

An Investigation of a Bilevel Energy Market Model

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Abstract. The two-level model of interaction between power producers (GC) and System Operator (SO) is considered. The upper level corresponds to GC which try to increase their profit by distortion information about the characteristics operating costs of their power plants. The lower level corresponds to SO which solves the generation scheduling problem on the basis of technical parameters of the power plants provided by the producers. SO is aimed at minimizing total production costs in Electric Power System (EPS). Two formulations of this model are considered. In the first case the bounds on generation and capacities of lines are considered insignificant. For this case we study properties of the objective function of the upper level and investigate the existence of the Nash equilibrium. In the second case we will take into account additional bounds on the power generation and power flows. In this case the standard replacement of the lower level problem by the Karush-Kuhn-Tucker (KKT) optimality conditions is proposed. In this paper, we present a simple numerical example to validate the proposed approach and demonstrate its main features.

Keywords: Electricity market, bilevel optimization, Nash equilibrium, noncooperative game, mixed-integer quadratic programming.

1 Introduction

A lot of investigations are devoted to research of the electricity market. Today an actual task is study of interaction between System Operator and power producers [1]. Producers in the electricity market are Generation Companies (GC).

Power producers provide their costs to the System Operator in the market conditions. SO solves a generation scheduling problem, minimizing total costs of electricity generation and calculating the nodal prices (dual variables) on the basis of technical parameters of the power plants provided by the producers. To increase their profit power producers deliberately distort real values of some technical parameters of the power plants thereby implicitly influencing the prices. This interaction between SO and GC can be modeled using a two-level mathematical programming problems [2, 3]. Earlier the bilevel programming technique was used for modeling the heat energy market [4].

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The two-level model of interaction between System Operator and producers is considered in this paper. The upper level corresponds to the profit maximization of Generation Companies with true costs functions. The lower level of the problem corresponds to SO efforts to schedule generation and calculate local marginal prices (LMP) on the basis of total production costs minimization. On the upper level there are several GC, each seeks to maximize its own profits, therefore on the upper level there is the equilibrium search problem.

In the considered model the limits on generation and capacities of lines are considered insignificant. For this case we research properties of the objective function of the upper level and prove the existence of the Nash equilibrium.

This paper presents a numerical example demonstrating the applicability of the proposed approach to the solution of the considered problem.

2 Problem formulation

The Electric Power System (EPS), consisting of n nodes and m lines is considered [1, 5]. We assume that the first k ($k < n$) nodes are power producers and the others ($n - k$) are consumers. Each power producer operates one power plant.

The network topology is given by the incidence $n \times m$ matrix A with entries

$$a_{ij} = \begin{cases} -1, & \text{if line } j \text{ enters node } i, \\ 0, & \text{if line } j \text{ is not connected to node } i, \\ 1, & \text{if line } j \text{ leaves node } i. \end{cases}$$

The power flows must satisfy Kirchhoff's first law

$$Ax = b,$$

where x_j is power flow on the line j , $b_i \geq 0$ is power generation for $i = \overline{1, k}$ and power demand (in this case $b_i \leq 0$) for $i = \overline{k+1, n}$. Note that matrix A has the following property: if $\lambda^T A = 0$, then $\lambda_1 = \lambda_2 = \dots = \lambda_n$.

In this paper the demand is inelastic, i.e. $b_i = \text{const}$, $i = \overline{k+1, n}$.

Generation of power producers is determined by quadratic costs function

$$c_i(b_i) = \alpha_i b_i^2 + \beta_i b_i + \gamma_i, \quad i = \overline{1, k}, \quad (1)$$

where $\alpha_i > 0$, $\beta_i > 0$, $\gamma_i > 0$.

System Operator solves the convex quadratic programming problem

$$\sum_{i=1}^k c_i(b_i) = \sum_{i=1}^k \alpha_i b_i^2 + \beta_i b_i + \gamma_i \rightarrow \min_{(x, b_1, \dots, b_k)}, \quad (2)$$

$$Ax = b. \quad (3)$$

Constraints (3) are always consistent, the objective function in (2) is strictly convex. This provides unique solution in variables b_1, \dots, b_k .

In (2) the total production costs in the EPS is considered. Solving this problem, SO determines the Lagrange multipliers associated with the constraints (3), which act as nodal equilibrium prices, and determines the amounts of generation for each power producer. Further on the basis of these data power producers define their own profit.

Such pricing system is aimed only to meet the demand with minimum costs and completely ignores the interests of producers. GC are aimed at maximizing their profits in market conditions. The minimum total production costs in the EPS is not the objective for them.

In this situation power producers deliberately distort real values of some technical parameters of the power plants. This means that parameter values $\alpha_i, \beta_i, \gamma_i$ in (2), reported to the SO, can be deliberately changed by power producers and may differ from the true values $\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i$ for the purpose of maximizing the profit with prices and amounts of generation, determined after the solution of the problem (2)-(3).

In this paper it is assumed that $\alpha_i = \hat{\alpha}_i, \gamma = \hat{\gamma}_i$, i.e. we study the impact of parameters β_i on GC profit. The problem of existence and finding the Nash equilibrium between GC is investigated. Therefore variable parameters β_i are used in problem formulation of SO, and for calculating producer's costs we use true values $\hat{\beta}_i$.

3 Search for the Nash equilibrium

To solve the problem (2)-(3), in which $\alpha_i = \hat{\alpha}_i, \gamma = \hat{\gamma}_i$ and vector $\beta = (\beta_1, \dots, \beta_k)^T$ is external parameter, we use the method of Lagrange multipliers. The Lagrange function has the form

$$L(b, x, \lambda) = \sum_{i=1}^k \hat{\alpha}_i b_i^2 + \beta_i b_i + \hat{\gamma}_i + \lambda^T (Ax - b).$$

Calculate the partial derivatives and equal them to zero:

$$\frac{\partial L}{\partial b_i} = 2\hat{\alpha}_i b_i + \beta_i - \lambda_i = 0, \quad i = \overline{1, k} \tag{4}$$

$$L_x = A^T \lambda = 0, \quad L_x = \left(\frac{\partial L}{\partial x_1}, \dots, \frac{\partial L}{\partial x_m} \right)^T. \tag{5}$$

Using the property of matrix incidence A , from (5) we obtain, that all prices λ_i are equal to the same price, which we denote by p ,

$$\lambda_1 = \lambda_2 = \dots = \lambda_n = p. \tag{6}$$

Thus, from (4) and (6) we have

$$2\hat{\alpha}_i b_i + \beta_i = p. \tag{7}$$

Note that the value of $2\hat{\alpha}_i b_i + \beta_i$ is the value of marginal costs of power producer i , because $c'_i(b_i) = 2\hat{\alpha}_i b_i + \beta_i$. Therefore in optimal solution all power producers facing the same level of marginal costs, which is equal to the established price.

Thus in the model all calculations are performed assuming a uniform price determined by GC's current costs.

Now we describe the profit for power producer i as a function of the zero marginal costs $\beta_i = c'_i(0)$. We assume, that in practice the impact of parameter β_i on profit is more significant.

As noted above the main aim of power producer is to maximize profit

$$p^*(\beta)b_i^*(\beta) - \hat{\alpha}_i b_i^*(\beta)^2 - \hat{\beta}_i b_i^*(\beta) - \hat{\gamma}_i \rightarrow \max_{\beta_i},$$

where $p^*(\beta)$ and $b_i^*(\beta)$ are the corresponding to a given parameter vector β optimal values of the dual and primal variables, obtained after solving the problem (2)-(3).

Profit functions of power producers

$$\pi_i(\beta) = p^*(\beta)b_i^*(\beta) - \left(\hat{\alpha}_i b_i^*(\beta)^2 + \hat{\beta}_i b_i^*(\beta) + \hat{\gamma}_i \right), \quad i = \overline{1, k}, \tag{8}$$

we rewrite in a form suitable for the further analysis.

From (7) we obtain

$$(2\hat{\alpha}_i b_i + \beta_i)b_i - \hat{\alpha}_i b_i^2 - \hat{\beta}_i b_i - \hat{\gamma}_i = \hat{\alpha}_i b_i^2 + \beta_i b_i - \hat{\beta}_i b_i - \hat{\gamma}_i,$$

where $\hat{\beta}_i$ is real zero costs of power producer i .

Since A is matrix incidence, then, summing all the equations of the system (3), we obtain the equation

$$0 = \sum_{i=1}^n b_i$$

or

$$b_1 + b_2 + \dots + b_k = |b_{k+1}| + \dots + |b_n| = b_T, \tag{9}$$

where b_T is total power demand.

From (7) and (9) we derive

$$p^*(\beta) = 2 \left[\frac{b_T + \frac{1}{2} \sum_{i=1}^k \frac{\beta_i}{\hat{\alpha}_i}}{\sum_{i=1}^k \frac{1}{\hat{\alpha}_i}} \right] \tag{10}$$

and

$$b_q^*(\beta) = \frac{2b_T + \sum_{i=1}^k \frac{\beta_i - \beta_q}{\hat{\alpha}_i}}{\sum_{i=1}^k \frac{1}{\hat{\alpha}_i}} \cdot \frac{1}{2\hat{\alpha}_q}. \tag{11}$$

Note that increase parameter β_i involves increase $p^*(\beta)$.

Analytical expression of profit functions as functions of parameters β turns out substitution of expressions (10) and (11) in (8) and has the form

$$\pi_q(\beta) = K_q \beta_q^2 + \xi_q(\beta), \quad q = \overline{1, k}, \tag{12}$$

where $\xi_q(\beta) = \xi_q(\beta_1, \dots, \beta_k)$ is affine function relatively to β_q , K_q is constant, defined below. As will be seen from further analysis, there is no need to explicitly write the view functions $\xi_q(\beta)$, moreover, this functions have a bulky appearance.

Let us make some remarks. We obtained analytical problem solution for SO (the lower level problem) and substituted this solution in objective functions of the upper level problems thus excluded the lower level problem from further consideration. As a result, we received non-cooperative k -person game with profit functions (12). It should be noted that the profit of power producer q depends not only on its own zero marginal costs, but also zero marginal costs other power producers. Besides, the increase of its own costs (parameter β_q) will reduce its own production, and thus, will lead to production increase of other power producers.

It is known that some games allow the reduction to optimization problem [6]. Such games are called potential and finding Nash equilibrium for them is significantly facilitated. The majority of practically interesting games are not potential and this is an obstacle to develop computational procedures, guaranteed determine the equilibrium point.

Lemma 1 ([7]). *The non-cooperative k -person game with profit functions (12) is not potential.*

Proof. The proof of this lemma can be found in [7].

We prove existence of the Nash equilibrium in this game.

Lemma 2 ([7]). *The values $K_q < 0 \forall q = \overline{1, k}$.*

Proof. By simple mathematical reformulation we get

$$\begin{aligned}
 K_q &= -\frac{1}{4} \cdot \frac{\hat{\alpha}_q^2 \left(\sum_{i=1}^k \frac{1}{\hat{\alpha}_i} \right)^2 - 1}{\hat{\alpha}_q^3 \left(\sum_{i=1}^k \frac{1}{\hat{\alpha}_i} \right)^2} = \\
 &= -\frac{1}{4} \left[\frac{\left(\sum_{i=1}^k \frac{\hat{\alpha}_q}{\hat{\alpha}_i} \right)^2 - 1}{\hat{\alpha}_q^3 \left(\sum_{i=1}^k \frac{1}{\hat{\alpha}_i} \right)^2} \right] = -\frac{1}{4} \left[\frac{\left(1 + \sum_{i \neq q}^k \frac{\hat{\alpha}_q}{\hat{\alpha}_i} \right)^2 - 1}{\hat{\alpha}_q^3 \left(\sum_{i=1}^k \frac{1}{\hat{\alpha}_i} \right)^2} \right] < 0.
 \end{aligned}$$

Therefore each function $\pi_q(\beta)$ is concave in β_q . We can assume that the parameters β_i vary within some reasonable limits: $\beta_i \in [0, \bar{\beta}]$, $\bar{\beta} = const > 0$. Then the set of strategy in our game is convex and compact. These conditions [8] guarantee the existence of the Nash equilibrium.

To find equilibrium we find partial derivatives

$$\frac{\partial \pi_q}{\partial \beta_q} = \frac{\partial p}{\partial \beta_q} \cdot b_q + \beta_q \cdot \frac{\partial b_q}{\partial \beta_q} - \hat{\beta}_q \cdot \frac{\partial b_q}{\partial \beta_q}. \tag{13}$$

Denote $\sigma = \sum_{i=1}^k \frac{1}{\hat{\alpha}_i}$, then

$$\frac{\partial p}{\partial \beta_q} = \frac{1}{\hat{\alpha}_q \sigma}, \quad \frac{\partial b_q}{\partial \beta_q} = \frac{1}{2\hat{\alpha}_q} \left[\frac{1}{\hat{\alpha}_q \sigma} - 1 \right].$$

Substitute the found partial derivatives in (13) and equate the received expression to zero.

$$\frac{\partial \pi_q}{\partial \beta_q} = \frac{1}{2\hat{\alpha}_q^2 \sigma} \cdot p - \beta_q \cdot \frac{1}{2\hat{\alpha}_q} - \hat{\beta}_q \cdot \frac{1}{2\hat{\alpha}_q} \left[\frac{1}{\hat{\alpha}_q \sigma} - 1 \right] = 0.$$

After standard reformulation we obtain

$$2b_T + \sum_{i=1}^k \frac{\beta_i}{\hat{\alpha}_i} - \beta_q \hat{\alpha}_q \sigma^2 = \hat{\beta}_q \hat{\alpha}_q \sigma^2 \cdot \left[\frac{1}{\hat{\alpha}_q \sigma} - 1 \right].$$

As a result, finding the equilibrium is reduced to the solution system of linear equations of the following form:

$$\sum_{i=1}^k \frac{\beta_i}{\hat{\alpha}_i} - \beta_q \hat{\alpha}_q \sigma^2 = \hat{\beta}_q \hat{\alpha}_q \sigma^2 \cdot \left[\frac{1}{\hat{\alpha}_q \sigma} - 1 \right] - 2b_T. \tag{14}$$

In vector-matrix form equation (14) can be written as follows:

$$D\beta = r, \tag{15}$$

where

$$D = \begin{pmatrix} \frac{1}{\hat{\alpha}_1} - \hat{\alpha}_1 \sigma^2 & \frac{1}{\hat{\alpha}_2} & \dots & \frac{1}{\hat{\alpha}_k} \\ \frac{1}{\hat{\alpha}_1} & \frac{1}{\hat{\alpha}_2} - \hat{\alpha}_2 \sigma^2 & \dots & \frac{1}{\hat{\alpha}_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\hat{\alpha}_1} & \frac{1}{\hat{\alpha}_2} & \dots & \frac{1}{\hat{\alpha}_k} - \hat{\alpha}_k \sigma^2 \end{pmatrix}, \quad r = \hat{\beta}_q \hat{\alpha}_q \sigma^2 \cdot \left[\frac{1}{\hat{\alpha}_q \sigma} - 1 \right] - 2b_T.$$

Lemma 3 ([7]). *If $k > 1$ then matrix D^{-1} exists and all its elements are negative.*

Proof. The complete proof can be found in [7].

Lemma 4 ([7]). *The elements of the vector r are negative.*

Theorem 1. *If $k > 1$ then the solution β^* of the system (15) exists and is unique, the components of the vector β^* are positive and*

$$\beta^* = -\frac{1}{\sigma^2} \left[\begin{pmatrix} \frac{1}{\hat{\alpha}_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\hat{\alpha}_k} \end{pmatrix} + \frac{1}{\sigma^2 - \sum_{i=1}^k \frac{1}{\hat{\alpha}_i^2}} \begin{pmatrix} \frac{1}{\hat{\alpha}_1^3} & \dots & \frac{1}{\hat{\alpha}_1 \hat{\alpha}_k^2} \\ \vdots & \ddots & \vdots \\ \frac{1}{\hat{\alpha}_k \hat{\alpha}_1^2} & \dots & \frac{1}{\hat{\alpha}_k^3} \end{pmatrix} \right] \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_k \end{pmatrix}.$$

Proof. The proof follows from the two last lemmas.

The existence of the Nash equilibrium under assumption of bounded parameters β_i variation was proved above. As follows from this theorem, there is no need to set upper bounds on β_i . It is possible even to admit also the negative values of these parameters, still the equilibrium is achieved at the interior point of non-negative orthant, which corresponds to the practical interpretation of β_i .

4 The two-level problem formulation with constraints on the power generation and power flows

In this section we will take into account additional bounds on x and b . Consider first the problem with one power producer. Without loss of generality, assume that the power producer is in node 1. The problem of the System Operator

$$\hat{\alpha}_1 b_1^2 + \beta_1 b_1 + \hat{\gamma}_1 \rightarrow \min_{(x,b)},$$

$$A_1 x = d_1 - b_1,$$

$$A_i x = d_i, i = \overline{2, n},$$

where A is the incidence matrix, A_i are the rows of matrix A , d is demand. It's obvious that

$$b_1^* = b_T = \sum_{i=1}^n d_i. \tag{16}$$

Thus the optimization is carried out only in the variable x . From the optimality conditions obtain

$$\lambda_1^* = 2\hat{\alpha}_1 b_T + \beta_1 = \lambda_2^* = \lambda_3^* = \dots = \lambda_n^*. \tag{17}$$

The power producer's profit subject to (16)-(17)

$$\lambda_1^* b_1^* - (\hat{\alpha}_1 (b_1^*)^2 + \hat{\beta}_1 b_1^* + \hat{\gamma}_1) = (2\hat{\alpha}_1 b_T + \beta_1) b_T - (\hat{\alpha}_1 b_T^2 + \hat{\beta}_1 b_T + \hat{\gamma}_1)$$

is linear, increasing relative to β_1 function. Therefore under the constraints

$$0 \leq \beta_1 \leq \bar{\beta}_1$$

profit reaches its maximum at the right end of the segment: $\beta_1^* = \bar{\beta}_1$.

Let us pass now to the general statement. The problem of the System Operator

$$\sum_{i=1}^n (\hat{\alpha}_i b_i^2 + \beta_i b_i + \hat{\gamma}_i) \rightarrow \min_{(x,b)}, \tag{18}$$

$$Ax = d - b, \tag{19}$$

$$\underline{x} \leq x \leq \bar{x}, \tag{20}$$

$$0 \leq b \leq \bar{b}. \tag{21}$$

The Lagrange function

$$\begin{aligned}
 L(b, x, \lambda, \mu^1, \mu^2, \eta^1, \eta^2) = & \sum_{i=1}^n (\hat{\alpha}_i b_i^2 + \beta_i b_i + \hat{\gamma}_i) + \\
 & + \sum_{i=1}^n \lambda_i (d_i - b_i - \sum_{j=1}^m a_{ij} x_j) - \sum_{j=1}^m \mu_j^1 (x_j - \bar{x}_j) + \sum_{j=1}^m \mu_j^2 (x_j - \bar{x}_j) - \\
 & - \sum_{i=1}^n \eta_i^1 b_i + \sum_{i=1}^n \eta_i^2 (b_i - \bar{b}_i).
 \end{aligned}$$

The optimality conditions represent the stationarity conditions, conditions of complementary slackness, constraints on the sign of the dual variables corresponding to inequality constraints:

$$\frac{\partial L}{\partial b_k} = 2\hat{\alpha}_k b_k + \beta_k - \lambda_k - \eta_k^1 + \eta_k^2 = 0, k = \overline{1, n}, \quad (22)$$

$$\frac{\partial L}{\partial x_s} = - \sum_{i=1}^n \lambda_i a_{is} - \mu_s^1 + \mu_s^2 = 0, s = \overline{1, m}, \quad (23)$$

$$\mu_j^1 (x_j - \bar{x}_j) = 0, \mu_j^2 (x_j - \bar{x}_j) = 0, j = \overline{1, m}, \quad (24)$$

$$\eta_i^1 b_i = 0, \eta_i^2 (b_i - \bar{b}_i) = 0, i = \overline{1, n}, \quad (25)$$

$$\mu_j^1 \geq 0, \mu_j^2 \geq 0, j = \overline{1, m}, \quad (26)$$

$$\eta_i^1 \geq 0, \eta_i^2 \geq 0, i = \overline{1, n}, \quad (27)$$

supplemented by the conditions (19)-(21).

If in some nodes there are no power producers then we set $\alpha_i = \beta_i = 0$ and $\bar{b}_i = 0$ for these nodes.

Now interaction between GC and SO is formulated as the bilevel problem with GC at the first level and SO at the second. We replace the lower level by the system of necessary optimality conditions (19)-(27) obtaining the one-level nonconvex problem.

It is known [9, 10] that the nonlinear constraints of the form

$$wv = 0, w \geq 0, v \geq 0$$

can be rewritten in the form of linear constraints by introducing an additional Boolean variable ϑ as follows:

$$w \leq M\vartheta, v \leq M(1 - \vartheta), w \geq 0, v \geq 0.$$

Let M be a sufficiently large constant. In the example below $M = 10^6$. Introduce the following Boolean variables: $\zeta_j^1, \zeta_j^2, \zeta_i^3, \zeta_i^4$. Then the constraints (24)-(27) are equivalent

$$\begin{aligned} \mu_j^1 &\leq M\zeta_j^1, (x_j - \underline{x}_j) \leq M(1 - \zeta_j^1), \\ \mu_j^2 &\leq M\zeta_j^2, (x_j - \bar{x}_j) \leq M(1 - \zeta_j^2), \\ \eta_i^1 &\leq M\zeta_i^3, (b_i - \underline{b}_i) \leq M(1 - \zeta_i^3), \\ \eta_i^2 &\leq M\zeta_i^4, (b_i - \bar{b}_i) \leq M(1 - \zeta_i^4), \\ \mu_j^1 &\geq 0, \mu_j^2 \geq 0, \eta_i^1 \geq 0, \eta_i^2 \geq 0, \\ \zeta_j^1 &\in \{0, 1\}, \zeta_j^2 \in \{0, 1\}, \zeta_i^3 \in \{0, 1\}, \zeta_i^4 \in \{0, 1\}. \end{aligned}$$

After the introduction of additional variables and replace the constraints an equivalent mixed-integer quadratic programming problem is obtained. To solve the problems of this class the solver CPLEX can be used.

The solution of this problem are variables of upper level and lower level problems. Optimization in a one-level problem is carried out on all variables: $(b_i, x_j, \lambda_i, \beta_i, \mu_j^1, \mu_j^2, \eta_i^1, \eta_i^2, \zeta_j^1, \zeta_j^2, \zeta_i^3, \zeta_i^4)$.

5 Numerical example

Consider the Electric Power System, consisting of two producers and two consumers. Initial data are given in Table 1.

Table 1. Initial data for the EPS nodes and lines

Node	Costs functions	Demand	Generation limits	Line	Flow limits, MW
1	$c_1(b_1) = 0.1b_1^2 + 80b_1$	85	30-500	1	0-260
2	$c_2(b_2) = 0.09b_2^2 + 100b_2$	-	50-300	2	0-320
3	-	412.4	-	3	-

The results of the calculation with $\beta_1 = 80, \beta_2 = 100$ are shown in Table 2. Power producers gain profits $\pi_1 = 8\,308.35$ and $\pi_2 = 3\,937.23$.

Table 2. The results of the calculation with exact characteristics of costs

Node	b_i	Prices	Line	x_j
1	288.24	137.65	1	203.24
2	209.16	137.65	2	-
3	-	-	3	412.40

Table 3 shows the solution obtained under the assumption that producers distort information about the characteristics operating costs of their power plants. Producers gain profits $\pi_1 = 20\,147.40$ and $\pi_2 = 12\,306.44$.

Table 3. The results of the calculation with distorted characteristics of costs

Node	b_i	β_i	Prices	Line	x_j
1	291.40	120.00	178.28	1	206.40
2	206.00	141.20	178.28	2	-
3	-	-	-	3	412.40

The results of numerical example demonstrate the effect achieved by power producers due to distortion information about the characteristics operating costs of their power plants. Producers increase the level of market prices by changing β_i . The prices increase provide producers the opportunity to obtain additional profit.

6 Conclusion

The electricity market model of interaction between power producers (GC) and System Operator (SO) was considered. This model is formulated as two-level mathematical programming problem. Thus on the upper level GC maximizes its own profits due to the distortion of technical parameters of the power plants, transmitted SO. Two formulations of this model are considered. In the first one the bounds on generation and capacities of lines are considered insignificant. For this case we prove the existence of the Nash equilibrium. The equilibrium is unique and is achieved at the interior point of non-negative orthant. In the second one the model takes into account additional bounds on the power generation and power flows. For this case the equivalent one-level mixed-integer quadratic programming problem is proposed.

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