal{S}$ in $\mathcal{P}$, where $\mathcal{S}$ is a set of ground literals. $\mathcal{S}$ is said to be *consistent* iff it does not contain a pair of complementary literals.

4.2 Three-Valued Łukasiewicz Semantics

We consider the three-valued Łukasiewicz Semantics [27], for which the corresponding truth values are $\top$, $\bot$ and U, which mean *true*, *false* and *unknown*, respectively. A *three-valued interpretation* I is a mapping from formulas to a set of truth values $\{\top, \bot, \mathsf{U}\}$. The truth value of a given formula under I is determined according to the truth tables in Table 4. We represent an interpretation as a pair $I = \langle I^\top, I^\bot \rangle$ of disjoint sets of atoms, where $I^\top$ is the set of all atoms that are mapped to $\top$ by I, and $I^\bot$ is the set of all atoms that are mapped to $\bot$ by I. Atoms which do not occur in $I^\top \cup I^\bot$ are mapped to U. Let $I = \langle I^\top, I^\bot \rangle$ and $J = \langle J^\top, J^\bot \rangle$ be two interpretations: $I \subseteq J$ iff $I^\top \subseteq J^\top$ and $I^\bot \subseteq J^\bot$. $I(F) = \top$ means that a formula F is mapped to true under I. $\mathcal{M}$ is a *model* of $\mathsf{g}\mathcal{P}$ if it is an interpretation, which maps each clause occurring in $\mathsf{g}\mathcal{P}$ to $\top$. I is the *least model* of $\mathsf{g}\mathcal{P}$ iff for any other model J of $\mathsf{g}\mathcal{P}$ it holds that $I \subseteq J$.

4.3 Reasoning with Respect to Least Models

For a given $\mathcal{P}$, consider the following transformation:

1. For each A where $\mathit{def}(A, \mathcal{P}) \neq \emptyset$, replace all
   $A \leftarrow \mathit{Body}_1, \ldots, A \leftarrow \mathit{Body}_m \in \mathit{def}(A, \mathcal{P})$ by $A \leftarrow \mathit{Body}_1 \vee \ldots \vee \mathit{Body}_m$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

[4] We consider weak completion semantics and, hence, a clause of the form $A \leftarrow \bot$ is turned into $A \leftrightarrow \bot$ provided that this is the only clause in the definition of A.

The obtained ground program is called *weak completion* of $\mathcal{P}$ or $wc\mathcal{P}$.[5]

It has been shown in [18] that logic programs as well as their weak completions admit a least model under Ł-logic. Moreover, the least Ł-model of $wc\mathcal{P}$ can be obtained as the least fixed point of the following semantic operator, which is due to Stenning and van Lambalgen [32]: $\Phi_{\mathcal{P}}(\langle I^{\top}, I^{\bot}\rangle) = \langle J^{\top}, J^{\bot}\rangle$, where

$$\begin{aligned} J^{\top} &= \{A \mid A \leftarrow Body \in def(A,\mathcal{P}) \text{ and } Body \text{ is } true \text{ under } \langle I^{\top}, I^{\bot}\rangle\} \\ J^{\bot} &= \{A \mid def(A,\mathcal{P}) \neq \emptyset \text{ and} \\ &\qquad Body \text{ is } false \text{ under } \langle I^{\top}, I^{\bot}\rangle \text{ for all } A \leftarrow Body \in def(A,\mathcal{P})\} \end{aligned}$$

Weak completion semantics (WCS) is the approach to consider weakly completed logic programs and to reason with respect to the least Ł-models of these programs. We write $\mathcal{P} \models_{wcs} F$ iff formula F holds in the least Ł-model of $wc\mathcal{P}$. In the remainder of this paper, $\mathcal{M}_{\mathcal{P}}$ denotes the least Ł-model of $wc\mathcal{P}$.

The correspondence between weak completion semantics and well-founded semantics [33] for tight programs, i.e. those without positive cycles, is shown in [10].

4.4 Integrity Constraints

A set of *integrity constraints* $\mathcal{IC}$ comprises clauses of the form $\bot \leftarrow Body$, where $Body$ is a conjunction of literals. Under a three-valued semantics, there are several ways on how to understand integrity constraints [23], two of them being the *theoremhood view* and the *consistency view*. Consider the $\mathcal{IC}$

$$\bot \leftarrow \neg p \wedge q$$

The theoremhood view requires that a model only satisfies the set of integrity constraints if for all its clauses, $Body$ is false under this model. In the example, this is only the case if p is true or if q is false in the model. In the consistency view, the set of integrity constraints is satisfied by the model if $Body$ is unknown or false in it. Here, a model satisfies $\mathcal{IC}$ already if either p or q is unknown.

In this paper we adopt the consistency view. Formally, given $\mathcal{P}$ and set $\mathcal{IC}$, $\mathcal{P}$ *satisfies* $\mathcal{IC}$ iff there exists I, which is a model for $\mathsf{g}\mathcal{P}$, and for each $\bot \leftarrow Body \in \mathcal{IC}$, we find that $I(Body) \in \{\bot, \mathsf{U}\}$.

5 Reasoning Towards an Appropriate Logical Form

5.1 Existential Import: Modeling Gricean Implicature

We assume that humans understand quantifiers with existential import, i.e. *for all* implies *there exists*. This is a reasonable assumption – called the Gricean Implicature [15] – as in natural language we normally do not quantify over things that do not exist. Furthermore, Stenning and van Lambalgen [32] have shown that humans require existential import for a conditional to be true. The program for A in Table 1 together with existential import is

$$\mathcal{P}_{\mathsf{A}} = \{b(X) \leftarrow a(X),\ a(o) \leftarrow \top\}$$

[5] Note that undefined atoms are not identified with $\bot$ as in the completion of $\mathcal{P}$ [3].

where the first clause represents 'all a are b' and the second clause states that there exists a constant, viz. o, for which $a(o)$ is true. The least Ł-model of $\mathcal{P}_{\mathsf{A}}$ is

$$\langle\{a(o), b(o)\}, \emptyset\rangle$$

which maps Aab, Iab, Iba and Aba to *true*, i.e. the programs which represent Aab, Iab, Iba and Aba are true under the least Ł-model of $\mathcal{P}_{\mathsf{A}}$. We will now show how we can represent the corresponding programs for Iab and Iba.

5.2 Positive and Negative Facts

The second and the third mood in Table 1, I and E, each implies two facts about something, e.g., about some constant o. The program for I in Table 1 is

$$\mathcal{P}_{\mathsf{I}} = \{a(o) \leftarrow \top,\ b(o) \leftarrow \top\}$$

where o is a constant for which it holds that $a(o)$ and $b(o)$ are true. Its least Ł-model is

$$\langle\{a(o), b(o)\}, \emptyset\rangle$$

which maps Iab, Iba, Aab, and Aba to *true*. Section 5.4 explains why whenever Iab is mapped to *true*, Iba is mapped to *true* as well, and vice versa. As o is the only object for which $a(o)$ and $b(o)$ is true, we can generalize over all constants. Accordingly, Aab and Aba hold as well. Similarily, the program for E is

$$\mathcal{P}_{\mathsf{E}} = \{a(o) \leftarrow \top,\ b(o) \leftarrow \bot\}$$

where o is a constant for which $a(o)$ is true and $b(o)$ is false. Its least Ł-model is

$$\langle\{a(o)\}, \{b(o)\}\rangle$$

which maps Eab and Oab to *true*. Like in the case of I, as o is the only object for which $a(o)$ is true and $b(o)$ is false, we can generalize over all constants. Therefore, Eab holds as well.

5.3 Negative Conclusions

The consequence in the third mood E is the negation of $b(X)$. As the weak completion semantics does not allow negative heads in clauses, we cannot represent this inference straightaway. Therefore, for every negative conclusion $\neg p(X)$ we introduce an auxiliary formula $p'(X)$ together with the clause $p(X) \leftarrow \neg p'(X)$. Accordingly, the program of the example for E in Table 1 together with the assumption of existential import, is

$$\mathcal{P}_{\mathsf{E}} = \{b'(X) \leftarrow a(X),\ b(X) \leftarrow \neg b'(X),\ a(o) \leftarrow \top\}$$

Its least Ł-model is

$$\langle\{a(o), b'(o)\}, \{b(o)\}\rangle$$

which maps Eab and Oab to *true*. With the introduction of these auxiliary atoms, the need for integrity constraints arises. A model $I = \langle I^\top, I^\bot \rangle$ for $\mathcal{P}$ that contains both $b(c)$ and $b'(c)$ in $I^\top$, where c is any constant in $\mathcal{P}$, should be invalidated. This condition can be represented by the integrity constraint

$$\mathcal{IC} = \{\bot \leftarrow b(X) \wedge b'(X)\}$$

and is to be understood as discussed in Section 4.4. For the following examples, whenever there exists a $p(X)$ and its $p'(X)$ counterpart in $\mathcal{P}$, we implicitly assume $\mathcal{IC} = \{\bot \leftarrow p(X) \wedge p'(X)\}$.

5.4 Symmetry

The results of the psychological experiments presented in [25] show that participants distinguish between the cases 'some a are c' or 'some c are a' (Iac and Ica). However, for the mood I, a and c can be interchanged in FOL, because by commutativity, $\exists X(a(X) \wedge c(X))$ is semantically equivalent to $\exists X(c(X) \wedge a(X))$. Likewise, the formalizations of Iac and Ica under WCS, i.e. $\{a(o) \leftarrow \top, c(o) \leftarrow \top\}$ and $\{c(o) \leftarrow \top, a(o) \leftarrow \top\}$ are semantically equivalent. Thus, neither FOL nor WCS can distinguish between Iac and Ica.

In FOL, $\forall X(a(X) \rightarrow b(X))$ is semantically equivalent to $\forall X(\neg b(X) \rightarrow \neg a(X))$ by modus tollens. Likewise, $\forall X(a(X) \rightarrow \neg b(X))$ is semantically equivalent to $\forall X(b(X) \rightarrow \neg a(X))$. For the representation under WCS and, in particular, given the additional fact representing the existential import, these two formulas are not semantically equivalent anymore. Eab is represented by

$$\{b'(X) \leftarrow a(X),\ b(X) \leftarrow \neg b(X),\ a(o) \leftarrow \top\}$$

whereas Eba is represented by

$$\{a'(X) \leftarrow b(X),\ a(X) \leftarrow \neg a(X),\ b(o) \leftarrow \top\}$$

both together with the corresponding integrity constraints.

6 Predictions by the Weak Completion Semantics

In this section we present the three problems IE1, AA3, and EA3 and show the generated conclusions under the weak completion semantics.

6.1 Syllogism IE1

The program representing syllogism IE1 assuming existential import is

$$\mathcal{P}_{\mathsf{IE1}} = \{a(o_1) \leftarrow \top,\ b(o_1) \leftarrow \top,\ c'(X) \leftarrow b(X),\ c(X) \leftarrow \neg c'(X),\ b(o_2) \leftarrow \top\}$$

The weak completion of $\mathsf{g}\mathcal{P}_{\mathsf{IE1}}$ is

$$\begin{aligned}\{\ & a(o_1) \leftrightarrow \top,\ b(o_1) \leftrightarrow \top,\ c'(o_1) \leftrightarrow b(o_1),\ c(o_1) \leftrightarrow \neg c'(o_1),\\ & b(o_2) \leftrightarrow \top,\ c'(o_2) \leftrightarrow b(o_2),\ c(o_2) \leftrightarrow \neg c'(o_2)\ \}\end{aligned}$$

Its least Ł-model is

$$\langle\{a(o_1), b(o_1), c'(o_1), b(o_2), c'(o_2)\}\{c(o_1), c(o_2)\}\rangle,$$

which maps Oac and Eac to *true*. These are exactly the conclusions drawn by the participants and, hence, there is a perfect match in this example. One should observe that Oac is the only valid conclusion in classical FOL, whereas Eac is not a valid conclusion in classical FOL. Likewise, neither PSYCOP nor verbal models nor mental models match the participant's choices (see Table 3).

6.2 Syllogism AA4

The program representing syllogism AA4 assuming existential import is

$$\mathcal{P}_{\mathsf{AA4}} = \{a(X) \leftarrow b(X),\ b(o_1) \leftarrow \top,\ c(X) \leftarrow b(X),\ b(o_2) \leftarrow \top\}$$

The weak completion of $\mathsf{g}\mathcal{P}_{\mathsf{AA4}}$ is

$$\begin{array}{l}\{\ a(o_1) \leftrightarrow b(o_1),\ c(o_1) \leftrightarrow b(o_1),\ b(o_1) \leftrightarrow \top, \\ \ \ a(o_2) \leftrightarrow b(o_2),\ c(o_2) \leftrightarrow b(o_2),\ b(o_2) \leftrightarrow \top\ \}\end{array}$$

Its least Ł-model is

$$\langle\{b(o_1), c(o_1), a(o_1), b(o_2), c(o_2), a(o_2)\}, \emptyset\rangle$$

and maps Iac, Ica, Aac, and Aca to *true*. The majority of the participants concluded Aac and NVC. Hence, there is a partial overlap in this example. One should observe that Iac and Ica are the only valid conclusions in classical FOL. From Table 3 we observe that in this example PSYCOP computes a perfect match, the conclusions computed by the mental model theory overlap, and there is no overlap between the participant's choices and the conclusions computed by the verbal model theory.

6.3 Syllogism EA3

The program representing syllogism EA3 assuming existential import is

$$\mathcal{P}_{\mathsf{EA3}} = \{b'(X) \leftarrow a(X),\ b(X) \leftarrow \neg b'(X),\ a(o_1) \leftarrow \top,\ b(X) \leftarrow c(X),\ c(o_2) \leftarrow \top\}$$

The weak completion of $\mathsf{g}\mathcal{P}_{\mathsf{EA3}}$ is

$$\begin{array}{l}\{\ b'(o_1) \leftrightarrow a(o_1),\ b(o_1) \leftrightarrow \neg b'(o_1) \vee c(o_1),\ a(o_1) \leftrightarrow \top, \\ \ \ b'(o_2) \leftrightarrow a(o_2),\ b(o_2) \leftrightarrow \neg b'(o_2) \vee c(o_2),\ c(o_2) \leftrightarrow \top\ \ \}\end{array}$$

Its least Ł-model is

$$\langle\{a(o_1), b'(o_1), c(o_2), b(o_2)\}, \{b(o_1)\}\rangle$$

which does not map any statement involving A, I, E, or O to *true*. Hence, WCS leads to NVC. The majority of the participants concluded Eac and Eca. Hence, there is no overlap in this example. One should observe that Eac and Eca are the only valid conclusions in classical FOL. Inspecting the results depicted in Table 3 we observe that PSYCOP, the verbal model theory as well as the mental model theory compute solutions which overlap with the participants choices.

	participants	FOL	PSYCOP	verbal models	mental models	WCS
IE1	Eac, Oac	Oac	Oac, Iac, Ica	Oac	Eac, Eca, Oca, Oac, NVC	Eac, Oac
EA3	Eac, Eca	Eac, Eca	Eac, Eca, Oac, Oca	NVC, Eca	Eac, Eca	NVC
AA4	Aac, NVC	Iac, Ica	Iac, Ica	NVC, Aca	Aca, Aac, Iac, Ica	Aca, Aac, Iac, Ica

Table 5. The conclusions drawn by the participants are highlighted in gray and compared to the predictions of the theories FOL, PSYCOP, verbal and mental models as well as WCS for the syllogisms IE1, EA3, and AA4.

6.4 Summary

We have formalized three examples under WCS and have compared them to FOL, PSYCOP, the verbal, and the mental model theory. The results are summarized in Table 5. The selected examples are typical in the sense that for some syllogisms the conclusions drawn by the participants and WCS are identical, for some syllogisms the conclusions drawn by the participants and WCS overlap, and for some syllogisms the conclusions drawn by the particpants and WCS are disjoint. Moreover, WCS differs from the other cognitive theories.

7 Discussion

Our goal is to compare WCS to existing cognitive theories on syllogistic reasoning. To this end, we need to evaluate the predictions of WCS concerning all 64 syllogisms and compare it to all the cognitive theories mentioned in [24].

We did not consider abnormalities in the specification of implications as suggested in [32]. If each abnormality is mapped to false, then the specification with abnormalities is semantically equivalent to the specification given in this paper. On the first sight it appears that all abnormalities are indeed mapped to false, but we should take a second look. In particular, because we need to break the symmetry between our current specifications of Iac and Ica. Furthermore, in the reported meta-study participants dealt with abstract reasoning problems. This may explain why we did not need abnormalities here. However modeling the belief bias in syllogistic reasoning can require the abnormality predicate [28,4].

Rules like $b(X) \leftarrow \neg b'(X)$ have been introduced as a technical means to deal with implications whose conclusion is negative. Such negative conclusions cannot be directly modelled in WCS as the model intersection property would be lost. This technical reason might be justified by the principle of truths [22], which states that only true items can be represented. Please note that these additional rules come without existential import. Adding such import will introduce a new constant for each of these rules, which may lead to different conclusions.

References

1. J.E. Adler and L.J. Rips. *Reasoning: Studies of Human Inference and Its Foundations*. Cambridge University Press, 2008.
2. L. J. Chapman and J. P. Chapman. Atmosphere effect re-examined. *Journal of Experimental Psychology*, 58(3):220–6, 1959.
3. K. L. Clark. Negation as failure. In H. Gallaire and J. Minker, editors, *Logic and Data Bases*, volume 1, pages 293–322. Plenum Press, New York, NY, 1978.
4. E.-A. Dietz. A computational logic approach to syllogisms in human reasoning. In Claudia Schon Ulrich Furbach, editor, *Proceedings of the Workshop on Bridging the Gap between Human and Automated Reasoning on the 25th International Conference on Automated Deduction (CADE-25)*, pages 17–31. CEUR-WS.org, 2015.
5. E.-A. Dietz and S. Hölldobler. A new computational logic approach to reason with conditionals. In F. Calimeri, G. Ianni, and M. Truszczynski, editors, *Logic Programming and Nonmonotonic Reasoning, 13th International Conference, LPNMR*, volume 9345 of *Lecture Notes in Artificial Intelligence*, pages 265–278. Springer, 2015.
6. E.-A. Dietz, S. Hölldobler, and R. Höps. A computational logic approach to human spatial reasoning. In *IEEE Symposium on Human-Like Intelligence (CIHLI)*, 2015. (to appear).
7. E.-A. Dietz, S. Hölldobler, and L. M. Pereira. On conditionals. In G. Gottlob, G. Sutcliffe, and A. Voronkov, editors, *Global Conference on Artificial Intelligence*, Epic Series in Computing. EasyChair, 2015.
8. E.-A. Dietz, S. Hölldobler, and M. Ragni. A computational logic approach to the suppression task. In N. Miyake, D. Peebles, and R. P. Cooper, editors, *Proceedings of the 34th Annual Conference of the Cognitive Science Society*, pages 1500–1505, Austin, TX, 2012.
9. E.-A. Dietz, S. Hölldobler, and M. Ragni. A computational logic approach to the abstract and the social case of the selection task. In *11th International Symposium on Logical Formalizations of Commonsense Reasoning*, 2013.
10. E.-A. Dietz, S. Hölldobler, and C. Wernhard. Modeling the suppression task under weak completion and well-founded semantics. *Journal of Applied Non-Classsical Logics*, 24(1–2):61–85, 2014.
11. J. S. Evans. *Bias in human reasoning - causes and consequences.* Essays in cognitive psychology. Lawrence Erlbaum, 1989.
12. J.S.B.T. Evans, J. L. Barston, and P. Pollard. On the conflict between logic and belief in syllogistic reasoning. *Memory & Cognition*, 11(3):295–306, 1983.
13. J.S.T. Evans. Thinking and believing. *Mental models in reasoning*, 2000.
14. A. Garnham and J. Oakhill. *Thinking and Reasoning*. Wiley, 1994.
15. H Paul Grice. Logic and conversation. reprinted in studies in the way of words, ed. hp grice.(1989), 1975.
16. S. Hölldober. *Logik und Logikprogrammierung*, volume 1: Grundlagen. Synchron Publishers GmbH, Heidelberg, 2009.
17. S. Hölldobler and C. D. Kencana Ramli. Logic programs under three-valued Łukasiewicz semantics. In P. M. Hill and D. S. Warren, editors, *Logic Programming, 25th International Conference, ICLP 2009*, volume 5649 of *Lecture Notes in Computer Science*, pages 464–478, Heidelberg, 2009. Springer.

18. S. Hölldobler and C. D. Kencana Ramli. Logics and networks for human reasoning. In *International Conference on Artificial Neural Networks, ICANN 2009, Part II*, pages 85–94, 2009.
19. P. N Johnson-Laird. *Mental models: Towards a cognitive science of language, inference, and consciousness*. Harvard University Press, 1983.
20. P. N. Johnson-Laird. *Mental models: towards a cognitive science of language, inference, and consciousness*. Harvard University Press, Cambridge, MA, 1983.
21. P. N. Johnson-Laird and R. M. J. Byrne. *Deduction*. Lawrence Erlbaum Associates, Inc, Hove and London, 1991.
22. P. N Johnson-Laird and F. Savary. Illusory inferences: A novel class of erroneous deductions. *Cognition*, 71(3):191–229, 1999.
23. A. C. Kakas, R. A. Kowalski, and F. Toni. Abductive logic programming. *Journal of Logic and Computation*, 2(6):719–770, 1993.
24. S. Khemlani and P. N. Johnson-Laird. Theories of the syllogism: A meta-analysis. *Psychological Bulletin*, pages 427–457, 2012.
25. Sangeet Khemlani and P. N. Johnson-Laird. Theories of the Syllogism: A Meta-Analysis. *Psychological Bulletin*, 138:427–457, 2012.
26. J. W. Lloyd. *Foundations of Logic Programming*. Springer-Verlag New York, Inc., New York, NY, USA, 1984.
27. Jan Łukasiewicz. O logice trójwartościowej. *Ruch Filozoficzny*, 5:169–171, 1920. English translation: On three-valued logic. In: Łukasiewicz J. and Borkowski L. (ed.). (1990). *Selected Works*, Amsterdam: North Holland, pp. 87–88.
28. L. M. Pereira, E.-A. Dietz, and S. Hölldobler. An abductive reasoning approach to the belief-bias effect. In C. Baral, G. De Giacomo, and T. Eiter, editors, *Principles of Knowledge Representation and Reasoning: Proceedings of the 14th International Conference*, pages 653–656. AAAI Press, 2014.
29. L. M. Pereira, E.-A. Dietz, and S. Hölldobler. Contextual abductive reasoning with side-effects. In I. Niemelä, editor, *Theory and Practice of Logic Programming (TPLP)*, volume 14, pages 633–648. Cambridge University Press, 2014.
30. Thad A Polk and Allen Newell. Deduction as verbal reasoning. *Psychological Review*, 102(3):533, 1995.
31. L. J. Rips. *The psychology of proof: Deductive reasoning in human thinking*. The MIT Press, Cambridge, MA, 1994.
32. K. Stenning and M. van Lambalgen. *Human Reasoning and Cognitive Science*. A Bradford Book. MIT Press, Cambridge, MA, 2008.
33. Allen Van Gelder, Kenneth A. Ross, and John S. Schlipf. The well-founded semantics for general logic programs. *Journal of the ACM*, 38(3):619–649, 1991.
34. R. S. Woodworth and S. B. Sells. An atmosphere effect in formal syllogistic reasoning. *Journal of Experimental Psychology*, 18(4):451–60, 1935.