Using Kernel Consolidation for Query Answering in Inconsistent OBDA

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Abstract

This paper investigates the definition of belief revision operators that correspond to the inconsistency tolerant semantics in the Ontology Based Data Access (OBDA) setting. By doing this, we aim at providing a more general characterisation, as well as the construction, of the above mentioned semantics. In fact, the main result of this paper is the idea of using kernel consolidation in Datalog$^+$ to achieve a new semantic for inconsistent tolerant ontology query answering.

Keywords. Ontology Based Data Access, Inconsistency-tolerant semantics, Kernel Consolidation.

1 Introduction

We position ourselves in the Ontology Based Data Access (OBDA) setting where a query is being asked over a set of knowledge bases defined over a common ontology. When the union of knowledge bases along with the ontology is inconsistent, several semantics have been defined (Bienvenu 2012; Lembo et al. 2010) which are tolerant to inconsistency. They all rely on computing *repairs*, i.e. maximal (in terms of set inclusion) subsets of the knowledge bases. Several inconsistency tolerant semantics (such as Intersection of All Repairs: IAR, All Repairs: AR, Intersection of Closed Repairs: ICR) have been studied (Bienvenu 2012; Lembo et al. 2010), from a productivity point of view and a complexity point of view.

In this paper we take a new approach and aim to define new characterisations of two such semantics IAR and ICR which are specially interesting because these give us a unique result and because of their data complexity. We argue that such characterisation can provide an alternative way of comparing the semantics and can provide new insights into their properties. Furthermore, such characterisation can be used when proposing a generalisation of inconsistency tolerant semantics.

In order to provide the new characterisation we define belief revision operators that correspond to IAR and ICR based on kernel consolidation. This characterization will be more general than the one proposed in (Croitoru and Rodriguez 2014). Please note that while a lot of work has been done in belief revision and OBDA, none of the approaches deal with the axiomatic characterisations of the inconsistency tolerant semantics. The paper is structured as follows: Section 2 introduces the rule based OBDA language used in the paper and three semantics for inconsistent tolerant query answering in this setting. In Section 3 we introduce the concepts of belief revision adapted to the OBDA context and we reproduce results from (Croitoru and Rodriguez 2014). In Section 4, we take an alternative point of view and consider a more general characterisation of inconsistency tolerant semantics using kernel consolidation. In addition, we provide a new axiomatic characterisation of the inconsistency tolerant semantics. Finally, Section 5 concludes the paper and comments on future work.

2 Rule Based Knowledge Representation

There are two major approaches in the literature used to represent an ontology for the OBDA problem: Description Logics (such as $\mathcal{EL}$ (Baader, Brandt, and Lutz 2005) and DL-Lite (Calvanese et al. 2007) families) and rule based languages. The most notable rule based language is the Datalog$^+$ (Calì, Gottlob, and Lukasiewicz 2009) language, a generalization of Datalog that allows for existentially quantified variables in the head of the rules. Despite Datalog$^+$ undecidability when answering conjunctive queries, there exist decidable fragments of Datalog$^+$ that are studied in the literature (Baget et al. 2011). These fragments generalize the above mentioned Description Logics families.

In this paper we represent the ontology via rules using the Datalog+ language. We consider a (potentially inconsistent) knowledge base composed of a set $\mathcal{F}$ of facts corresponding to existentially closed conjunctions of atoms[1], which can contain n-ary predicates; a set of negative constraints $\mathcal{N}$ which represent the negation of a fact and an ontology composed of a set of rules $\mathcal{R}$ that represent general implicit knowledge that can introduce new variables in their head (conclusion).

A rule is *applicable* to set of facts $\mathcal{F}$ if and only if the set entails the hypothesis of the rule. If rule R is appli-

[1]For technical reasons we consider $\mathcal{F}$ as a set of atoms and not as a conjunction of atoms. While the two are equivalent when dealing with consistent knowledge bases, this is no longer the case when dealing with inconsistent knowledge bases. This work convention is very important in the remaining parts of the paper.

cable to the set F, the application of R on F produces a new set of facts obtained from the initial set with additional information from the rule conclusion. We then say that the new set is an *immediate derivation* of F by R denoted by $R(F)$. Let F be a set of facts and let $\mathcal{R}$ be a set of rules. A set F_n is called an *$\mathcal{R}$-derivation* of F if there is a sequence of sets (*derivation sequence*) $(F_0, F_1, \ldots, F_n)$ such that: (i) $F_0 \subseteq F$, (ii) F_0 is $\mathcal{R}$-consistent, (iii) for every $i \in \{1, \ldots, n-1\}$, it holds that F_i is an immediate derivation of F_{i-1}. Given a set $\{F_0, \ldots, F_k\}$ and a set of rules $\mathcal{R}$, the closure of $\{F_0, \ldots, F_k\}$ with respect to $\mathcal{R}$, denoted $\mathtt{Cl}_{\mathcal{R}}(\{F_0, \ldots, F_k\})$, is defined as the smallest set (with respect to $\subseteq$) which contains $\{F_0, \ldots, F_k\}$, and is closed for $\mathcal{R}$-derivation (that is, for every $\mathcal{R}$-derivation F_n of $\{F_0, \ldots, F_k\}$, we have $F_n \subseteq \mathtt{Cl}_{\mathcal{R}}(\{F_0, \ldots, F_k\})$). Finally, we say that a set $\mathcal{F}$ and a set of rules $\mathcal{R}$ *entail* a fact G (and we write $\mathcal{F}, \mathcal{R} \models G$) iff the closure of the facts by all the rules entails G (i.e. if $\mathtt{Cl}_{\mathcal{R}}(\mathcal{F}) \models G$). Given a set of facts $\{F_1, \ldots, F_k\}$, and a set of rules $\mathcal{R}$, the set of facts is called *$\mathcal{R}$-inconsistent* if and only if there exists a constraint $N = \neg\mathcal{F}$ such that $\mathtt{Cl}_{\mathcal{R}}(\{F_1, \ldots, F_k\}) \models \mathcal{F}$. A set of facts is said to be *$\mathcal{R}$-consistent* iff it is not $\mathcal{R}$-inconsistent. A knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is said to be *consistent* if and only if $\mathcal{F}$ is $\mathcal{R}$-consistent. A knowledge base is *inconsistent* if and only if it is not consistent.

Several semantics have been proposed to handle consistency based on the concept of data *repairs* (Bienvenu 2012; Lembo et al. 2010; Lukasiewicz, Martinez, and Simari 2013). Once the repairs are computed, various strategies can be adapted to answer a query. We can consider *all repairs* (AR-semantics), the *intersection of all repairs* (IAR-semantics) or the *intersection of closed repairs* (ICR-semantics).

Definition 1 *(Bienvenu 2012; Lembo et al. 2010) Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **AR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{AR} \alpha$ iff for every repair $A' \in \mathcal{R}epair(\mathcal{K})$, it holds that $\mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$.*

Definition 2 *(Bienvenu 2012; Lembo et al. 2010) Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **IAR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{IAR} \alpha$ iff $\mathtt{Cl}_{\mathcal{R}}(\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})}) \models \alpha$.*

Definition 3 *(Bienvenu 2012; Lembo et al. 2010) Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **ICR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{ICR} \alpha$ iff $\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} \mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$.*

It has been shown (Lembo et al. 2010; Lukasiewicz, Martinez, and Simari 2013) that deciding if $\mathcal{K} \models_{AR} \alpha$ is coNP-complete in data complexity. On the contrary, the same authors have shown that deciding if either $\mathcal{K} \models_{IAR} \alpha$ or $\mathcal{K} \models_{ICR} \alpha$ is polynomially tractable. For this reason, many authors prefer to adapt either IAR or ICR semantics for their applications.

3 Belief Revision Operators

Definition 4 *A selection function is a function γ such that for every set $\mathcal{F}$ of formulae and any fact α it holds: $\gamma(\mathcal{F} \perp \alpha)$ is a non-empty subset of $\mathcal{F} \perp \alpha$ if this set is non-empty, otherwise, it is $\gamma(\mathcal{F} \perp \alpha) = \{\mathcal{F}\}$.*

Definition 5 *(Updated AGM85 (Alchourrón, Gärdenfors, and Makinson 1985)) Let $\mathcal{F}$ be a set of facts in a knowledge base $\mathcal{K}$. Let $\mathcal{F} \perp \alpha$ and γ be the set of all maximal subsets of $\mathcal{F}$ that do not imply α and a selection function, respectively. The* partial meet contraction *on $\mathcal{F}$ that is generated by γ is the operation $\sim_\gamma$ such that for all facts α:*

$$\mathcal{F} \sim_\gamma \alpha = \cap\gamma(\mathcal{F} \perp \alpha)$$

Two limiting cases have been thoroughly studied: when γ gives back either only one element of $\mathcal{F} \perp \alpha$ or all members of $\mathcal{F} \perp \alpha$. In the first case, we are talking about *Maxichoice contraction* and in the second it is called *Full meet contraction*(FMC).

There are other special and interesting cases when the selection function is based on a relation (that may be considered as a preference relation).

Definition 6 *A selection function γ for a belief base $\mathcal{F}$ in a knowledge base $\mathcal{K}$, and the contraction operator based on it, are*

1. *relational if and only if there is a binary relation $\sqsubseteq$ such that for every fact α, if $\mathcal{F} \perp \alpha$ is non-empty, then*

$$\gamma(\mathcal{F} \perp \alpha) = \{A \in \mathcal{F} \perp \alpha | C \sqsubseteq A \textit{ for all } C \in \mathcal{F} \perp \alpha\}$$

2. *transitively relational if and only if there is such a relation that is transitive.*

Based on partial meet contraction, one can define a partial meet consolidation as $\mathcal{F} \sim_\gamma \perp$ which is the intersection of the "most preferred" maximal consistent subsets of $\mathcal{F}$, i.e. $\mathcal{F}! = \mathcal{F} \sim_\gamma \perp = \cap\gamma(\mathcal{F} \perp \perp)$ where $\perp$ denotes logical contradiction.

Partial meet consolidation has been axiomatically characterized as follows:

Theorem 1 *(Adapted (Hansson 1991)) An operation is a partial meet consolidation if and only if for all sets $\mathcal{F}$ of facts the following are satisfied:*

Consistency: *$\mathcal{F}!$ is $\mathcal{R}$-consistent.*

Inclusion: *$\mathcal{F}! \subseteq F$.*

Relevance: *If $\alpha \in \mathcal{F} \restriction F!$, then there is some $\mathcal{F}'$ with $\mathcal{F}! \subseteq \mathcal{F}' \subseteq \mathcal{F}$, such that $\mathcal{F}'$ is $\mathcal{R}$-consistent and $\mathcal{F}' \cup \{\alpha\}$ is $\mathcal{R}$-inconsistent.*

*In addition, it is a **full meet consolidation** if and only if it also satisfies:*

Core identity: *$\beta \in \mathcal{F}!$ if and only if $\beta \in \mathcal{F}$ and there is no $\mathcal{F}' \subseteq \mathcal{F}$ such that $\mathcal{F}'$ is $\mathcal{R}$-consistent but $\mathcal{F}' \cup \{\beta\}$ is $\mathcal{R}$-inconsistent.*

*On the other hand, an operator is a **maxi-choice consolidation** if and only if it satisfies the postulates consistency, inclusion, and:*

Fullness: *If $\beta \in \mathcal{F}$ and $\beta \in \mathcal{F}!$ then $\mathcal{F}! \cup \{\beta\}$ is $\mathcal{R}$-inconsistent.*

There are different ways to characterize belief base functions. In fact, we would say that there is an axiomatic characterization and an infinite number of constructions, four of them were extensively studied in the literature: using remainder sets, using kernel sets, epistemic entrenchment and spheres system. In (Croitoru and Rodriguez 2014), it was defined operators of consolidation that given an inconsistent knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, return a new consistent knowledge base $\mathcal{K}! = (\mathcal{F}!, \mathcal{R}, \mathcal{N})$. In addition, a characterization of IAR and ICR semantics was given in terms of FMC such that:

1. If ! is a Full Meet Consolidation for $\mathcal{F}$ then we get the IAR semantics.
2. If ! is a Full Meet Consolidation for $\mathtt{Cl}_{\mathcal{R}}(\mathcal{F})$ then we get the ICR semantics.

Furthermore, in (Croitoru and Rodriguez 2014), it was shown the reciprocal is also true. In addition, the notion of full meet consolidation was used to show an axiomatic characterization of IAR and ICR semantics.
In the following section, we will define the operators corresponding to the inconsistency tolerant semantics above using kernel sets.

4 Kernel Consolidation

In this section, we are considering an alternative point of view which is based on the simple hypothesis that a fact α is $\mathcal{R}$-entailed from a subset $\mathcal{A}$ of $\mathcal{F}$ if and only if it contains some minimal α-entailing subset of $\mathcal{A}$. Hence, in this context, in order to remove α from $\mathcal{F}$, it is only necessary to remove at least one fact of each minimal α-entailing subset of $\mathcal{F}$. In formal terms, this approach was formalized by Hansson in (Hansson 1994) and we have adapted it to the ontology based data access setting as follows:

Definition 7 *Let $\mathcal{F}$ be a set of facts in a knowledge base $\mathcal{K}$ and α a fact. Then $\mathcal{F} \perp\!\!\!\perp \alpha$ is the set such that $A \in \mathcal{F} \perp\!\!\!\perp \alpha$ if and only if:*

1. $A \subseteq \mathcal{F}$.
2. $\alpha \in \mathtt{Cl}_{\mathcal{R}}(A)$.
3. If $B \subset A$, then $\alpha \notin \mathtt{Cl}_{\mathcal{R}}(B)$.

$\mathcal{F} \perp\!\!\!\perp \alpha$ is a kernel set, and its elements are the α-kernels of $\mathcal{F}$.

A contraction operation $\sim$ can be based on the simple principle that no α-kernel should be included in $\mathcal{F} \sim \alpha$. This can be obtained with an incision function, a function that selects at least one element from each α-kernel for removal. It was defined by Hansson and adapted for our setting here as follows:

Definition 8 *An incision function σ for $\mathcal{F}$ is a function such that for every α:*

i) *$\sigma(\mathcal{F} \perp\!\!\!\perp \alpha) \subseteq \cup(\mathcal{F} \perp\!\!\!\perp \alpha)$.*
ii) *If $\emptyset \neq \mathcal{A} \in \mathcal{F} \perp\!\!\!\perp \alpha$, then $\mathcal{A} \cap \sigma(\mathcal{F} \perp\!\!\!\perp \alpha) \neq \emptyset$.*

An operation that removes exactly those elements that are selected for removal by an incision function is called an operation of kernel contraction and it responds to the following definition:

Definition 9 *Let σ be an incision function for $\mathcal{F}$. The kernel contraction $\approx_\sigma$ for $\mathcal{F}$ is defined as follows:*

$$\mathcal{F} \approx_\sigma \alpha = \mathcal{F} \upharpoonright \sigma(\mathcal{F} \perp\!\!\!\perp \alpha)$$

It turns out that all partial meet contractions on belief bases are kernel contractions, but the converse relationship does not hold, i.e. there are kernel contractions that are not partial meet contractions. In other words, kernel contraction is a generalization of partial meet contraction.
However, the next observation gives a direct connection between them (see (Hansson 1991)):

Observation 1

$$\cap(\mathcal{F} \perp \alpha) = \mathcal{F} \upharpoonright \cup(\mathcal{F} \perp\!\!\!\perp \alpha)$$

The above definition of kernel contraction can be straightforwardly transferred to consolidation. Thus, *Kernel Consolidation‡* is defined in the following way:

$$\mathcal{F}\ddagger = \mathcal{F} \approx_\sigma \perp = \mathcal{F} \upharpoonright \sigma(\mathcal{F} \perp\!\!\!\perp \perp)$$

Hence, if a set of facts $\mathcal{F}$ is inconsistent with a new fact f, a straight solution to gain consistency is to repair the joined theory $\mathcal{F} \cup \{f\}$, by removing from it the minimal number of facts that support the contradiction. This simple idea underlies kernel sets plus incision function as mentioned above.

Example 1 *((Lembo et al. 2010)) We consider a simple knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ describing the "Formula One Teams" domain, where:*

$\mathcal{F}$ $\{Mechanic(felipe), Driver(felipe)\}$.
$\mathcal{R}$ $\{Mechanic(x) \rightarrow TeamMember(x), Driver(y) \rightarrow TeamMember(y)\}$.
$\mathcal{N}$ $\{Driver(z), Mechanic(z) \rightarrow\}$.

In words, $\mathcal{K}$ specifies that drivers and mechanics are team members, but drivers are not mechanics (and viceversa, because $\mathcal{N}$ states that the two concepts Driver and Mechanic are disjoint). The facts $Mechanic(felipe)$ and $Driver(felipe)$ asserts that $felipe$ is both a driver and a mechanic. It is easy to see that $\mathcal{K}$ is unsatisfiable, since $felipe$ violates the disjointness between driver and mechanic. There is only one α-kernel in this case: $\mathcal{F} \perp\!\!\!\perp \perp = \{Mechanic(felipe), Driver(felipe)\}$. Therefore, the incision function σ must select a non-empty subset of $\mathcal{F} \perp\!\!\!\perp \perp$. There are three different possibilities, i.e. $\sigma(\mathcal{F} \perp\!\!\!\perp \perp)$ is any of the following: $\{Mechanic(felipe)\}$; $\{Driver(felipe)\}$; $\{Mechanic(felipe), Driver(felipe)\}$.

According to Observation 1 and the results given in (Croitoru and Rodriguez 2014), IAR and ICR semantics correspond to an incision function that chooses $\{Mechanic(felipe), Driver(felipe)\}$. But overall, we are able to obtain more interesting results. For instance, if we consider that $felipe$ is more important in the team as Driver than as Mechanic, then we should only remove $Mechanic(felipe)$ in order to recover consistency. Intuitively, this kind of incision function can be thought of as selecting for deletion the least valuable or important elements of each kernel. It is therefore reasonable to let σ be based in a binary relation that represents comparative epistemic value. The study of this kind of relational kernel consolidation will be left for future research.

Axiom Compliance

In this section we go one step further in the definition of kernel consolidation operators through a set of postulates. In order to give logical properties of that kind of consolidation operators, we first rephrase Hansson's postulates within our framework. Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, the original postulates can be rewritten in the following way:

Consistency: $\mathcal{K}\ddagger = (\mathcal{F}\ddagger, \mathcal{R}, \mathcal{N})$ is consistent.

Inclusion: $\mathcal{K}\ddagger = (\mathcal{F}\ddagger, \mathcal{R}, \mathcal{N}) \sqsubseteq \mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$.

Core-retainment: If $f \in \mathcal{F} \upharpoonright \mathcal{F}\ddagger$, then there is $\mathcal{F}' \subseteq \mathcal{F}$ such that $\mathcal{F}'$ is $\mathcal{R}$-consistent and $\mathcal{F}' \cup \{f\}$ does not.

The first postulate says that the result of every consolidation is always consistent. Inclusion says that the new knowledge base should not contain anything that was not already in the original set. The postulate of core-retainment tries to capture the intuition that if a fact has to be removed, then this fact is relevant to imply $\bot$. The representation theorem for this kind of consolidation operators can be formulated as follows:

Theorem 2 *An operator $\ddagger$ is an operation of kernel consolidation if and only if it satisfies Consistency, Inclusion and Core-retainment.*

Proof *Checking that operations of kernel consolidation satisfy postulates: Consistency and Inclusion follow directly from definition. To see that kernel consolidation satisfies Core-retainment, let $f \in \mathcal{F} \upharpoonright \mathcal{F}\ddagger$. Then $f \in \sigma(\mathcal{K}\perp\!\!\!\perp \bot)$. Since $\sigma(\mathcal{F}\perp\!\!\!\perp \bot) \subseteq \cup(\mathcal{F}\perp\!\!\!\perp \bot)$, there should be some set $\mathcal{D}$ such that $f \in \mathcal{D} \in \mathcal{F}\perp\!\!\!\perp \bot$. Let $\mathcal{F}' = \mathcal{D} \upharpoonright \{f\}$. Thus $\mathcal{F}' \not\vdash \bot$ and $\mathcal{F}' \cup \{f\} \vdash \bot$. Hence, $\ddagger$ satisfies Core-retainment.*

On the contrary direction, let $\ddagger$ be an operator that satisfies the three postulates mentioned in the theorem. We need to show there exists a incision function σ such that $\mathcal{F}\ddagger = \mathcal{F} \upharpoonright \sigma(\mathcal{F}\perp\!\!\!\perp \bot)$. For that, we define $\sigma(\mathcal{F}\perp\!\!\!\perp \bot) = \mathcal{F} \upharpoonright \mathcal{F}\ddagger$. Clearly, it follows from inclusion that $\mathcal{F}\ddagger = \mathcal{F} \upharpoonright \sigma(\mathcal{F}\perp\!\!\!\perp \bot)$. Therefore, it only remains to verify that σ is an incision function. Of course, σ is a function. Then, we need to show that it satisfies the two conditions of Definition 8.
For the first condition, we are going to show that $\sigma(\mathcal{F}\perp\!\!\!\perp \bot) \subseteq \cup(\mathcal{F}\perp\!\!\!\perp \bot)$. We take $f \in \sigma(\mathcal{F}\perp\!\!\!\perp \bot)$. From that and Core-retainment, we conclude there exists $\mathcal{F}' \subseteq \mathcal{F}$ such that $\mathcal{F}'$ is $\mathcal{R}$-consistent and $\mathcal{F}' \cup \{f\}$ does not. Hence, $f \in \cup(\mathcal{F}\perp\!\!\!\perp \bot)$.
For the second condition, we need to show for any $\mathcal{D} \in \mathcal{F}\perp\!\!\!\perp \bot$ that $\mathcal{D} \cap \sigma(\mathcal{F}\perp\!\!\!\perp \bot) \neq \emptyset$. By consistency, $\mathcal{F}\ddagger \not\vdash \bot$.Then, we conclude that $\mathcal{D} \nsubseteq \mathcal{F}\ddagger$, and thus, there is some $f' \in \mathcal{D}$ such that $f' \notin \mathcal{F}\ddagger$. Since $\mathcal{D} \subset \mathcal{F}$ it follows that $f' \in \mathcal{F} \upharpoonright \mathcal{F}\ddagger$, i.e. $f' in \sigma(\mathcal{F}\perp\!\!\!\perp \bot)$. Therefore, $f' \in \mathcal{D} \cap \sigma(\mathcal{F}\perp\!\!\!\perp \bot)$, which is enough to show that ii) in Definition 8 is satisfied.
This concludes the proof.

5 Conclusion and future work

In this paper we have shown how to get a more general characterisation of the tolerance inconsistence semantics in the OBDA setting. We did this by covering the semantics using known kernel consolidation operators from belief revision. Note that by using the equivalence showed in (Croitoru and Vesic 2013) we also obtained here an axiomatic characterisation of some argumentation semantics in a particular logic instantiated case. Such result can serve as basis for an axiomatic characterisation of argumentation semantics in general. It is worth mentioning that our approach does not depend on the existence of negation as usual in Belief Revision. That is important because many description logics do not admit negation of all kinds of axioms. We are currently investigating the link between preference based inconsistency tolerant reasoning in the OBDA setting and kernel consolidation.

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