# Selection of the Optimal Parameters of the Process for Thermal Laser Treatment of Metals for Creating the Molten Pool with a Required Depth

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Abstract. In the paper we examine the problem of optimization of parameters of technological process for obtaining a molten pool with a needed depth on the surface of the metal sample under the thermal local laser treatment. A mathematical model of heat distribution during thermal laser treatment is simplified to a linear problem of heating a half-space, that can be used for plates of finite thickness. Despite a number of assumptions, this approach gives satisfactory qualitative and quantitative understanding of the heating stage. The accepted model allows us to control the quality criteria of the surface layer and, above all, its main feature – the depth of the molten pool by selecting the optimal parameters of laser radiation. To solve the optimization problem the algorithm based on the scanning method of the search space (the zero-order method of conditional optimization) has been built. The obtained results can be used both as preliminary recommendations for setting variable parameters of the real technological process and as prediction of phase and structural states of the irradiated material.

**Keywords:** optimization of technological process, thermal laser treatment, power, molten pool, melting temperature, boiling temperature

## Introduction

In the modern industry the laser treatment of the materials has already become quite common. Such processes as laser cutting, welding, punching, marking, etc. have been widely used. The application of fiber lasers is the most effective, due to their small size, ease of use and the possibility of pervasion into difficult accessible places [1–3]. In addition, the fiber lasers of the new generation are of high quality of optical radiation, of high power and high speed of the scanning on the material surface. Modern laser technologies are complicated and depend on many factors. The same technological process can not proceed under the absolutely same conditions. Therefore, the development of

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a set of recommendations for the specific equipment to ensure successful achievement of the technological process results is an important task.

One of the popular laser thermal processing (laser scanning on the material surface) is the formation of a molten pool with different configurations and sizes in the surface layer of the sample [3]. Thermal material parameters (density, heat capacity, thermal conductivity, thermal diffusivity, melting temperature, boiling temperature, etc.), the geometric sizes of the sample, and the technological possibilities of the laser (lower and upper power limits of the laser treatment, the minimum and maximum spot radius of the laser radiation, etc.) affect the outcome of the process. Also an important factor is the action time (the exposure time of laser radiation). Experimental selection of variable parameters for obtaining a molten pool of a required size is a laborious, expensive and sometimes impossible process. Therefore, mathematical modeling and solving optimization problems, which arise in the solution of this issue, are always relevant.

In this paper the problem of choosing the optimal parameters of the thermal laser treatment for obtaining a molten pool with geometric sizes, which are the closest to the required, is examined. The calculations are carried out in the framework of the mathematical model of heat distribution under the local heating of the material (structural steel A) by laser treatment. The source of laser treatment is considered to be an impulse one. The source treatment power varies in the range from 20 to 1000 watts, the treatment time – from 0.01 to 0.1 sec, the laser spot radius of 0.0025 to 0.005 meters. Since the increasing of the laser source power and the treatment time lead to growing the depth of the molten pool and uncontrolled rising in the temperature on the surface of material until its boiling and vaporization, one of the main constraints on the process result is the condition that the material must not boil at the surface of the molten pool.

## 1 The mathematical model of heat distribution during thermal laser treatment

Conducting technological operation is determined by the specific impact of the laser radiation on the material (depends on the selected device) and specified features of a technological problem. The power, wavelength and time of radiation exposure are usually considered as the main parameters [4] among many characteristics of laser radiation. Here we examine the process of thermal action of the radiation, heating the object to a specific temperature, therefore the *laser radiation power* P is assumed as the main characteristic. The pulse power and mean power are considered for pulsed lasers. The mean power depends on the pulse time and frequency. The working object is heated to the temperature T, which is determined by the power density Q of absorbed radiation. The distribution of Q on the irradiated surface S can be specified in the various forms. For example [5], the absorbed power density Q with the uniform distribution law is directly proportional to the product of the radiation power  $P_0$  by a coefficient of surface absorption of laser radiation A and inversely proportional to the square of the irradiated zone S. For calculations a Gaussian distribution law of the absorbed power density within the irradiated zone usually used most often [6]:

$$Q = Aq_{\max}e^{-\kappa r^2},\tag{1}$$

where  $q_{\text{max}}$  – the highest heat flux to the beam axis, r – the radial distance from the beam axis to a point of the irradiated surface,  $\kappa$  – the coefficient characterizing the shape of a Gaussian probability curve on the irradiated surface ( $\kappa = 1/r_0^2$ , where  $r_0$  – the radius of the laser spot). The value of  $q_{\text{max}}$  in (1) depends on the concentration of heat flux zone. If this area is  $r \in [0, \infty)$ , then  $q_{\text{max}} = P_0/(\pi r_0^2)$  [5]. To calculate the local effects at  $r \in [0, r_0]$  the highest heat flux is determined by the dependence

$$q_{\max} = \frac{P_0}{\pi r_0^2 (1 - e^{-1})}.$$
(2)

Heating of a body under the action of laser radiation is described by the differential heat equation [6]:

$$\frac{\partial T(r,Z,t)}{\partial t} = \nabla^2 T(r,Z,t),\tag{3}$$

where  $r = \sqrt{X^2 + Y^2}$ ,  $\{X, Y, Z\}$  – Cartesian coordinates with origin at the center of the laser spot on the material surface, the axis Z coincides with the beam axis, perpendicular to the surface of the heated material and directed into the interior (Fig. 1), t – the time,  $\nabla^2$  – Laplace operator.



Fig. 1. Scheme of the molten pool obtained by thermal laser radiation: H – the depth of molten pool (melt depth); R – isotherm radius on the sample surface corresponding to the melting temperature  $T_{\rm m}$ ;  $r_0$  – the radius of the laser spot on the sample surface; h – the sample thickness

Equation (3) is valid for strong absorption of radiation, when the depth  $\delta$  of light pervasion into the material is much smaller than the thickness h of the heated layer:  $h \gg \delta$  ( $\delta = \alpha^{-1}$ ,  $h \gg a\tau$ ,  $\alpha$  – the light absorption coefficient, a – the heat diffusivity of the material,  $\tau$  – the pulse width, h – the material thickness). For metals the optical radiation with wave length  $\lambda = 10^{-1} \div 10^3$  m is absorbed in a layer with thickness  $\delta = 10^{-6} \div 10^{-5}$  cm.

The boundary condition on the surface Z = 0 specifies the action of surface source [6]:

$$-k \left. \frac{\partial T(r, Z, t)}{\partial Z} \right|_{Z=0} = \begin{cases} (1 - K_{\text{ref}})q(r, t), \, r \leqslant r_0, \\ 0, \qquad r > r_0, \end{cases}$$
(4)

where q(r,t) – the radiation power density on the body surface,  $K_{\text{ref}}$  – the surface reflection coefficient for the metal in the zone of influence of the laser radiation  $(1 - K_{\text{ref}} = A)$ ,  $(1 - K_{\text{ref}})q(r,t)$  – the absorbed specific radiation flux, k – the thermal conductivity of the material.

The solution of the problem (3)-(4) under the condition

$$\partial T(r, Z, t)|_{r=\infty} = \partial T(r, Z, t)|_{Z=\infty} = \partial T(r, Z, t)|_{t=0} = T_0$$
(5)

allows to specify the relationship between the temperature T(r, Z, t) of the irradiated material with the power density of the laser radiation q(r, t). In (5)  $T_0$  denotes the initial temperature of the irradiated body. The real space-time distribution of the laser radiation q(r, t) is usually approximated by various functions [6]. Here we use the dependence

$$q(r) = q_{\max}e^{-r^2/r_0^2}, \quad q(t) = \text{const.}$$
 (6)

Assuming impulse heating, when  $r_0 \gg \sqrt{a\tau}$ , a solution of (3)–(6) is the dependence

$$T(r,Z,t) = \frac{Ar_0}{k\sqrt{\pi}}q_{\max} \arctan\left[\frac{2\sqrt{at}}{r_0}\right] + T_0, \quad r = 0, \quad Z = 0.$$
(7)

Formula (7) is also used to calculate the threshold power density at a certain threshold temperature  $T_{\rm th}$  (melting point, boiling point, etc.):

$$q_{\rm th} = (T_{\rm th} - T_0) \left( \frac{Ar_0}{k\sqrt{\pi}} \arctan\left[\frac{2\sqrt{at}}{r_0}\right] \right)^{-1}, \quad r = 0, \quad Z = 0.$$
(8)

On the surface of the irradiated material  $(r \neq 0, Z = 0)$  the solution of problem (3)–(6) is the dependence

$$T(r, Z, t) = \frac{2A\sqrt{at}}{k}q_{\max}e^{-r^2/r_0^2}\operatorname{Erfc}\left[\frac{Z}{2\sqrt{at}}\right] + T_0,$$
(9)

where  $\operatorname{Erfc}[Z/(2\sqrt{at})]$  – the additional integral function of the probability integral.

At the depth of the irradiated material  $(r \neq 0, Z \neq 0)$  we obtained the solution of problem (3)–(6) for the temperature

$$T(r, Z, t) = \frac{2A\sqrt{at}}{k} q_{\max} e^{-r^2/r_0^2} \operatorname{Erfc}\left[\frac{Z}{2\sqrt{at}}\right] + T_0.$$
 (10)

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The relations (10) and (2) allow us to write the relationship between temperature T and basic parameters of the laser treatment  $(P_0, r_0, t)$ :

$$T(P_0, r_0, t, r, Z) = \frac{2A\sqrt{at}}{k} \frac{P_0}{\pi r_0^2 (1 - e^{-1})} e^{-r^2/r_0^2} \operatorname{Erfc}\left[\frac{Z}{2\sqrt{at}}\right] + T_0.$$
(11)

From the formula (11) the dependence of the molten bath radius r, corresponding to some given values of Z and T, can be explicitly obtain:

$$r(P_0, r_0, t, Z, T) = r_0 \sqrt{\ln\left[\frac{2A\sqrt{at}}{k(T - T_0)} \frac{P_0}{\pi r_0^2 (1 - e^{-1})} \operatorname{Erfc}\left[\frac{Z}{2\sqrt{at}}\right]\right]}.$$
 (12)

From (12) at Z = 0 the isotherm radius R for a predetermined threshold value  $T = T_{\rm th}$  (melting temperature, boiling temperature, etc.) on the irradiated surface is obtained:

$$R(P_0, r_0, t) = r_0 \sqrt{\ln\left[\frac{2A\sqrt{at}}{k(T_{\rm th} - T_0)} \frac{P_0}{\pi r_0^2 (1 - e^{-1})}\right]}.$$
(13)

#### 2 The optimization problem

Here the problem of choosing the optimal parameters of a technological process for creating the molten pool by pulsed laser thermal treatment is formulated. For this purpose a formalism similar to [8–10] is used. The laser treatment power  $P_0$ , the spot radius  $r_0$  and the action time t of the laser radiation are taken as variable parameters. The vector of varied parameters is defined as

$$\mathbf{x} = (x_1, x_2, x_3),\tag{14}$$

where  $x_1 = P_0$ ,  $x_2 = r_0$ ,  $x_3 = t$ . The variable parameters (14) are limited by the inequalities

$$x_i^{\min} \leqslant x_i \leqslant x_i^{\max}, \quad i = \overline{1, 3}.$$
 (15)

In (15) the minimum and maximum values for each parameter  $x_i$  must be set by taking into account the technical capabilities of the equipment. In this paper the characteristics of industrial fiber laser with a maximum power of 1000 W [7] are selected for modeling.

Output parameters characterizing the creation process of the molten pool are denoted as

$$\mathbf{y}(\mathbf{x}) = (y_1, y_2, y_3),$$
 (16)

where  $y_1 = T_c(\mathbf{x})$  – the temperature in the spot center on the plate surface (calculated from (11) at Z = r = 0),  $y_2 = R(\mathbf{x})$  – the radius of the molten spot on the plate surface (calculated from (13) at  $T_{th} = T_f$ ) and  $y_3 = H(\mathbf{x})$  – the depth of the molten pool (implicit function, calculated from equation  $T(\mathbf{x}, H(\mathbf{x})) = T_f$  at r = 0). For output parameters (16) the following requirements are set:

$$y_j^{\min} < y_j(\mathbf{x}) < y_j^{\max}, \quad j = \overline{1, 3}.$$
(17)

The limits for the searching parameter  $y_j$  in (17) are set to satisfy the feasibility conditions of results of the technological process:

1) the material temperature in the center of the laser spot on the sample surface must be greater than the melting point and less then the boiling point:

$$T_{\rm f} < T_{\rm c}(\mathbf{x}) < T_{\rm b};$$

2) the radius of the molten pool on the sample surface must be greater than zero and less than the radius of the laser spot:

$$0 < R(\mathbf{x}) < r_0$$

3) the depth of of the molten pool must be greater than zero and less than the target value  $H^*$ :

$$0 < H(\mathbf{x}) < H^*.$$

The problem is to choose the parameters  $\mathbf{x}$  so that the feasibility conditions of the technological process would be satisfied and the depth of the molten pool  $H(\mathbf{x})$  would be close to a given value  $H^*$ .

Using the notation adopted above, the statement of the problem can be written in the form [8-10]:

$$\mathbf{x} = \arg\min_{\mathbf{x}\in D_x} |y_3(\mathbf{x}) - H^*|,\tag{18}$$

where the allowable set  $D_x$  in the space of variable parameters, for which the constraints (15) and the conditions of feasibility of the process (17) are satisfied, has the form

$$D_x = \left\{ \mathbf{x} \mid \mathbf{x}^{\min} \leqslant \mathbf{x} \leqslant \mathbf{x}^{\max}, \ \mathbf{y}^{\min} < \mathbf{y}(\mathbf{x}) < \mathbf{y}^{\max} \right\}.$$
(19)

Thus, the solving of the problem (18)–(19) lies in searching such set of variable parameters  $\mathbf{x} = (x_1, x_2, x_3)$ , at which the minimum of the objective function  $\Delta H(\mathbf{x})$  is achieved:

$$\Delta H(\mathbf{x}) = |y_3(\mathbf{x}) - H^*|.$$
<sup>(20)</sup>

For solving the optimization problem with constraints of a special kind [11] a scanning method of the search space (the zero-order conditional optimization) on a discrete set of values  $\mathbf{x}$  and  $\mathbf{y}(\mathbf{x})$  is used. Step  $dx_i$  for each of the variable parameters  $x_i$  is assumed constant. The solution algorithm is presented in block diagram form in Fig. 2. The use of first or second-order optimization methods in this case is rather difficult because of the complex form of model relations.

BLOCK 1 performs the calculation of the discrete set of the output parameters  $y_j(\mathbf{x})$ (j = 1, 2, 3) at each grid point  $(x_1, x_2, x_3)$ , at the same time the conditions of successful feasibility of technological process (17) are checked for each  $y_j$ . If all conditions (17) are met, the current point of space  $\mathbf{x}$  and the corresponding  $\mathbf{y}$  are suitable and recorded to the row n in the two-dimensional array  $\{\mathbf{x}; \mathbf{y}\}$  (n - counter of suitable iterations).

BLOCK 2, separately allocated on the scheme as part of BLOCK 1, contains the evaluation of the values  $y_3 = H(\mathbf{x})$ , given in implicit form by the equation  $T(\mathbf{x}, H(\mathbf{x})) = T_f$ , which can be solved with respect to H by dichotomy method.

In the BLOCK 3, the values  $\Delta H = |y_3^k(\mathbf{x}^k) - H^*|$  are calculated for the all saved sets  $\{\mathbf{x}^k; \mathbf{y}^k\}$  (k = 1, 2, ...n). Then, the minimum is determined among all of  $\Delta H$ , and



Fig. 2. The block diagram for solving of the problem about an optimal choice of variable parameters by scanning method of the search space

the corresponding set number is saved in the variable "Record". Thus, the solution of the problem (18)–(19) is a set of variable parameters  $\mathbf{x}^{\text{Record}}$ , delivering a minimum of the objective function (20).

## 3 Specifying of the basic parameters

For the computational experiment the necessary thermal characteristics of the chosen material (Table 1) and laser processing parameters (Table 2) are specified. The characteristics of the material (structural steel A) are defined according to [12]. The parameters in Table 2 correspond to industry ytterbium fiber lasers [7].

No.	Characteristic	Symbol	Value	Unit
1.	Matherial density	ρ	7870	$\mathrm{kg/m^{3}}$
2.	Heat capacity	c	465	$\rm J/(kg^{\circ}C)$
3.	Thermal conductivity	k	47	$W/(m^{\circ}C)$
4.	Thermal diffusivity	a	k/(c ho)	$\mathrm{m}^2/\mathrm{s}$
5.	The sample height (thickness)	h	0.004	m
6.	The sample length	d	0.15	m
7.	The sample width	b	0.10	m
8.	The melting temperature	$T_{\rm m}$	1535	$^{\circ}\mathrm{C}$
9.	The boiling temperature	$T_{ m b}$	2862	$^{\circ}\mathrm{C}$
10.	The initial temperature of the material	$T_0$	20	$^{\circ}\mathrm{C}$
11.	The coefficient of surface absorption of laser	A	0.75	
	radiation			

 Table 1. The thermophysical material characteristics

Table 2. The parameters of the laser treatment

No.	Parameter	Symbol	Value (min max)	Unit
1.	The action time of the laser treatment	t	0.01 0.1	s
2.	The radius of the laser spot	$r_0$	$0.0025 \dots 0.005$	m
3.	The laser radiation power	$P_0$	201000	W

### 4 The results and conclusions

The solution of the choice problem of an optimum set of technological process parameters is obtained for the given material characteristics (Table 1) and the laser treatment parameters (Table 2). The values  $dx_i$  (i = 1, 2, 3), allowing us to achieve the

desired accuracy in the calculation of the objective function, are taken as the following:  $dx_1 = 20, dx_2 = 0.00025, dx_3 = 0.01$ . At the defined limit of the molten pool depth  $H^* = 0.0005$  m the allowable set  $D_x$  consists of 197 vectors  $\mathbf{x} = (x_1, x_2, x_3)$  which satisfy the feasibility conditions of the process (17). The Table 3 shows a part of the resulting discrete set of suitable collections of variable and output parameters. It is obvious that the solution of the optimization problem for a given value of  $H^*$  is a vector  $\mathbf{x}^1 = (960; 0.00275; 0.08) (P_0 = 960 \text{ W}, r_0 = 0.00275 \text{ m} \text{ and } t = 0.08 \text{ s})$ , minimizing the objective function  $\Delta H(\mathbf{x})$ .

	Parameters						
	$x_1$	$x_2$	$x_3$	$y_1$	$y_2$	$y_3$	$ y_3 - H^* $
No.	$(P_0)$	$(r_0)$	(t)	$(T_{\rm c})$	(R)	(H)	$(\Delta H)$
1.	960	0.00275	0.08	2087.9	0.001534	0.000490	1.02692E-05
2.	980	0.003	0.1	2003.2	0.001557	0.000481	1.86615E-05
3.	680	0.0025	0.1	2001.6	0.001295	0.000480	1.99738E-05
•••		• • •	• • •	•••	• • •	•••	•••
195.	840	0.003	0.08	1540.4	0.000179	6.39343E-06	0.000494
196.	880	0,00325	0.1	1537.4	0.000129	3.15857 E-06	0.000497
197.	520	0.0025	0.1	1535.3	0.000036	4.11987 E-07	0.000500

Table 3. The suitable sets  $\{\mathbf{x}, \mathbf{y}(\mathbf{x})\}$  at  $H^*=0.001$  m

Additionally to the shown above solutions, the algorithm presented in this paper is used for calculating the maximum depth of a molten pool which satisfies all the necessary conditions and can be obtained on the surface of the selected material (Table 1) by the laser with given specifications (Table 2). For this purpose the value  $H^*$  in (20) is taken equal to the sample thickness h = 0.004 m. The obtained isothermal diagram of the molten pool with the maximum possible depth H = 0.000978 m is shown in Fig. 3 and corresponds to the solution  $\mathbf{x} = (960; 0.0025; 0.1)$  ( $P_0 = 960$  W,  $r_0 = 0.0025$  m and t = 0.1 s).

To obtain a deeper molten pool it is necessary to increase the maximum values of variable parameters of the laser. However, this method, firstly, is not always technically possible, and secondly, may cause an unwanted effect – boiling of the material at the sample surface.

Thus, the paper offers a version of solving the problem of choosing the optimal process parameters of laser thermal treatment to create the molten metal pool with a predetermined depth. The obtained results can be used as a pre-theoretical foundation for experimental testing of real laser system. The calculating algorithm takes into account the basic restrictions on the technical parameters of the laser and the necessary conditions for realization of the process.

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Fig. 3. The diagram of temperature distribution across the processed sample height. The dark area limited by dashed isotherm  $T_{\rm m}$  corresponds to the molten pool with a depth H and radius R on the sample surface

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