Cost-Effective Strip Covering with Identical Directed Sensors

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Abstract. We study the problem of constructing a cost-effective regular cover of a strip with identical sectors. Three effective coverage models are considered and their comparative analysis is performed which allows to obtain an upper bound for the minimum number of the sectors per unit length of the strip.

Keywords: sensor networks; video monitoring; regular covering

Introduction

Sensor networks are designed to monitor the areas and/or the objects. Each sensor in the network collects data within a certain area, which is called a *coverage domain* of the sensor. In the case of monitoring the plane region each point of the region should be covered, i.e. it (point) should belong to the coverage domain of at least one sensor. Sensing energy consumption is proportional to the coverage area, and multiple coverage involves unnecessary loss of the energy. Therefore, the problem of constructing an energy efficient sensor network is reduced to the problem of finding the least dense cover [1–3]. The most studied are the covers of the plane [4–7]. In particular, in [4] the least dense cover with identical disks is proposed, its density is $2\pi/\sqrt{27} \approx 1.2091$. In [5] a cover with disks of two types is proposed, its density tends to 1.0189 with unlimited growth of the number of disks, which radii tends to zero.

The number of the papers devoted to the covering of the bounded regions is substantially less. The first attempts to construct a strip covers were made in [8, 9] (the strip can be considered as a semi-bounded domain).

Evidently that the density of a cover is at least 1, and the deviation from 1 characterizes the effectiveness of the cover. More types of figures used in the cover, the lower density of the coverage can be. Of course, it is legitimate to compare the covers, which use the same set of figures. For simplicity of analysis the researchers, as a rule, consider the *regular* covers, in which the whole area is split into the equal polygons (*tiles*), and all the tiles are covered equally [2–5, 8, 10]. To evaluate the quality of the regular cover it is sufficient to consider the coverage of *one* tile. In [3] we introduced a classification of the regular covers, that has allowed to compare the covers in the same class.

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The need to monitor the strip occurs when observing the objects such as roads, pipelines, perimeters of the objects, etc. In the case when camera is located at a certain height above the surface, on the surface it covers an ellipse. The coverage problems with ellipses are considered, for example, in [10, 11]. If the camcorder lens is positioned horizontally, then the coverage area in this case is a sector. The angle and radius of the sector are determined by the characteristics of the device, and can take different feasible values. The sectors are used in the covers in the several papers [12–22]. In [21], we considered the problem of minimizing the number of identical sectors per unit of the covered area in the case when the vertices of the sectors which cover one tile are located at the same point. In [22], we considered the problem of constructing the least dense cover of a strip with identical sectors.

In this paper, as well as in [22], we consider the problem of constructing an optimal cover of the strip with equal sectors, but according to the criterion the minimum number of sectors per unit length of the strip. Since the sectors are equal, such objective function can be considered as a *cost function*.

The paper is organized as follows. In the next section a formulation of the problem is provided. The covering models are proposed in the section 3, and for each model the objective function is provided. Section 4 presents a comparative analysis of the proposed coverings, allowing to select a concrete cover depending on the parameters of the sector. Section 5 concludes the paper.

1 Problem Formulation

To denote the sector, we use the couple (R, α), where $R > 0$ is the radius and $\alpha \in (0, \pi/2]$ is the angle of the sector. Let strip be given and, without loss of generality, let its width be equal to 1. Assume that there is an unlimited collection of the identical sectors (R, α) each of which can be arbitrary *placed* and *oriented*.

Definition 1. *A collection C of the placed and oriented sectors is called a* ***cover*** *of the strip S if every point of S belongs to at least one sector in C.*

Definition 2. *A cover C of the strip S is called* ***regular*** *if S is split into the equal rectangles (tiles), and all tiles are covered identically.*

The height of a tile is 1, but its length depends on the cover. Let us denote the minimal length of a tile in the cover C as $L(C)$, and the number of sectors covering one tile denote as $Q(C)$.

Problem P. *It is required to construct a regular cover $C = C(R, \alpha)$ of the strip with equal sectors (R, α) in which the ratio $\sigma(C) = Q(C)/L(C)$ is minimal.*

2 The Coverage Models

As an approximate solutions of the problem P, we consider the same three models of covers as in [22]. But instead of the density function we consider the objective function $\sigma(C(R, \alpha))$.

2.1 Model M1

Suppose that $0.5 < R \sin\alpha \leq 1$. Let us define the *pair of sectors* inside the strip such that two of their sides (one side of each sector) lie on the opposite boundaries of the strip, while the other two are tangent to each other. The pair of sectors cover the rectangle (tile) $GBCF$ in Fig. 1 that is a part of the strip. Cover M1 is constructed with these pairs of sectors as shown in Fig. 1, so the number of sectors covering one tile is $Q(M1) = 2$. In this regular cover the tile is the rectangle $GBCF$ whose height coincides with the strip width and equals to 1. At that, the strip is subdivided into identical tiles, and all tiles are covered in the same fashion by the pairs of sectors.

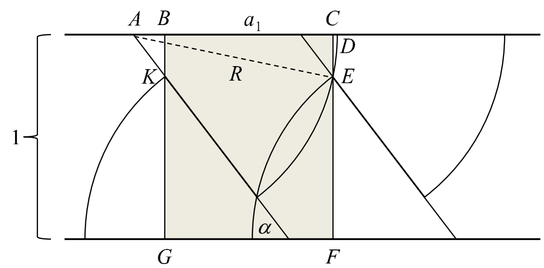

Fig. 1. Model M1 [22].

Lemma 1. The objective function for the cover M1 is

$$\sigma(M1(R,\alpha)) = \frac{2\sin\alpha}{\sin\alpha\sqrt{R^2-(1-R\sin\alpha)^2}-(1-R\sin\alpha)\cos\alpha}, \tag{1}$$

and it takes the minimum value equals $2\sin\alpha$ when $R\sin\alpha = 1$.

Proof. In [22], we proved the expression

$$L(M1) = a_1 = \sqrt{R^2-(1-R\sin\alpha)^2} - \frac{1-R\sin\alpha}{\tan\alpha}.$$

Then the objective function is

$$\sigma(M1(R,\alpha)) = \frac{Q(M1)}{L(M1)} = \frac{2\sin\alpha}{\sin\alpha\sqrt{R^2-(1-R\sin\alpha)^2}-(1-R\sin\alpha)\cos\alpha}.$$

Denote $x = 1 - R\sin\alpha \geq 0$. Then $R = (1-x)/\sin\alpha$ and

$$\sigma(M1(x,\alpha)) = \frac{2\sin\alpha}{\sqrt{(1-x)^2 - x^2\sin^2\alpha} - x\sin^2\alpha\cos\alpha}.$$

Since the functions $\sin\alpha$, $\sin^2\alpha$ and $\sin^2\alpha\cos\alpha$ are all positive, the function $\sigma(M1(x,\alpha))$ is increasing by x. Then (1) takes its minimum when $R\sin\alpha = 1$. By substituting this value in the formula (1), we get a minimum equals $2\sin\alpha$. The proof is over.

Remark 1. *If the sector (R, α) is given, then we cannot choose arbitrary R and α, but the value $2\sin\alpha$ is the lower bound for the functional (1), which decreases with decreasing α.*

If the sensor is a camcorders, then the greater the angle the smaller the radius and vice versa, but the area of the sector can be fixed, for example,

$$R^2\alpha/2 = S = const. \tag{2}$$

If so, then we cannot set always $R\sin\alpha = 1$ $(x = 0)$. In this case we can take the minimal possible positive $x = 1 - R\sin\alpha = 1 - \sqrt{2S/\alpha}\sin\alpha$. Then it is necessary to find maximum for $\sqrt{2S/\alpha}\sin\alpha$ which is at most 1. The function $f(\alpha) = \sin\alpha/\sqrt{\alpha}$ is concave and positive having maximum equals $\bar{f} \approx 0.8512$ when $\alpha = \bar{\alpha} \approx \pi/2.6953 \approx 66.78°$. Therefore, if $\sqrt{2S}\bar{f} \leq 1$, then we set $\alpha = \bar{\alpha}$, else set $\alpha = \hat{\alpha}$, where $\hat{\alpha}$ is such that $\sqrt{2S/\hat{\alpha}}\sin\hat{\alpha} = 1$.

Substituting the equality (2) into the formula (1), we obtain

$$\sigma(M1(S,\alpha)) = \frac{2\sin\alpha}{\sin\alpha\sqrt{\frac{2S}{\alpha} - \left(1 - \sqrt{\frac{2S}{\alpha}}\sin\alpha\right)^2} - \left(1 - \sqrt{\frac{2S}{\alpha}}\sin\alpha\right)\cos\alpha}.$$

For any feasible value of S the function $\sigma(M1(S,\alpha))$ is increasing. Then the minimal feasible angle α gives minimum to the $\sigma(M1(S,\alpha))$. For example, if $S = 2$, then

$$\min_{\{\alpha:\frac{\alpha}{8\sin^2\alpha} < S \leq \frac{\alpha}{2\sin^2\alpha}\}} \sigma(M2(S,\alpha)) \approx 0.2523$$

when $\alpha \approx \pi/49.9458 \approx 3.6°$.

2.2 Model M2

Suppose now that $R\sin\alpha \geq 1$, and let us consider the cover in Fig. 2 and denote it as M2. The cover model M2 has much in common with model M1. One side of each sector of the pair of sectors lies on the strip boundary, and the sectors of the same pair do not intersect, but in M2 a portion of each sector goes beyond the strip.

Lemma 2. The objective function for the cover M2 is

$$\sigma(M2(R,\alpha)) = \frac{2\sin\alpha}{(\sqrt{R^2-1}+R)\sin\alpha - \cos\alpha}, \tag{3}$$

and it takes its minimum when R and α are the greatest possible.

Proof. In [22], we proved the expression

$$L(M2) = a_2 = R + \sqrt{R^2-1} - \frac{1}{\tan\alpha}.$$

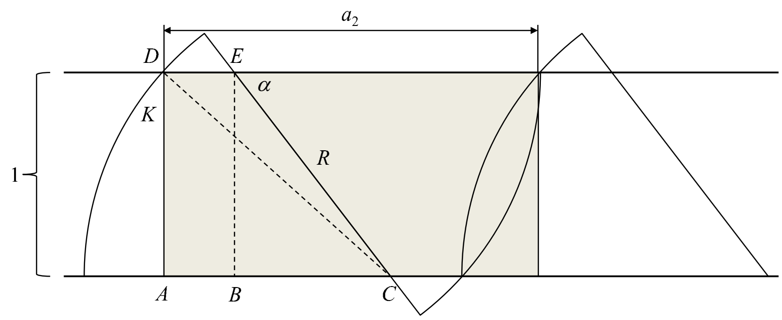

Fig. 2. Model M2 [22].

Then the objective function for M2 is

$$\sigma(M2(R,\alpha)) = \frac{Q(M2)}{L(M2)} = \frac{2\sin\alpha}{(\sqrt{R^2-1}+R)\sin\alpha - \cos\alpha}.$$

Function $\sigma(M2(R,\alpha))$ decreases with increasing α and R. Then (3) takes its minimum when $\alpha = \pi/2$ and R is the greatest possible. The proof is over.

Remark 2. *If the sector* (R,α) *is given, then we cannot take always both* R *and* α *the greatest possible.*

If equality (2) holds, then $R = \sqrt{2S/\alpha}$, and

$$\sigma(M2(S,\alpha)) = \frac{2\sin\alpha}{\sin\alpha\left(\sqrt{2S/\alpha-1}+\sqrt{2S/\alpha}\right) - \cos\alpha},$$

assuming that $2S/\alpha \geq 1$ and $\sin\alpha\left(\sqrt{2S/\alpha-1}+\sqrt{2S/\alpha}\right) - \cos\alpha > 0$. For each feasible value of S the function $\sigma(M2(S,\alpha))$ has one minimum. For example, if $S = 2$, then

$$\min_{\left\{\alpha : S \geq \frac{\alpha}{2\sin^2\alpha}\right\}} \sigma(M2(S,\alpha)) \approx 0.5055$$

when $\alpha \approx \pi/12.2958 \approx 14.64°$.

2.3 Model M3

Let none of the sector side lie on the boundary of the strip, and let $R\cos(\alpha/2) \geq 1$. We denote the cover in Fig. 3 as M3. Each of the sectors leans upon the boundary of the strip by at least one end of the arc, and the sector axis is at some angle to the strip boundary, whereas the tangent sectors in the pair is directed oppositely.

Let us introduce the parameter β, the angle between the sector axis and the line orthogonal to the strip boundaries (Fig. 3). Then $0 \leq \beta \leq \pi/2 - \alpha/2 - \arcsin(1/R)$.

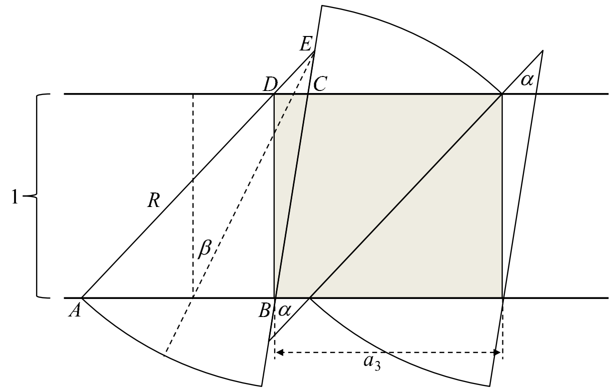

Fig. 3. Model M3 [22].

Lemma 3. The objective function for the cover M3 is

$$\sigma(M3(R,\alpha)) = \frac{2}{\sin\alpha}\min\left\{\frac{\cos^2(\alpha/2)}{2R\cos(\alpha/2)-1};\ \frac{\sin^2(\alpha+\arcsin\frac{1}{R})}{2R\sin(\alpha+\arcsin\frac{1}{R})-1}\right\}, \quad (4)$$

and its value is at least

$$\frac{1}{R\sqrt{2}-1}.$$

Proof. In [22], we proved that

$$|AB| = \frac{R\sin\alpha}{\cos(\beta-\alpha/2)}$$

and

$$|DC| = |AB|\left(1-\frac{1}{R\cos(\beta-\alpha/2)}\right) = \frac{R\sin\alpha}{\cos(\beta-\alpha/2)}\left(1-\frac{1}{R\cos(\beta-\alpha/2)}\right).$$

Then the length of the tile is

$$L(M3) = a_3 = |AB| + |DC| = \frac{\sin\alpha(2R\cos(\beta-\alpha/2)-1)}{\cos^2(\beta-\alpha/2)}$$

and

$$\sigma(M3(R,\alpha,\beta)) = \frac{2}{a_3} = \frac{2\cos^2(\beta-\alpha/2)}{\sin\alpha(2R\cos(\beta-\alpha/2)-1)}.$$

Function $\sigma(M3(R,\alpha,\beta))$ is concave with respect to β for each values of α and R, then it reaches minimum at the boundary of the feasible region, i.e.

$$\sigma(M3(R,\alpha)) = \min\{f_1(R,\alpha);\ f_2(R,\alpha)\},$$

where

$$f_1(R,\alpha) = \frac{2\cos^2(\alpha/2)}{\sin\alpha(2R\cos(\alpha/2)-1)}$$

and

$$f_2(R,\alpha) = \frac{2\sin^2(\alpha+\arcsin\frac{1}{R})}{\sin\alpha(2R\sin(\alpha+\arcsin\frac{1}{R})-1)}.$$

The functions $f_1(R,\alpha)$ and $f_2(R,\alpha)$ are decreasing, first $f_1(R,\alpha) \geq f_2(R,\alpha)$, then vice-versa. Then

$$\min_{\alpha\leq\pi/2} \sigma(M3(R,\alpha)) = f_1(R,\pi/2) = \frac{1}{R\sqrt{2}-1}.$$

This completes the proof of Lemma 3.

If (2) holds, then

$$\sigma(M3(S,\alpha,\beta)) = \frac{2\cos^2(\beta-\alpha/2)}{\sin\alpha\left(2\sqrt{2S/\alpha}\cos(\beta-\alpha/2)-1\right)},$$

$$0 \leq \beta \leq \pi/2-\alpha/2-\arcsin\sqrt{\frac{\alpha}{2S}},\ \alpha\leq 2S.$$

Function (4) is concave with respect to β, then it takes minimum value on the boundary of the feasible region, i.e.

$$\min_{0\leq\beta\leq\pi/2-\alpha/2-\arcsin\sqrt{\frac{\alpha}{2S}}} \sigma(M3(S,\alpha,\beta)) =$$

$$\min\left\{\sigma(M3(S,\alpha,0)), \sigma\left(M3\left(S,\alpha,\pi/2-\alpha/2-\arcsin\sqrt{\frac{\alpha}{2S}}\right)\right)\right\} =$$

$$\frac{2}{\sin\alpha}\min\left\{\frac{\cos^2(\alpha/2)}{\sqrt{8S/\alpha}\cos(\alpha/2)-1};\ \frac{\sin^2\left(\alpha+\arcsin\sqrt{\frac{\alpha}{2S}}\right)}{\sqrt{8S/\alpha}\sin\left(\alpha+\arcsin\sqrt{\frac{\alpha}{2S}}\right)-1}\right\}.$$

Let us fix any feasible S. When α is small, the decreasing function

$$f_3(S,\alpha) = \frac{2\cos^2(\alpha/2)}{\sin\alpha(\sqrt{8S/\alpha}\cos(\alpha/2)-1)}$$

is greater than the increasing function

$$f_4(S,\alpha) = \frac{2\sin^2(\alpha+\arcsin\sqrt{\frac{\alpha}{2S}})}{\sin\alpha(\sqrt{8S/\alpha}\sin(\alpha+\arcsin\sqrt{\frac{\alpha}{2S}})-1)}.$$

Then in order to minimize the objective function one should take minimal feasible angle, and function $f_4(S,\alpha)$ gives the minimum.

For example, if $S=2$, then

$$\min_{0\leq\beta\leq\pi/2-\alpha/2-\arcsin\sqrt{\frac{\alpha}{2S}}} \sigma(M3(S,\alpha,\beta)) = f_4(2,\alpha) \to 0.5$$

when $\alpha \to 0$.

3 Comparative Analysis of Models M1, M2, and M3

The objectives of this section is to find out which of the three considered cover models M1, M2, or M3 has minimum objective function $\sigma(M1(R,\alpha))$, $\sigma(M2(R,\alpha))$ or $\sigma(M3(R,\alpha))$ for arbitrary R and α.

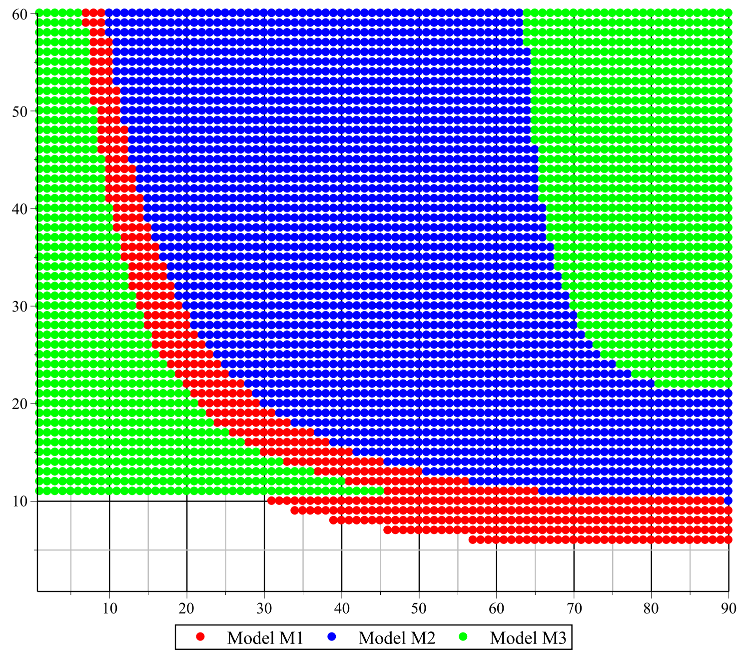

Fig. 4. The preference regions for the covers M1 (red), M2 (blue), and M3 (green).

Definition 3. *By the **best cover** we understand the cover among models M1, M2, and M3 with least objective function.*

Since the analytical calculation of the objective functions turned out to be a hard problem, the subsequent results were obtained numerically using Maple 17.02 package.

At each point (α, R), $\alpha \in [1°, 90°]$, $0 < R \leq R_{max}$ we calculate the values of the objective functions $\sigma(M1(R,\alpha))$, $\sigma(M2(R,\alpha))$ and $\sigma(M3(R,\alpha))$ and select the minimum among them. The model of the strip cover that corresponds to this value is the best among M1, M2, and M3.

The preference areas are indicated in Fig. 4 for each coverage model. Given different values of the regions of admissibility of the sector parameters, we obtain different zones, but the character of the pattern will not change. The image in Fig. 4 is obtained when $R = 0.1, 0.2, \ldots, R_{max} = 6.0$ and $\alpha = 1°, 2°, \ldots, 90°$.

It is obvious that for any fixed value of the angle α the values of the objective functions decrease with increasing radius R. Therefore, assuming the radius $R = R_{max}$ we reduce maximally the value of the objective function. Fig. 5 depicts a graph of the

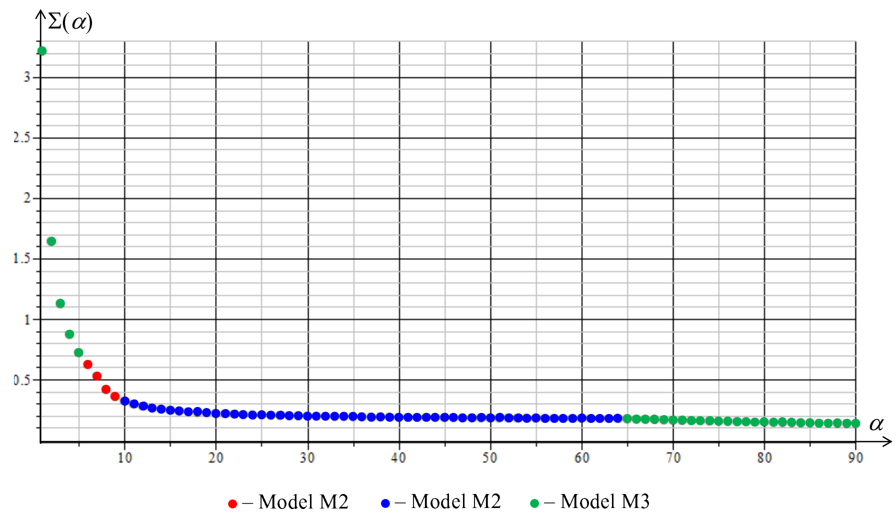

Fig. 5. Function $\Sigma(\alpha)$.

function

$$\Sigma(\alpha) = \min\left\{\sigma(M1(R_{max}, \alpha)), \sigma(M2(R_{max}, \alpha)), \sigma(M3(R_{max}, \alpha))\right\},$$

when $R_{max} = 6$. Its minimum equals $\Sigma(\alpha) = \sigma(M3(R_{max}, \alpha)) \approx 0.1678$ when $\alpha = 90°$. If, for example, $R_{max} = 2$, then $\Sigma(\alpha) = \sigma(M2(R_{max}, \alpha)) \approx 0.5359$ when $\alpha = 90°$.

Suppose equality (2) holds. Then we cannot set $R = R_{max}$, because $R = \sqrt{2S/\alpha}$ depends on S and α. Depending on the value of S, we get different results, but the character of graphics remains like in Fig. 6 and Fig. 7.

Fig. 7 depicts a graph of the function

$$\Sigma_S(\alpha) = \min\left\{\sigma(M1(S, \alpha)), \sigma(M2(S, \alpha)), \sigma(M3(S, \alpha))\right\},$$

when $S = 2$. Its minimum equals $\Sigma_S(\alpha) = \sigma(M1(S, \alpha)) \approx 0.5049$ when $\alpha = 14°$. If, for example, $S = 6$, then $\Sigma_S(\alpha) = \sigma(M2(S, \alpha)) \approx 0.1669$ when $\alpha = 5°$.

4 Conclusion

Since the identical directed sensors (when the coverage domain is the sector) have equal cost, in this paper we consider the problem of constructing the optimal cover of a strip with identical sectors object to the minimum number of sensors used to cover a strip.

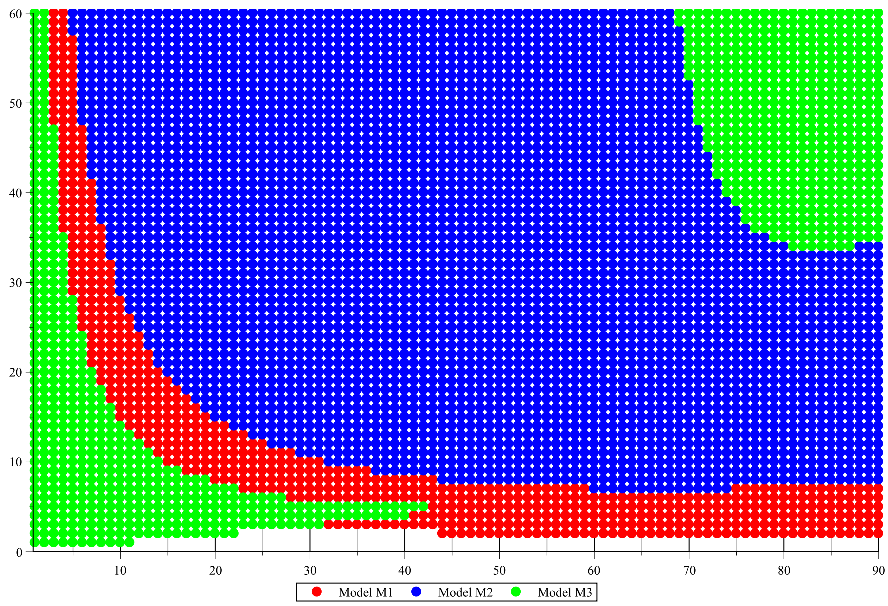

Fig. 6. The preference regions for the covers M1 (red), M2 (blue), and M3 (green) when equality (2) holds.

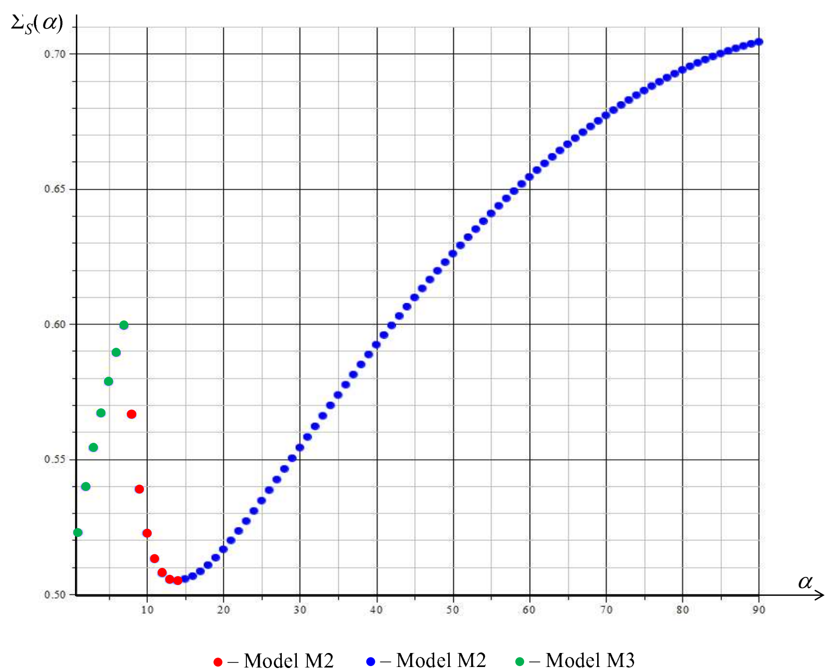

Fig. 7. Function $\Sigma_S(\alpha)$.

Three regular covers are studied, and their cost-effective comparative analysis is carried out.

Technique, which we have used, is similar to what was used in [22]. But the results we obtain are new. Using our results, one can choose the cost-effective coverage model for any sector (R, α), $0 < R \leq R_{max}$, $\alpha \in [1°, 90°]$.

Moreover, we have considered the case when the angle and radius of the sector are related the natural relation (2), where S is the area of the sector. If the area S is fixed, then by increasing the angle α of the sector radius R decreases, and vice versa. In particular, we can find the optimal angle and the best coverage model for any S.

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