

Markov Processes in Modeling Life Cycle of Economic Clusters

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Abstract. The object is economic clusters' life cycle (ECLC). The aim is to model its regulations algorithm. The hypothesis: ECLC transitions are stochastic processes independent of past. The method is Markov processes with discrete time. The algorithm is implemented in programming language Eclipse. The results are: 1) mathematical model of ECLC with life-cycle stages, stochastic transitions, transition values. For mature clusters their borders and actors are defined; 2) ECLC modeling algorithm with given stochastic transition matrix considering influence of different environments; 3) software "Modeling _Cluster_v1" for Androids to model online cluster development based on ECLC regulations. The novelties: for the first time ECLC model considers stochastic character of life-cycle stages simulating the number of actors at each stage, cluster's size, diameter and delimitation. Practical application: Program is applied to education, for interest groups involved in cluster development and projects to model of clusters' evolution trajectories under different environmental changes.

Keywords: stage in the life cycle, life cycle patterns, operational research, computer simulations of life cycle

1 Introduction

The development of operational research in different subject areas seems to be quite relevant nowadays. One area where such methods are of great demand is the Science of Economics. This is due to the fact that improved management of the large-scale highly-sophisticated economic systems is associated today with the latest mathematical tools since traditional economic methods used to solve the problems in the field demonstrate the tunnel vision and poor performance.

One of the most promising economic systems of this kind is a cluster structure. However, despite the wide use of cluster approach to different areas and in different

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countries, its comprehensive and detailed description and analysis in scientific literature are absent; the processes of clusters' formation and development remain poorly studied. As a consequence, it is difficult to consider and evaluate properly the development and management of cluster structures in the projects targeted for that. However, it is absolutely necessary in order for the clusters to ensure maximum generation of positive externalities. One of the most pressing issues in this regard is their life cycle.

Further development of the theory of economic clusters and their practical implementation within the cluster approach requires mathematical and computer modeling.

The bibliographic survey reveals that in recent years some attempts have been made to simulate the life cycle [1, 2, 3, 4, 5, 6], degradation and collapse of cluster structures [7], [4], [8], etc.

In these works nondeterministic model [3], Lotka-Volterra model [6], network theory [8] are used.

However, as the analysis sums up, operational research methods have not been implemented yet to the study and modeling the selected aspects of the economic clusters, even though the methods have high heuristic perspectives. The considerations mentioned above, define the objective of the study to search for and apply the methods of operational research, which allows accurate and adequate simulating economic clusters' life cycle.

2 Theory and Methodology

In the article the economic clusters are understood as non-institutionalized association of independent economic entities in a joint arrangement based on proximity (territorial, sectorial, cultural), complementarity (product, resource, process), interconnectivity (material, nonmaterial, informational) [9, p. 162]; then the life cycle of the economic clusters is considered as the shift or change leading to transition in its basic state and pattern [9, p. 238].

Like any functional institutions, economic clusters have a life cycle, which represents a certain set and a sequence of stages, each of which is characterized by the special institutional structure and the status of the cluster.

As a tool for simulating the life cycle of the cluster structure the Markov processes with discrete time are used. The choice of the method is determined by the fact that the life cycle of the economic clusters is a stochastic process independent of past. Hence, the discrete Markov processes with high confidence can describe the transition in the life-cycle stages.

It should be noted that at present the Markov processes are applied to the studies in various fields: in naval shipbuilding industry for planning and control the outfitting process in ships [10]; for industrial environments to predict the long-term evolution of composting processes on an industrial scale [11]; for optical diagnostic purposes to simulate angular distribution of photons through turbid slabs [12]; in genetic studies for modeling the complex dynamics of mobile genetic elements within genomes [13]; in chemistry to simulate interaction potential between particles in a colloidal system and to control their assembly into a close-packed crystalline objects [14]; in cattle farming for unbiased prediction of the future performance of the ani-

mals [15]; in building industry for cracking prediction on civil structures [16]; in medical science for optimization of anemia treatment in hemodialysis patients [17]; and even in music for enforcement of structure and repetition within music for bagana [18].

In economics according to the survey of the scientific literature the Markov processes are used to model optimal dynamic resource allocations between various companies to prevent defaults [19], to regulate social exchanges toward producing social equilibrium [20], to implement a social choice function [21], to maximize the expected discounted value of the total future profits [22], to solve optimal lending problem to certain types of borrowers [23], to examine the desirable sizes and policies of a strategic petroleum reserve for oil consumption countries [24], to show the optimality of a so-called save-up-to level policy and the existence of the optimal initial stock [25], to examine the financial optimality of disaster risk reduction measures [26] etc. But the Markov methods used to model clusters or in any other complex social-economic systems have not yet been found.

3 Model

In the mathematical model of the economic clusters there are three types of actors: 1) Producers [products]; 2) Providers [resources]; 3) Consumers [products].

Each actor in the cluster implements the goal which determines its behavior in the market and is characterized by a number of indicators.

1) Producers constitute the basis of the market and are characterized by the two groups of indicators: a) indicators measuring the productive potential; b) indicators defining the rules of behavior.

a) The group of the indicators measuring the productive potential of Producers includes:

- innovative activity – I_p discovering the potential of Producer for innovative activity is determined by the method given in [27]. The range of values of the indicator is 0-40. The higher the value, the more easily Producer ventures into innovative markets;
- internal capacity – M_p , measuring the financial, manufacturing and human potential of Producer is determined by the expert method based on the analysis of his financial conditions, productive and human capacities. Evaluation criteria: high, medium, low;
- potential for adaptation and management – C_p , reflecting the ability of Producer to adapt to changes in the environment is determined by the expert method on the basis of the analysis of the management and the capacity to adapt to the external environment. Evaluation criteria: high, medium, low.

b) The group of indicators that define the rules of behavior of Producers is specified by the tuple or set of rules:

$$B_p = \langle b_{pr}, b_{pp}, b_{bs}, b_{bm} \rangle \quad (1)$$

where b_{pr} – rules for acquiring all necessary resources for the manufacture of products and their storage, including purchase price, volume, quality, warehousing system; b_{pp} – rules of production, simulating the production cycle and the quality of products;

b_{bs} – rules of storage and realization of products that simulate warehousing of the finished products, cost and sales rules; b_{bm} – rules of behavior on the market, determining the Producer's goodwill, advertising policy, dealing with clients.

2) Providers supplying resources for Producers are the second group of actors in the economic cluster. Since the basic indicators for the analysis of interaction between Producers and Providers are the rules for acquiring resources included in the set of the Producers' behavior rules, Providers in the model defines the indicators of the resources supplied. This is due to the fact that the relations of Producers and Consumers play the main part in the processes of cluster formation and development and, consequently, in implementing or patterning the life cycle of the economic clusters. The activities of Providers under these conditions are aimed at rapid offtake of the resources supplied.

Providers are characterized by the following indicators:

- price of a resource unit – PR_r in model monetary units;
- amount of resource produced by one supplier – V_r defined in model volumetric units;
- quality of the resource – Q_r defined by the expert method based on the analysis of resources. Evaluation criteria: high, medium, low.

3) Consumers are characterized by the two groups of indicators: a) indicators determining capacity; b) indicators defining the rules for inflow of funds and purchase of products.

a) The group of the indicators determining the potential of Consumers includes:

- amount of money that Consumer is willing to spend on the purchase of products – F_s in model monetary units;
- volume of purchased products – V_s is defined as the ratio of available funds from Consumer to the value of the products.

b) The group of indicators that define the rules of Consumer's behavior is specified by the set of rules:

$$B_s = \langle b_{bb}, b_{fs} \rangle \quad (2)$$

where b_{bb} – rules for the purchase of products on the market, including the Producer's evaluation based on the rules of the group b_{bm} and the group b_{bs} , as well as the quality of the products manufactured (or sold); b_{fs} – funding rules.

The economic clusters in the model discussed appear to be a system arranging the three interrelated components: production, resources and consumption. Each component is represented by the special type of actors.

Production component serves as the core for the cluster system, determining the processes of cluster formation, the development of a uniform cluster structure or pattern and implementation of a certain life-cycle trajectory of a cluster [28]. The core of the economic cluster is a set of homogeneous enterprises producing goods with similar characteristics (Producers). The companies included in the cluster core form a compact set in (n)-dimensional space of characteristics. Once the core of the cluster is formed, it attracts Providers and Consumers. With it, between all the actors in the cluster the relations and ties of various levels of sustainability with the parameters and rules for cluster behavior are inevitably formed. Thus, the economic cluster can be selected when used the clustering methods of the (n)-dimensional space based on the analysis of distances between actors of the cluster [28].

Economic clusters are characterized by the following parameters:

- availability of a cluster core, and relationships (informational, resource, communicative) between Producers;
- the number of Producers in the cluster core structure;
- the diameter of the cluster, which is defined on the basis of calculating the location of the cluster actors in space characteristics. The higher the level of development of the cluster, the smaller is the diameter;
- the number of Providers in the cluster defined on the basis of the analysis of the conditions of enterprises entering the diameter of the cluster;
- the number of Consumers in the cluster, defined in the same way as the previous indicator;
- the synergistic effect defined as the increase in production volumes with the increase of professional communication within the cluster without changing the production capacity of Producers.

The economic clusters are self-organizing natural systems, in their life cycle they go through several stages: 1) diffuse group; 2) emerging cluster; 3) growing cluster; 4) mature cluster; 5) stagnant cluster.

Shift in the economic life-cycle stages of a cluster is shown in Fig. 1.

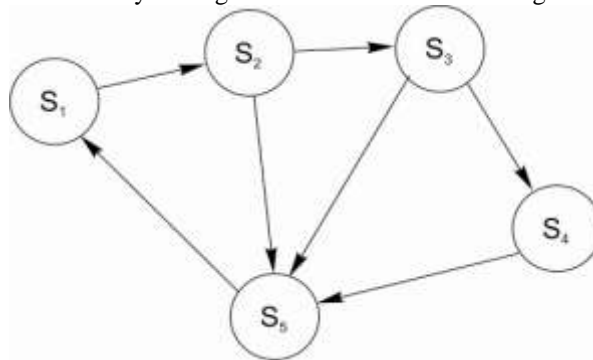


Fig. 1. Shift of economic life-cycle stages of cluster

Characteristics of life-cycle stages of economic cluster are provided in Table 1.

Table 1. Characteristics of life-cycle stages of economic cluster

Stage name	Diffuse group	Emerging cluster	Growing cluster	Mature cluster	Stagnant cluster
Designation	S_1	S_2	S_3	S_4	S_5
Existence of Cores and Links	Missing	Available			Missing
Number of Producers in the Cluster Core	Missing, representing diffuse group	Constant number, depending on industry and market	Constant number		
Cluster Diameter	-	Maximum size depending on industry and market	Decreasing	Constant	Increasing
Number of Providers in the Cluster	Missing, representing diffuse group	Increasing depending on industry and market	Increasing		Decreasing
Number of Consumers in the Cluster				Decreasing	
Synergistic Effect	Missing		Available	Missing	

Indicators characterizing Producers in economic cluster are listed in Table 2.

Table 2. Indicators characterizing Producers in economic cluster

Stage name	Diffuse group	Emerging cluster	Growing cluster	Mature cluster	Stagnant cluster
Designation	S_1	S_2	S_3	S_4	S_5
Innovative Activity I_p	Changing taking into account specific activity of each enterprise	Changing taking into account overall trend of cluster development, variance indicator reduced		Changing slightly or changing stochastically in each company, variance indicator increased	Changing taking into account the specific activity of each enterprise
Internal Potential M_p					Changing taking into account specific activity of each producer
Potential for Adaptation and Management C_p					
Behavior of Producers	Determined by market trends, weak relations between Producers	Clustering processes increasing, market influence decreasing, behavior depending on the interaction between Producers of cluster	Clustering processes greatly influencing, behavior depending on the interaction between Producers of cluster	Clustering influence decreasing, market impact increasing, Producers' behavior in the cluster becoming more stochastic	Determined by market trends, weak relations between Producers

Indicators characterizing Providers in economic cluster are given in Table 3.

Table 3. Indicators characterizing Providers in economic cluster

Stage name	Diffuse group	The emerging cluster	Growing cluster	Mature cluster	Stagnant cluster
Designation	S_1	S_2	S_3	S_4	S_5
Resource Unit Price	Depending on first cost and market		Resource price averaging and depending on resource acquisition rules asked by providers	Changing slightly, depending on market	
Resource Volume Produced by one Provider	Determined by the capacity of market		Determined by needs of Producers in cluster	Determined by needs of Producers in cluster and capacity of market	Determined by capacity of market
Quality of Resource	Quality of resources heterogeneous, depending on Provider	Quality of resources averaging and depending on rules of resource acquisition, asked by Producer in cluster		Not changing or slightly changing	

Consumers' indicators in economic cluster are listed in Table 4.

Table 4. Consumers' indicators in economic cluster

Stage name	Diffuse group	Emerging cluster	Growing cluster	Mature cluster	Key cluster
Designation	S_1	S_2	S_3	S_4	S_5
Funding	Depending on individual Consumer				
Volume of Purchased Products	Depending on amount of funding of Consumer				
Rules for Purchase of Products on Market	Formed to meet individual preferences and wishes of Consumer	Averaging for Consumers in cluster are developing taking into influence of clustering processes	Similar to Consumers in cluster	Changing to reflect influence of market and competition	Formed to meet individual preferences and wishes of Consumer
Funding Rules	Individual				

Changing stages in the life cycle of the economic clusters operates in external environment.

Classification of external environmental conditions is given in Table 5.

Table 5. Classification of external environmental conditions of economic clusters

Condition	External status economic cluster		
	Favorable	Neutral	Unfavorable
Characteristics of External Conditions	A favorable political, economic, social and technological conditions, optimal market competition	Little impact or no impact of external factors on the processes of clustering	High or low level of competition on market, negatively affecting the activities of actors in the processes of clustering
Shift Trends in Cluster Life Cycle Stages	Rapid clusters' development and state of growth for a long time	Gradual clusters' development, the possible existence of early stagnation	Great probability of clustering processes completion at early stages of cluster development and increase in period of stagnation in cluster

As noted above, the life cycle of the economic clusters is a probabilistic stochastic process independent of past. This enables modeling and description of the transition processes in stages or phases of their cluster life cycle by means of the discrete Markov process. Matrix of transitions between stages in the cluster life cycle is as follows:

$$\|p_{ij}\| = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{15} \\ p_{21} & p_{22} & \dots & p_{25} \\ \dots & \dots & \dots & \dots \\ p_{51} & p_{52} & \dots & p_{55} \end{pmatrix} \quad (3)$$

Calculations for each for each quantum of modeling time are performed according to the algorithms used for Markov process modeling [29]. The possible trajectories of life-cycle stage changes in economic clusters are calculated on the basis of the appropriate calculating matrixes on the indices which have the maximum probability meaning if the particular cluster is studied at a certain life-cycle stage. There can be several such trajectories. After that, the indices of the number of the actors of the cluster and the change in its characteristics within the modeled time can be calculated.

The transition matrix between life-cycle stages in the cluster is formed with the help of expert analysis and the results of the research in the functioning of the real economic clusters. For example, innovative economic clusters are developing more intensively and transforming into the stage of growth much more rapidly but as well they can be transforming into the stage of stagnation with the same high speed. It is due to the speed of the processes developing in them, the innovative ones being included. Agro-industrial clusters, for instance, are characterized by slower development as traditionalism and slow innovations are quite typical for them. Most of the Agro-industrial clusters in the world as well in Russia are mature clusters and they seldom transfer to the stagnation stage. Such regulations may be taken into account when building transition matrices between the life-cycle stages of economic clusters of various types.

Calculation of the probability of economic cluster development at each stage of its life cycle can be performed for each quantum of modeling time considering specific environment.

4 An Algorithm of Modeling the Life Cycle of the Economic Clusters

Modeling algorithm of the economic clusters' life cycle includes the following stages:

Stage 1. Setting a baseline of initial parameters for modeling life cycle of economic cluster, including the number of actors in each group (Producers, Providers, Consumers).

Stage 2. Specifying the number of quanta or tacts of modeling time.

Stage 3. Setting parameters for the life cycle of economic cluster.

Stage 4. Specifying external environment for economic cluster.

Stage 5. Developing a matrix of transitions between stages of the life cycle of economic cluster.

For each condition of an external environment for the cluster the default range of the probabilities of transition from one life-cycle stage to another is specified. Developing matrices (2) is carried out on the basis of the random number generator considering the specified range of values.

Stage 6. Visualization of the results of the Stage 5. If there is no need to adjust the matrix (2), you should go to the Stage 8, if it is necessary to correct the matrix (2), you should go to the Stage 7.

Stage 7. User adjustment and correction of the matrix values (2).

Stage 8. Calculation of the matrix values (2) for each quantum of modeling time.

Stage 9. Visualization of calculation in the Stage 8.

Stage 10. The choice of the most probable cluster life-cycle stages for each quantum of modeling time. The selection is done by ranking the values p_{ij} considering the specified range of values. If all values p_{ij} miss the specified range, you should go to the Stage 5, if not – you should go to Stage 11.

Stage 11. Visualization of calculation of the Stage 10.

Stage 12. Delimitation of the cluster and cluster actors for each quantum of modeling time using algorithm FOREL [30].

Stage 13. Calculation of the life cycle of economic cluster for each quantum of modeling time.

Stage 14. Calculation of indicators for all actors in the economic cluster, and for each quantum of modeling time. In the transition from the first to the subsequent stages of the economic cluster life cycle averaging the cluster indicators for the cluster actors takes place by reducing the confidence interval and, vice versa, when you return from the subsequent stages of the life cycle of the cluster to the previous ones, there is an increase in the dispersion of values or in the variance indicator for each of the indicators of cluster actors. Also in the transition from one stage of the economic cluster life cycle to another the number of cluster actors can vary.

Stage 15. Cognitive visualization of results of the calculations using the Dashboard technology [31].

5 The Simulation Program of the Life Cycle of the Economic Clusters

Based on the algorithm of simulating the economic clusters' life-cycle the Program "Modeling_Cluster_v1" for Android-based devices is developed. The choice of the Android operating system is due to the independence of this device from hardware and to its performance characteristics as well.

In Fig. 2 a screenshot of the main window of the Program is given.



Fig. 2. Main Window of Program "Modeling_Cluster_v1"

6 Conclusions

The following results were obtained in the article presented, namely:

1. mathematical model of life cycle of the economic clusters including the life-cycle stages and transitions between them; probability values of transitions. For stages with a mature economic cluster, the problem of determining the boundary or delimitation parameters of the cluster is solved, its actors are defined, changing in its number at different stages of the life cycle is considered;
2. modeling algorithm of the life cycle of the economic clusters with given transition probability matrices between the stages of the cluster life cycle, for the impacts of different types of environment can be specified. The algorithm allows calculating the most probable trajectory of the life cycle of the economic clusters, as well as determining their sizes and actors at each stage;
3. software product "Modeling_Cluster_v1" implemented for Android-based devices which allows online management of the cluster development on the basis of the identified patterns of transition between the stages of the economic clusters life cycle to implement modeling its evolution taking into account environment impact.

Simulation results are presented in the form of Dashboard, visualizing the changes of stages in the life cycle of economic clusters and the behavior of its actors at each stage.

Thus, such methods as the Markov processes with discrete time provide modeling the life cycle of the economic clusters. This made possible to identify the regulations in the development of the specific economic clusters and determine both their sizes and characteristics of their actors at each stage of the life cycle.

The novelty of the results obtained is that for the first time the model of the economic cluster life cycle takes into account the probability character of transitions between the stages, allows model the number of actors participating in each of the stages, the size, the diameter and the delimitation of the cluster.

The software product may be used for the educational purposes, as well as for executive bodies and interest groups involved in the development and implementation of cluster programs and projects, also for modeling possible trajectories of economic cluster evolution in changing economic environments.

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