

Robust Image Watermarking Technique Based on Genetic Algorithm Optimization and Even Odd Modulation

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Abstract. Robustness is one of the important issues in watermarking. In particular, robustness against JPEG compression is especially actual. Nowadays there are many different applications requiring image compression, such as Internet, satellite imaging, remote sensing, multimedia, preservation of art work and so on. In this paper, an effective watermarking technique based on the genetic algorithm optimization and even-odd modulation in frequency domain is presented. The genetic algorithm finds the optimum watermark strength and location for its embedding in a spectrum of the cover image. Experimental results have demonstrated high values of robustness and fidelity of the proposed scheme.

Keywords: image watermarking, genetic algorithm optimization, even-odd modulation, robustness, fidelity, frequency domain, JPEG compression, discrete cosine transform

1 Introduction

Digital watermarking refers to the process of embedding an authentication message called watermark into some content, for instance, the digital image, which uniquely identifies its ownership. Regardless of the particular applications any watermarking scheme must meet two main requirements. The visual quality of the watermarked image should be high and the watermark should withstand against any image processing in one way or another. Such processing may include lossy compression, filtering, added noise and many others. In watermarking terminology, any processing that may prevent detection of the watermark is called an attack. In general case the image is sent on the Internet as the compressed image to optimally use the bandwidth of the network. In such applications the image compression is essential to save storage space and transmission time.

The embedded watermark should possess two key qualities, robustness and fidelity. Robustness implies the strength of the watermark to sustain any attacks. Fidelity accounts for the quality of the watermarked image.

At present there are two popular techniques in watermarking: when an image is processed in spatial domain and in transform domain.

Spatial domain based watermarking focuses on modifying the pixels of images. It directly loads the values of a watermark into the image pixels. However spatial domain techniques have weak robustness against common image processing and attacks which may easily destroy the watermark [3].

Transform domain watermarking techniques are suitable first of all in such applications where robustness is of prime concern. The idea of transformed watermarking is reduced to embedding a watermark by altering the coefficients in frequency domain obtained with a help of discrete Fourier transform (DFT) [4], discrete cosine transform (DCT) [5] and discrete wavelet transform (DWT) [1,2].

Since the human eyes are more sensitive to low frequency distortions, the watermark should be embedded into the high frequency coefficients to attain better perceptual invisibility. However, the watermark hidden in the high frequency domain might be discarded after lossy compression and other attacks. So to make effective withstand against such attacks it is preferable to embed the watermark into the lower frequency range of the image.

To achieve its high robustness against different attacks and in the same time to provide the fidelity the even-odd modulation (EOM) technique is developed. It has been shown in this work that the EOM technique provides very high fidelity of the image even in a case when the watermark is embedded into the lower frequency range of the image. So in order to increase robustness of the scheme against such dangerous attacks as JPEG compression it is decided to change the values of the lower frequency coefficients while embedding the watermark to the image.

Another way to improve the performance of watermarking schemes is to make use of artificial intelligent techniques. The watermarking can be viewed as an optimization problem. Therefore, it can be solved by Genetic Algorithm (GA), which allows improving the fidelity of the watermarked image while keeping the robustness of the scheme against image manipulations. There are a lot of publications devoted to GA. For example, Huang and Wu [6] proposed a watermarking scheme based on the DCT and GA. They embed the watermark into the image by selectively modifying the middle-frequency coefficients of the image. The GA is applied to search for the locations to embed the watermark in the DCT coefficient blocks such that the quality of the watermarked image is optimized.

In papers [8-12] the surveys of digital watermarking with GA are presented.

In this paper, a new blind scheme of digital image watermarking is proposed. To achieve high robustness of it against different attacks and in the same time to provide high fidelity the even-odd modulation technique is developed, which embeds the watermark in low frequency DCT coefficients by changing their parity. To find the best position for the watermark embedding the GA is used which allows to improve the robustness and fidelity of the scheme. Comparison has been made between the proposed scheme and others presented in the literature to examine the image fidelity and to evaluate robustness.

2 Watermark embedding procedure

The proposed scheme uses the DCT, which creates a frequency spectrum of the entire original gray-level image I of size $M \times N$. This spectrum is divided into three main regions namely low frequencies sub-band (LF), middle frequencies sub-band (MF) and high frequencies sub-band (HF). The watermark W of size $G \times H$ is embedded within the LF sub-band of the area Ψ by modifying the coefficients of the LF sub-band as shown in Fig.1.

Assume that the coefficients $a(m,n)$ of the obtained spectrum are represented by matrix A of size $M \times N$, where $m=\overline{1, M}$, $n=\overline{1, N}$. Then in general case any coefficient may be written as an integer decimal number:

$$a(m,n) = a(m,n)_{p-1}10^{p-1} + a(m,n)_{p-2}10^{p-2} + \dots + a(m,n)_k10^k + \dots + a(m,n)_110^1 + a(m,n)_010^0,$$

where P is the total quantity of digits,

k is the number of digit in the coefficient $a(m,n)$, counting from right to left, $0 \leq k < P$.

Let the digital watermark W be a binary matrix of size $G \times H$ formed by a pseudo-random number generator (PRNG).

Assume that the embedding of the watermark W within the region Ψ is carried out in such a way that if $w(g,h)=1$ then it must comply with even coefficient $a(m,n)$ and, respectively, if $w(g,h)=0$ then it must comply with odd coefficient $a(m,n)$. Let also admit that the coordinates m' and n' of upper left angle of the region Ψ are given as it is shown in Fig. 1.

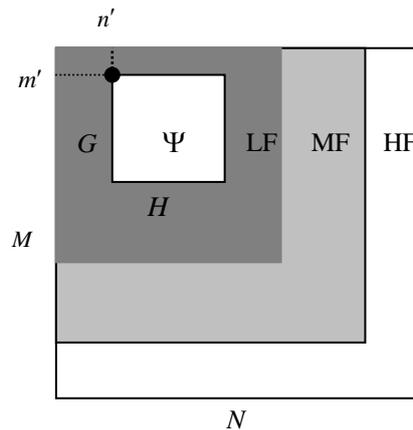


Fig. 1. Matrix A of size $M \times N$, (LF - low frequencies, MF - middle frequencies, HF - high frequencies)

Then the embedding process may be described as follows:

if $w(g,h)=1$ and $a(m'-1+g,n'-1+h)$ is odd then $a(m'-1+g,n'-1+h)$ turns into even number,

if $w(g,h)=1$ and $a(m'-1+g,n'-1+h)$ is even then $a(m'-1+g,n'-1+h)$ remains unchanged,

if $w(g,h)=0$ and $a(m'-1+g,n'-1+h)$ is odd then $a(m'-1+g,n'-1+h)$ remains unchanged,

if $w(g,h)=0$ and $a(m'-1+g,n'-1+h)$ is even then $a(m'-1+g,n'-1+h)$ turns into odd number,

where $g=\overline{1,G}$ and $h=\overline{1,H}$.

However the smallest noise can change the parity of the coefficient. Really, it is just enough to add "1" to the coefficient to change its parity. In this case the robustness of the watermarking scheme when it exposed to a smallest noise will be extremely low. To increase the robustness of the scheme it is proposed to change the parity of the more significant digits of the coefficients. For example, one watermark bit may be embedded into the second digit $a(m,n)_1$ of the coefficient. Then the noise with the same value will not change the parity of the digit $a(m,n)_1$. So it is easy to make a conclusion that the more significant digit of the coefficient is used to embed a bit of the watermark, the more difficult to change its parity and, consequently, the scheme will be more robustness to noise. However it should be noted that there may be a situation where the addition to the coefficient $a(m,n)$ of any number might change the parity of more significant digits. Let, for example, the coefficient $a(m,n)$ has a value 3999 (that is $p=4$) and the watermark bit has been embedded into the digit $a(m,n)_3$. Then a noise with any level will change the parity of this digit.

To avoid such situations it is proposed to modify the coefficient $a(m,n)$ in the following way. Suppose that the digit $a(m,n)_3$ was chosen for watermark bit embedding. To "protect" it against influence of noise it is proposed to assign to the previous digit $a(m,n)_2$ a value that is greater than zero but less than nine, for example, the value of four. After such modification the number 3999 will be turned to number 3499. Then if $a(m,n)=3499$ the digit $a(m,n)_3$ will not change its parity when the noise levels are in the range from -499 to 500. After the watermark has been embedded the inverse DCT is made using the modified matrix \hat{A} and as a result the watermarked image \hat{I} is obtained.

Note that the choice of the concrete digit for embedding of watermark's bit plays the same role as the choice of an amplifier coefficient value in watermarking [3,16]:

$$\hat{S} = S + \alpha W,$$

where S is a matrix representing a spectrum of the original image;

\hat{S} is a matrix representing a spectrum of the watermarked image;

α is an amplifier coefficient.

3 Watermark extraction procedure

The aim of this procedure is to form a matrix \hat{W} , which in ideal case should be identical to the matrix W . In connection with that the direct DCT is first performed over the image \hat{I} . Admit that the watermark bits have been embedded to the k -th digit of the coefficients within the region Ψ . Then by known coordinates m' and n' the upper left angle of the region Ψ is found and within that region the corresponding digits of the coefficients are analyzed to form elements $\hat{w}(g, h)$ of the matrix \hat{W} :

if $a(m'-1+g, n'-1+h)$ is even then $\hat{w}(g, h)=1$, otherwise $\hat{w}(g, h)=0$, where $g=1, \overline{G}$ and $h=1, \overline{H}$.

So as a result the matrix \hat{W} will be formed, which represents the extracted watermark.

4 Genetic algorithm

Any optimization problem is modeled in GA by defining the chromosomal representation, fitness function (FF) and the GA operators [7-11]. It starts with some randomly selected population made of individuals, each corresponding to a solution of the problem. An individual in the population is called chromosome. The FF is used to evaluate the quality of each solution so that chromosomes with high quality will survive and form a population for the next generation. The GA recombines a new generation to find the best optimal solution. The three GA operators, crossover, selection, and mutation, are applied to the chromosomes repeatedly to determine the In the watermarking the amplification coefficient α is used in a way to provide balance between fidelity and robustness of the watermarked scheme. Since these requirements are conflicting, then the value of α should be an optimization parameter. The value of α determines the strength of the watermark and can be varied within some range. In the proposed scheme the strength of the watermark is determined by choosing the concrete digits of the frequency coefficients.

The FF is calculated as a combination of the Peak Signal Noise Ratio (PSNR) and the Normalized Coefficient (NC):

$$FF = PSNR + 100NC.$$

The PSNR evaluates the fidelity of the watermarked image (quality index) and is computed as

$$PSNR = 10 \log_{10} \frac{B^2}{\frac{1}{M \times N} \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} [s_1(l, k) - s_2(l, k)]^2},$$

where B is a maximum pixel brightness, for example, 255;

$s_1(l, k)$ and $s_2(l, k)$ are pixels brightness of the images I и \hat{I} , respectively.

The NC evaluates the robustness of the watermarking scheme and is computed as

$$NC = \frac{\sum_{g=1}^G \sum_{h=1}^H w(g, h) \hat{w}(g, h)}{\sqrt{\sum_{g=1}^G \sum_{h=1}^H w^2(g, h)} \sqrt{\sum_{g=1}^G \sum_{h=1}^H \hat{w}^2(g, h)}}$$

where $w(g, h)$ and $\hat{w}(g, h)$ represent the original w and the extracted \hat{w} watermarks, respectively.

The NC determines the similarity between the original watermark w and the extracted watermark \hat{w} from the attacked watermarked image (robustness). The FF is increased proportionately with the increasing of PSNR and NC. The NC has been multiplied by factor 100 since its normal values are in the range 0 - 1, while the values of PSNR may lie in a wide range (mainly from 30 to 100). The FF is computed for all chromosomes in the population and the best chromosomes with the corresponding fitness value are saved. Genetic operators like mutation and crossover are performed on the selected parents to make new offspring which are included in the population to form the next generation.

The chromosomes with low fitness values are discarded through this process. The discarded chromosomes will be replaced by new offspring after executing the crossover and mutation genetic operators.

Crossover is responsible for producing better offspring by inheriting high-quality genes from their parents. In this work a crossover rate of 0.7 is used.

Mutation refers to the random alteration of the value in some positions of some chromosomes. The mutation is usually selected with a probability between 0 and 1, so that only a small portion of the genes in the chromosomes will be selected to be mutated. In this study the mutation rate equal to 0.05 is selected.

The whole process of robustness and fidelity optimization using GA can be described as follows.

1. A cover image and a watermark image are chosen.
2. The cover image is transformed by DCT into frequency domain and the area Ψ (see Fig. 1) of the obtained spectrum is divided into eight parts every of which has size 8×16 .
3. The watermark is also divided into eight parts every of which has size 8×16 (see Fig. 3).
4. Coming from a given value of PSNR the digit of low-frequency coefficients is set in which the bits of watermark must be embedded.
5. The first generation of GA chromosomes based on the watermark parameters is created. A different watermarked image is generated for each chromosome.
6. The fidelity of each watermarked image is evaluated by calculating the corresponding value of PSNR.
7. A chosen attack on the watermarked image is applied.
8. The watermark from the attacked image is extracted.
9. The robustness is evaluated by calculating NC between the original and extracted watermarks .

10. The FF for the obtained PSNR and NC values is computed.
11. The chromosomes with the best FF values are selected.
12. The new population is generated by performing the crossover and mutation functions on the selected chromosomes.
13. The steps from 5 to 13 are repeated until a predefined iteration number has been reached.

The entire process is repeated for several generations until the best solutions are obtained as showed in Fig. 2.

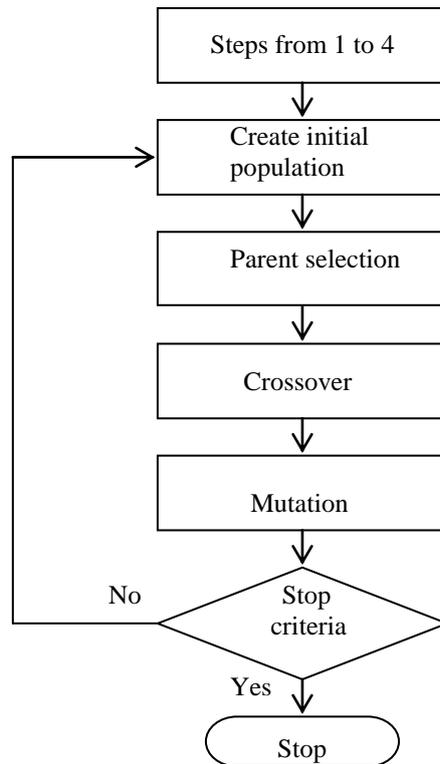


Fig. 2: The chart of a typical GA

5 Simulation and results

The proposed scheme is implemented in MATLAB software. The results obtained during different experiments are compared with the results presented in [1,2,5,14-18]. Therefore for correct comparison of results those 8-bit gray-level test images of size 512×512 and the watermarks that have been used in these works were chosen.

The binary watermarks of dimensions 16×32 and 32×32 pixels are formed by PRNG. To evaluate the quality of the watermarked image obtained by the proposed scheme the PSNR is used.

The strength of the watermark is varied by embedding its bits into different digits of frequency coefficients. Before embedding the watermark is divided into eight parts as shown in Fig. 3.

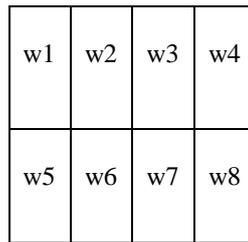


Fig. 3. Watermark divided into eight parts which must be embedded into the Ψ area

The embedding positions of the different parts of a watermark within the low frequency spectrum of the cover image are determined by chromosome. Fig. 4 shows the sample of chromosome.

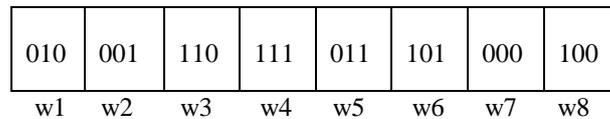


Fig. 4. A 24-bit chromosome

The performance of the extraction scheme was evaluated by measuring fidelity and robustness. The NC was used to measure the image quality of the watermark after extraction.

The Bit Error Rate (BER) is also employed to measure the image quality of the watermark after extraction. The BER has been calculated as the number of incorrectly decoded bits divided by the total number of embedded bits in the cover image:

$$BER = \frac{1}{G \times H} \sum_{g=1}^G \sum_{h=1}^H V(g, h),$$

$$\text{where } V(g, h) = \begin{cases} 1, & \text{if } w(g, h) \neq \hat{w}(g, h) \\ 0, & \text{if } w(g, h) = \hat{w}(g, h) \end{cases}.$$

In accordance with the proposed scheme the watermark bit should be embedded to the coefficient digit in order to change its parity. Naturally, more significant digits

of the number representing a coefficient are less susceptible to noise. But at the same time, the more significant digit is selected for embedding a watermark bit, the higher distortion of a number representing the coefficient. In order to determine these relationships three versions of the proposed scheme were tested in the following way.

First of all note that for convenience, it is desirable to introduce the values of coefficients in integer form while after the DCT the coefficients are represented in a real form as shown in Table 1. Therefore these real numbers are converted into integers by multiplying them by 10000 (see Table 2).

During the experimental research the watermark bits have been embedded into different digits of frequency coefficients, namely into the digit when $k=7$ (version A, $\tilde{a}aaaaaaa$), into the digit when $k=6$ (version B, $a\tilde{a}aaaaaa$) and into the digit when $k=3$ (version C, $aaaa\tilde{a}aaa$).

The robustness of the proposed scheme under JPEG compression was compared with other schemes. Four images were selected in the experiment. The quality factor (or, for short, QF) was used to determine the degree of loss in the compression process. In general, JPEG recommends QF of 75–95 for visually indistinguishable distortions, and QF of 50–75 for merely acceptable quality.

Table 1. Coefficients obtained after the DCT

-390.3282	-47.4748	584.4324	-135.2312
-334.2655	844.9531	535.6707	-1044.2649
1164.7977	-348.4653	39.8763	-84.5748
-1445.1743	1310.2793	-848.0618	-334.0621

Table 2. Coefficients in integer form

-3903282	-474748	5844324	-1352312
-13342655	8449531	5356707	-10442649
11647977	-3484653	398763	-845748
-14451743	13102793	-8480618	-3340621

The experimental results obtained with the proposed scheme and with other schemes collected from the original papers [1,2,5,14-16] are shown in Table3. The

values of NC and PSNR presented here are obtained using standard gray-level image Lena comprising 512×512 pixels, in conjunction with 16×32 watermark.

Table 3. Dependence of NC vs. factor QF

QF	[1]	[2]	[14]	[15]	[16]	Proposed scheme		
						A	B	C
NC								
1.5	-	-	-	-	-	0.59	0.59	0.69
3	-	-	-	-	-	0.67	0.61	0.61
5	-	-	-	-	-	0.85	0.62	0.61
10	-	0.42	0.35	0.39	0.61	0.98	0.65	0.62
15	-	0.56	0.52	0.72	0.84	0.99	0.76	0.62
20	-	0.70	0.69	0.91	0.96	0.99	0.97	0.63
25	-	0.80	0.81	0.96	0.99	1	0.99	0.64
30	0.24	0.86	0.85	0.98	0.99	1	0.99	0.64
40	0.37	0.94	0.92	0.99	1	1	1	0.66
50	0.49	0.97	0.95	1	1	1	1	0.69
70	0.75	1	0.96	1	1	1	1	0.99
90	1	1	0.99	1	1	1	1	1

Lossy compression is so common procedure in signal processing that watermark robustness to this attack is highly desirable. From Table 3 and Table 4 it is seen that the proposed scheme has the best robustness while keeping a high quality. All 512 bits have been accurately extracted when QF is up to 20. Even when QF decreases to 1.5, nearly 60% watermark bits can be correctly extracted, which demonstrates a significant advantage over other schemes.

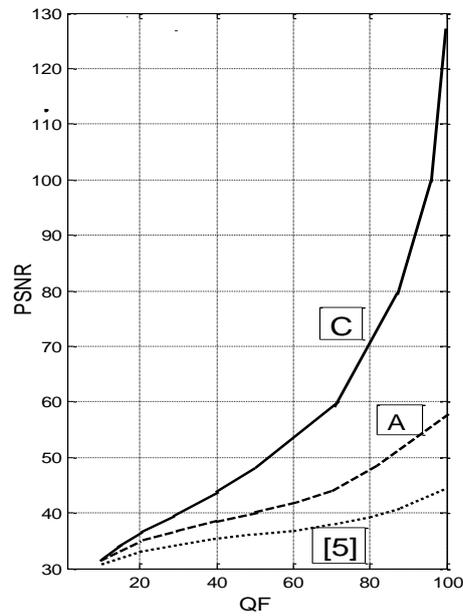
Table 3 also shows that the proposed scheme (version A) guarantees the watermark detection with $NC=1$ for all values of JPEG attacks ranging from 20%.

Table 4 presents the average values of PSNR obtained with the proposed scheme after processing 4 images of size 512×512 with watermark of size 16×32 and results in [2,5,15,16]. It is seen that the obtained values of PSNR are in between 58.7 dB and 143.1. So the results obtained with the proposed scheme demonstrate the highest quality of the images after the watermark has been embedded.

Another comparison was carried out applying the gray-level Lena image of size 512×512 with the watermark of 32×32 . The JPEG compression was implemented to the watermarked image and the experimental results are presented in Fig. 5, which reflects the dependence of PSNR vs. QF when using different versions of the proposed scheme and the scheme described in [5].

Table 4. The PSNR after watermark embedding

Papers	[2]	[15]	[5]	[16]	Proposed scheme		
					A	B	C
PSNR (dB)	42.2	41.5	43.3	42.2	58.7	99.5	143.1

**Fig. 5.** The plots of some simulation results reflecting dependence of PSNR vs. QF for different schemes.

The values of PSNR received for four different images of size 512×512 when the watermark of size 32×32 was embedded are shown in Table 5. It is easy to see that the obtained results are much better than in [5,17].

Table 5. The values of PSNR for four different images

Quality index	Tested images	[17]	[5]	Proposed scheme		
				A	B	C
PSNR	Man	30.85	43.32	56.7	89.9	121.9
	Couple	33.62	44.42	56.7	88.9	120.5
	Plane	33.69	44.42	57.8	87.6	118.7
	Lake	31.57	42.48	56.8	89.1	122.2

The results of the watermark robustness against various attacks are presented in Table 6. It is seen that the values of BER and NC obtained by schemes [5,18] are better than in a case of the developed scheme. Still this scheme resists these four kinds of attacks in a certain scope.

Table 6. Different attacks

Papers		Type of attack				
		Cropping		Gaussian noise 0.005	Median 5×5	Low pass filter 3×3
		1/8	½			
BER	[18]	0.05	0.28	0.23	0.27	0.09
	[5]	0.04	0.24	0.06	0.17	0
	Proposed scheme (A)	0.28	0.33	0.26	0.23	0.25
NC	[18]	0.954	0.811	0.855	0.821	0.938
	[5]	0.967	0.864	0.670	0.900	1
	Proposed scheme (A)	0.670	0.576	0.721	0.839	0.772

Conclusion

A blind watermarking scheme for gray-level still images using 2D DCT has been proposed. The scheme embeds the watermark in low frequency DCT coefficients by changing their parity. It was found that the use of the EOM technique and GA improved the robustness against JPEG compression and some other attacks such as additive noise, cropping and filtering. Comparison has been made between the proposed scheme and others presented in the literature to examine the image fidelity and evaluate robustness. The results show that the developed scheme can reach higher values of PSNR and NC, respectively.

Further researches can be concentrated on the development of the proposed scheme by using the characteristics of the human visual system. It would be also interesting to make the evaluation of GA convergence depending on a certain class of images.

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