Dual Model of Power Market with Generation and Line Capacity Expansion

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Abstract. The investigated model is a mathematical model in which operating power, installed power, power flows between nodes in electric power system (EPS) are optimized for the last year of the calculation period. The model is static, multinodal and it is represented as a large dimension linear programming problem. The aim of this study is analysis of the relationships between dual variables as the nodal and line prices.

Keywords: Electric Power System (EPS), primal and dual linear programming problems, nodal prices, dual variables.

1 Introduction

In our paper we study a model which describes the development of Electric Power System (EPS) in a long term period. From mathematical point of view the model is represented by a linear programming problem. The model is static. The statistic formulation have been used in Melentiev Energy Systems Institute SB RAS for 20 years. It showed itself to good advantage.
There is Kirchhoff's first law in the model, but there is no Kirchhoff's second law. It is due to the fact that the model is advanced. For this reason reactive energy is not considered.
The problem has been successfully used for a quite long period of time [2]. In 2011 a market version of the model was created and tested on real data from central Russia [5]. A necessity of investigation of power interstate connections caused a preliminary research performed in [1], where only the primal model was used. The aim of this paper consists in deriving the dual formulation and providing some interpretation of dual variables.

2 Description of the Model

To describe the model we need first to describe sets, parameters, variables, objective function and constraints.
Sets:

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J — set of nodes;
I — set of types of stations;
S — set of seasons (winter, spring, summer, autumn);
T — set of hours in days (0-23);
$R \subset S$ — set of seasons, in which an annual maximum load is achieved;
$Q_s \subset T$ — set of time intervals, in which an annual maximum load in season $s \in S$ is achieved.

Node parameters:
y^0_{ji} and $\overline{y}_{ji}$ — initial and maximum installed powers of station of type $i \in I$ in node $j \in J$;
x^0_{jist} and $\overline{x}_{jist}$ — minimum and maximum allowable operating powers of station of type $i \in I$ in node $j \in J$ in hour $t \in T$ of season $s \in S$;
v_{ji} — unit variable costs of station of type $i \in I$ in node $j \in J$;
k_{ji} and b_{ji} — relative capital investments and unit fixed costs of station of type $i \in I$ in node $j \in J$;
D_{jst} — consumer load in node $j \in J$ in hour $t \in T$ of season $s \in S$.

Line parameters:
$\kappa_{jj'}$ and $\beta_{jj'}$ — relative capital investments and unit fixed costs for new and developing lines between nodes j and j';
$\overline{a}_{jj'}$ — maximum power line capacity between nodes $j \in J$ and $j' \in J$;
$\pi_{jj'}$ — unit line losses between nodes $j \in J$ and $j' \in J$.

Other parameters:
ϵ — power reserve ratio (i.e. this is a power reserve coefficient for stations repair, emergency situations and etc.);
τ^{w}_s (τ^{h}_s) — equivalent number of working days (holidays) in season $s \in S$ (i.e. such a number of days which when multiplied by season maximum load results in electrical energy consumption equal to an accepted season value);
f — capital recovery factor (CRF), $f = \frac{\rho(1+\rho)^M}{(1+\rho)^M - 1}$, where ρ — discount rate, M — number of years, in which the capital is returned.
Capital recovery factor is calculated on the condition of capital recovery in equal parts G during M years with discount rate ρ.

Node variables:
y_{ji} — installed power of station of type $i \in I$ in node $j \in J$;
$x^{\mathrm{w}}_{jist}(x^{\mathrm{h}}_{jist})$ — operating power of station of type $i \in I$ in node $j \in J$ in hour $t \in T$ of season $s \in S$ on working days (holidays).

Line variables:
$a_{jj'}$ — power line capacity between nodes $j \in J$ and $j' \in J$;
$u^{\mathrm{w}}_{jj'st}(u^{\mathrm{w}}_{j'jst})$ — operating power flow in the set from node $j \in J$ to node $j' \in J$ (from node $j' \in J$ to node $j \in J$) in hour $t \in T$ of season $s \in S$ on working days;
$u^{\mathrm{h}}_{jj'st}(u^{\mathrm{h}}_{j'jst})$ — operating power flow in the set from node $j \in J$ to node $j' \in J$ (from node $j' \in J$ to node $j \in J$) in hour $t \in T$ of season $s \in S$ on holidays;
$\widetilde{u}_{jj'st}(\widetilde{u}_{j'jst})$ — "emergency" power flow in the set from node $j \in J$ to node $j' \in J$ (from node $j' \in J$ to node $j \in J$) in hour $t \in Q_s$ of season $s \in R$.

The objective function:
The objective function is a function of the total costs of the whole EPS and it has the form of:

$$\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^{\mathrm{w}}v_{ji}x_{jist}^{\mathrm{w}}+\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^{\mathrm{h}}v_{ji}x_{jist}^{\mathrm{h}}+ \tag{1}$$

$$+f\sum_{j\in J}\sum_{i\in I}k_{ji}(y_{ji}-y_{ji}^0)+\sum_{j\in J}\sum_{i\in I}b_{ji}y_{ji}+ \tag{2}$$

$$+f\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\kappa_{jj'}(a_{jj'}-a_{jj'}^0)+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\beta_{jj'}a_{jj'}\rightarrow\min\ . \tag{3}$$

The components of the objective function are total (annual) costs for operating power (1), costs for introduction of new capacities and fixed costs for its maintenance (2), costs for line capacity development and corresponding fixed costs (3).
Constraints:
On electric power station development:

$$y_{ji}^0\le y_{ji}\le\overline{y}_{ji},\quad j\in J,\quad i\in I\ . \tag{4}$$

On power line development:

$$a_{jj'}^0\le a_{jj'}\le\overline{a}_{jj'},\quad j\in J,\quad j'\in J\quad j'>J\ . \tag{5}$$

On operating power on working days and holidays respectively:

$$x_{jist}^0\le x_{jist}^{\mathrm{w}}\le\overline{x}_{jist},\quad j\in J,\quad i\in I,\quad s\in S,\quad t\in T; \tag{6}$$

$$x_{jist}^0\le x_{jist}^{\mathrm{h}}\le\overline{x}_{jist},\quad j\in J,\quad i\in I,\quad s\in S,\quad t\in T\ . \tag{7}$$

On flows in power lines on working days and holidays respectively:

$$0\le u_{jj'st}^{\mathrm{w}}\le a_{jj'},\quad j\in J,\quad j'\in J,\quad j'\neq J,\quad s\in S,\quad t\in T; \tag{8}$$

$$0\le u_{jj'st}^{\mathrm{h}}\le a_{jj'},\quad j\in J,\quad j'\in J,\quad j'\neq J,\quad s\in S,\quad t\in T\ . \tag{9}$$

On "emergency" flows in power lines:

$$0\le\widetilde{u}_{jj'st}\le a_{jj'},\quad j\in J,\quad j'\in J,\quad j'\neq J,\quad s\in R,\quad t\in Q_s\ . \tag{10}$$

On operating power balance of power stations on working days and holidays (it is based on Kirchhoff's first law):

$$\sum_{i\in I}x_{jist}^{\mathrm{w}}-\sum_{\substack{j'\in J\\ j'\neq j}}u_{jj'st}^{\mathrm{w}}+\sum_{\substack{j'\in J\\ j'\neq j}}u_{j'jst}^{\mathrm{w}}(1-\pi_{j'j})=D_{jst},\quad j\in J,\quad s\in S,\quad t\in T; \tag{11}$$

$$\sum_{i\in I}x_{jist}^{\mathrm{h}}-\sum_{\substack{j'\in J\\ j'\neq j}}u_{jj'st}^{\mathrm{h}}+\sum_{\substack{j'\in J\\ j'\neq j}}u_{j'jst}^{\mathrm{h}}(1-\pi_{j'j})=D_{jst},\quad j\in J,\quad s\in S,\quad t\in T\ . \tag{12}$$

On installed power in peak load hours:

$$\sum_{i\in I} y_{ji} - \sum_{\substack{j'\in J\\ j'\neq j}} \widetilde{u}_{jj'st} + \sum_{\substack{j'\in J\\ j'\neq j}} \widetilde{u}_{j'jst}(1-\pi_{j'j}) \geq D_{jst} + \epsilon \cdot D_{jst}, \qquad j\in J, \quad s\in R, \quad t\in Q_s \ . \tag{13}$$

The mathematical model is a linear programming problem. One needs to find a minimum of the objective function (1)–(3) with the constraints (4)–(13).

3 The Dual Problem

We derive the dual problem using technique from [7] based on the Lagrange function for problem (1)–(13). First, a part of Lagrange function, corresponding to considered group of constraints, is written. Then, the resulting constraint will be converted by rearrangement of the summands.
Dual function consists of several parts. The dual part corresponding to constraints on electric power station development, left part of constraints (4), dual variables $\underline{\xi}_{ji}$:

$$\sum_{j\in J}\sum_{i\in I} \underline{\xi}_{ji}(y^0_{ji} - y_{ji}) = \sum_{j\in J}\sum_{i\in I}\left[-\underline{\xi}_{ji}\right] y_{ji} + \sum_{j\in J}\sum_{i\in I} \underline{\xi}_{ji} y^0_{ji} \ . \tag{14}$$

The dual part corresponding to constraints on electric power station development, right part of constraints (4), dual variables $\overline{\xi}_{ji}$:

$$\sum_{j\in J}\sum_{i\in I} \overline{\xi}_{ji}(y_{ji} - \overline{y}_{ji}) = \sum_{j\in J}\sum_{i\in I} \overline{\xi}_{ji} y_{ji} - \sum_{j\in J}\sum_{i\in I} \overline{\xi}_{ji}\overline{y}_{ji} \ . \tag{15}$$

The dual part corresponding to constraints on power line development, left part of constraints (5), dual variables $\underline{\nu}_{jj'}$:

$$\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \underline{\nu}_{jj'}(a^0_{jj'} - a_{jj'}) = \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \left[-\underline{\nu}_{jj'}\right] a_{jj'} + \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \underline{\nu}_{jj'} a^0_{jj'} \ . \tag{16}$$

The dual part corresponding to constraints on power line development, right part of constraints (5), dual variables $\overline{\nu}_{jj'}$:

$$\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \overline{\nu}_{jj'}(a_{jj'} - \overline{a}_{jj'}) = \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \overline{\nu}_{jj'} a_{jj'} - \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}} \overline{\nu}_{jj'}\overline{a}_{jj'} \ . \tag{17}$$

The dual part corresponding to minimum power loading on working days, left part of constraints (6), dual variable $\underline{\mu}^{\mathrm{w}}_{jist}$:

$$\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T} \underline{\mu}^{\mathrm{w}}_{jist}(x^0_{jist} - x^{\mathrm{w}}_{jist}) = \tag{18}$$

$$= \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \left[-\underline{\mu}^{\mathrm{w}}_{jist} \right] x^{\mathrm{w}}_{jist} + \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \underline{\mu}^{\mathrm{w}}_{jist} x^{0}_{jist} \ .$$

The dual part corresponding to maximum power loading on working days, right part of constraints (6), dual variables $\overline{\mu}^{\mathrm{w}}_{jist}$:

$$\sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{w}}_{jist} (x^{\mathrm{w}}_{jist} - \overline{x}_{jist}) = \tag{19}$$

$$= \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{w}}_{jist} x^{\mathrm{w}}_{jist} - \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{w}}_{jist} \overline{x}_{jist} \ .$$

The dual part corresponding to minimum power loading on holidays, left part of constraints (7), dual variable $\underline{\mu}^{\mathrm{h}}_{jist}$:

$$\sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \underline{\mu}^{\mathrm{h}}_{jist} (x^{0}_{jist} - x^{\mathrm{h}}_{jist}) = \tag{20}$$

$$= \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \left[-\underline{\mu}^{\mathrm{h}}_{jist} \right] x^{\mathrm{h}}_{jist} + \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \underline{\mu}^{\mathrm{h}}_{jist} x^{0}_{jist} \ .$$

The dual part corresponding to maximum power loading on holidays, right part of constraints (7), dual variables $\overline{\mu}^{\mathrm{h}}_{jist}$:

$$\sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{h}}_{jist} (x^{\mathrm{h}}_{jist} - \overline{x}_{jist}) = \tag{21}$$

$$= \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{h}}_{jist} x^{\mathrm{h}}_{jist} - \sum_{j \in J} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \overline{\mu}^{\mathrm{h}}_{jist} \overline{x}^{\mathrm{h}}_{jist} \ .$$

The dual part corresponding to constraints on flows in power lines on working days, left part of constraints (8), dual variables $\underline{\gamma}^{\mathrm{w}}_{jj'st}$:

$$\sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{s \in S} \sum_{t \in T} \left[-\underline{\gamma}^{\mathrm{w}}_{jj'st} \right] u^{\mathrm{w}}_{jj'st} \ . \tag{22}$$

The dual part corresponding to constraints on flows in power lines on working days, right part of constraints (8), dual variables $\overline{\gamma}^{\mathrm{w}}_{jj'st}$:

$$\sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{s \in S} \sum_{t \in T} \overline{\gamma}^{\mathrm{w}}_{jj'st} (u^{\mathrm{w}}_{jj'st} - a_{jj'}) = \tag{23}$$

$$= \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{s \in S} \sum_{t \in T} \overline{\gamma}^{\mathrm{w}}_{jj'st} u^{\mathrm{w}}_{jj'st} - \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{s \in S} \sum_{t \in T} \overline{\gamma}^{\mathrm{w}}_{jj'st} a_{jj'} \ .$$

The dual part corresponding to constraints on flows in power lines on holidays, left part of constraints (9), dual variables $\underline{\gamma}^{\mathrm{h}}_{jj'st}$:

$$\sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{s \in S} \sum_{t \in T} \left[-\underline{\gamma}^{\mathrm{h}}_{jj'st} \right] u^{\mathrm{h}}_{jj'st} \ . \tag{24}$$

The dual part corresponding to constraints on flows in power lines on holidays, right part of constraints (9), dual variables $\overline{\gamma}^{h}_{jj'st}$:

$$\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{h}}_{jj'st}(u^{\mathrm{h}}_{jj'st}-a_{jj'}) = \tag{25}$$

$$=\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{h}}_{jj'st}u^{\mathrm{h}}_{jj'st}-\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{h}}_{jj'st}a_{jj'} \ .$$

The dual part corresponding to constraints on "emergency" flows in power lines, left part of constraints (10), dual variables $\underline{\eta}_{jj'st}$:

$$\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}\left[-\underline{\eta}_{jj'st}\right]\widetilde{u}_{jj'st} \ . \tag{26}$$

The dual part corresponding to constraints on "emergency" flows in power lines, right part of constraints (10), dual variables $\overline{\eta}_{jj'st}$:

$$\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}\overline{\eta}_{jj'st}(\widetilde{u}_{jj'st}-a_{jj'}) = \tag{27}$$

$$=\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}\overline{\eta}_{jj'st}\widetilde{u}_{jj'st}-\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}\overline{\eta}_{jj'st}a_{jj'} \ .$$

The dual part corresponding to constraints on operating power balance of power stations on working days (11), dual variables $\lambda^{\mathrm{w}}_{jst}$:

$$\sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{w}}_{jst}\left(\sum_{i\in I}x^{\mathrm{w}}_{jist}-\sum_{\substack{j'\in J\\ j'\neq j}}u^{\mathrm{w}}_{jj'st}+\sum_{\substack{j'\in J\\ j'\neq j}}u^{\mathrm{w}}_{j'jst}(1-\pi_{j'j})-D_{jst}\right) = \tag{28}$$

$$=\sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{w}}_{jst}\sum_{i\in I}x^{\mathrm{w}}_{jist}-\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}(\lambda^{\mathrm{w}}_{jst}-(1-\pi_{j'j})\lambda^{\mathrm{w}}_{j'st})\times$$

$$\times u^{\mathrm{w}}_{jj'st}-\sum_{j\in J}\sum_{s\in S}\sum_{t\in T}D_{jst}\lambda^{\mathrm{w}}_{jst} \ .$$

The dual part corresponding to constraints on operating power balance of power stations on holidays (12), dual variables $\lambda^{\mathrm{h}}_{jst}$:

$$\sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{h}}_{jst}\left(\sum_{i\in I}x^{\mathrm{h}}_{jist}-\sum_{\substack{j'\in J\\ j'\neq j}}u^{\mathrm{h}}_{jj'st}+\sum_{\substack{j'\in J\\ j'\neq j}}u^{\mathrm{h}}_{j'jst}(1-\pi_{j'j})-D_{jst}\right) = \tag{29}$$

$$= \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{h}}_{jst}\sum_{i\in I}x^{\mathrm{h}}_{jist} - \sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}(\lambda^{\mathrm{h}}_{jst} - (1-\pi_{j'j})\lambda^{\mathrm{h}}_{j'st})\times$$
$$\times u^{\mathrm{h}}_{jj'st} - \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}D_{jst}\lambda^{\mathrm{h}}_{jst}\ .$$

The dual part corresponding to constraints on installed power in peak load hours (13), dual variables σ_{jst}:

$$\sum_{j\in J}\sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst}\Bigg(D_{jst}+\epsilon\cdot D_{jst}-\sum_{i\in I}y_{ji}+\sum_{\substack{j'\in J\\ j'\neq j}}\widetilde{u}_{jj'st}-\sum_{\substack{j'\in J\\ j'\neq j}}\widetilde{u}_{j'jst}\times \tag{30}$$
$$\times(1-\pi_{j'j})\Bigg)=\sum_{j\in J}\sum_{s\in R}\sum_{t\in Q_s}[-\sigma_{jst}]\sum_{i\in I}y_{ji}+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}(\sigma_{jst}-$$
$$-(1-\pi_{j'j})\sigma_{j'st})\widetilde{u}_{jj'st}+\sum_{j\in J}\sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst}\left(D_{jst}+\epsilon\cdot D_{jst}\right)\ .$$

Thus, the dual Lagrange function consists of the following components: the objective function (1)–(3), the dual parts corresponding to all constraints of the problem, (14)–(30).

The dual Lagrange function is the following:

$$\Theta(\lambda^{\mathrm{w}},\lambda^{\mathrm{h}},\sigma,\underline{\xi},\overline{\xi},\underline{\nu},\overline{\nu},\underline{\mu}^{\mathrm{w}},\overline{\mu}^{\mathrm{w}},\underline{\mu}^{\mathrm{h}},\overline{\mu}^{\mathrm{h}},\underline{\gamma}^{\mathrm{w}},\overline{\gamma}^{\mathrm{w}},\underline{\gamma}^{\mathrm{h}},\overline{\gamma}^{\mathrm{h}},\underline{\eta},\overline{\eta}) = \tag{31}$$
$$=\min_{x,y,u,\widetilde{u},a}\Big\{L(x,y,u,\widetilde{u},a,\lambda^{\mathrm{w}},\lambda^{\mathrm{h}},\sigma,\underline{\xi},\overline{\xi},\underline{\nu},\overline{\nu},\underline{\mu}^{\mathrm{w}},\overline{\mu}^{\mathrm{w}},\underline{\mu}^{\mathrm{h}},\overline{\mu}^{\mathrm{h}},\underline{\gamma}^{\mathrm{w}},\overline{\gamma}^{\mathrm{w}},\underline{\gamma}^{\mathrm{h}},$$
$$\overline{\gamma}^{\mathrm{h}},\underline{\eta},\overline{\eta})\Big\} = \Theta_1(\lambda^{w},\lambda^{h},\sigma,\underline{\xi},\overline{\xi},\underline{\nu},\overline{\nu},\underline{\mu}^{\mathrm{w}},\overline{\mu}^{\mathrm{w}},\underline{\mu}^{\mathrm{h}},\overline{\mu}^{\mathrm{h}},\underline{\gamma}^{\mathrm{w}},\overline{\gamma}^{\mathrm{w}},\underline{\gamma}^{\mathrm{h}},\overline{\gamma}^{\mathrm{h}},\underline{\eta},\overline{\eta})+$$
$$+\Theta_2(\lambda^{\mathrm{w}},\lambda^{\mathrm{h}},\sigma,\underline{\xi},\overline{\xi},\underline{\nu},\overline{\nu},\underline{\mu}^{\mathrm{w}},\overline{\mu}^{\mathrm{w}},\underline{\mu}^{\mathrm{h}},\overline{\mu}^{\mathrm{h}},\underline{\gamma}^{\mathrm{w}},\overline{\gamma}^{\mathrm{w}},\underline{\gamma}^{\mathrm{h}},\overline{\gamma}^{\mathrm{h}},\underline{\eta},\overline{\eta}),$$

where

$$\Theta_1(\lambda^{\mathrm{w}},\lambda^{\mathrm{h}},\sigma,\underline{\xi},\overline{\xi},\underline{\nu},\overline{\nu},\underline{\mu}^{\mathrm{w}},\overline{\mu}^{\mathrm{w}},\underline{\mu}^{\mathrm{h}},\overline{\mu}^{\mathrm{h}},\underline{\gamma}^{\mathrm{w}},\overline{\gamma}^{\mathrm{w}},\underline{\gamma}^{\mathrm{h}},\overline{\gamma}^{\mathrm{h}},\underline{\eta},\overline{\eta}) = \tag{32}$$
$$=\min_{x,y,u,\widetilde{u},a}\Bigg\{\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}(\tau^{\mathrm{w}}_s v_{ji}-\lambda^{\mathrm{w}}_{jst}-\underline{\mu}^{\mathrm{w}}_{jist}+\overline{\mu}^{\mathrm{w}}_{jist})x^{\mathrm{w}}_{jist}+$$
$$+\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}(\tau^{\mathrm{h}}_s v_{ji}-\lambda^{\mathrm{h}}_{jst}-\underline{\mu}^{\mathrm{h}}_{jist}+\overline{\mu}^{\mathrm{h}}_{jist})x^{\mathrm{h}}_{jist}+$$
$$+\sum_{j\in J}\sum_{i\in I}(fk_{ji}+b_{ji}-\sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst}-\underline{\xi}_{ji}+\overline{\xi}_{ji})y_{ji}+$$
$$+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}(f\kappa_{jj'}+\beta_{jj'}-\underline{\nu}_{jj'}+\overline{\nu}_{jj'}-\sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{w}}_{jj'st}-\sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{h}}_{jj'st}-$$
$$-\sum_{s\in R}\sum_{t\in Q_s}\overline{\eta}_{jj'st})a_{jj'}+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}(\lambda^{\mathrm{w}}_{jst}-\lambda^{\mathrm{w}}_{j'st}(1-\pi_{j'j})-\underline{\gamma}^{\mathrm{w}}_{jj'st}+$$

$$+\overline{\gamma}^{\mathrm{w}}_{jj'st})u^{\mathrm{w}}_{jj'st} + \sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in S}\sum_{t\in T}(\lambda^{\mathrm{h}}_{jst} - \lambda^{\mathrm{h}}_{j'st}(1-\pi_{j'j}) - \underline{\gamma}^{\mathrm{h}}_{jj'st} + \overline{\gamma}^{\mathrm{h}}_{jj'st})u^{\mathrm{h}}_{jj'st} +$$

$$+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{s\in R}\sum_{t\in Q_s}(\sigma_{jst} - \sigma_{j'st}(1-\pi_{j'j}) - \underline{\eta}_{jj'st} + \overline{\eta}_{jj'st})\widetilde{u}_{jj'st}\Bigg\},$$

$$\Theta_2(\lambda^{\mathrm{w}}, \lambda^{\mathrm{h}}, \sigma, \underline{\xi}, \overline{\xi}, \underline{\nu}, \overline{\nu}, \underline{\mu}^{\mathrm{w}}, \overline{\mu}^{\mathrm{w}}, \underline{\mu}^{\mathrm{h}}, \overline{\mu}^{\mathrm{h}}, \underline{\gamma}^{\mathrm{w}}, \overline{\gamma}^{\mathrm{w}}, \underline{\gamma}^{\mathrm{h}}, \overline{\gamma}^{\mathrm{h}}, \underline{\eta}, \overline{\eta}) = \tag{33}$$

$$= \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{w}}_{jst}D_{jst} + \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{h}}_{jst}D_{jst} +$$

$$+\sum_{j\in J}\sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst}D_{jst}(1+\epsilon) + \sum_{j\in J}\sum_{i\in I}\underline{\xi}_{ji}y^0_{ji} - \sum_{j\in J}\sum_{i\in I}\overline{\xi}_{ji}\overline{y}_{ji} -$$

$$-f\sum_{j\in J}\sum_{i\in I}k_{ji}y^0_{ji} + \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\underline{\nu}_{jj'}a^0_{jj'} - \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\overline{\nu}_{jj'}\overline{a}_{jj'} -$$

$$-f\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\kappa_{jj'}a^0_{jj'} + \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\underline{\mu}^{\mathrm{w}}_{jist}x^0_{jist} - \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\overline{\mu}^{\mathrm{w}}_{jist}\overline{x}_{jist} +$$

$$+\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\underline{\mu}^{\mathrm{h}}_{jist}x^0_{jist} - \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\overline{\mu}^{\mathrm{h}}_{jist}\overline{x}_{jist}\ .$$

To get the dual problem expressions-cofactors before every primal variable could be zero. Thus one get constraints of the dual problem. The remaining part without variables is an objective function of the dual problem. Also conditions for the nonnegativity are written. As a result we get the following representation of the dual problem:

$$\sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{w}}_{jst}D_{jst} + \sum_{j\in J}\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{h}}_{jst}D_{jst} + \tag{34}$$

$$+\sum_{j\in J}\sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst}D_{jst}(1+\epsilon) + \sum_{j\in J}\sum_{i\in I}\underline{\xi}_{ji}y^0_{ji} - \sum_{j\in J}\sum_{i\in I}\overline{\xi}_{ji}\overline{y}_{ji} -$$

$$-f\sum_{j\in J}\sum_{i\in I}k_{ji}y^0_{ji} + \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\underline{\nu}_{jj'}a^0_{jj'} - \sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\overline{\nu}_{jj'}\overline{a}_{jj'} -$$

$$-f\sum_{j\in J}\sum_{\substack{j'\in J\\ j'>j}}\kappa_{jj'}a^0_{jj'} + \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\underline{\mu}^{\mathrm{w}}_{jist}x^0_{jist} - \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\overline{\mu}^{\mathrm{w}}_{jist}\overline{x}_{jist} +$$

$$+\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\underline{\mu}^{\mathrm{h}}_{jist}x^0_{jist} - \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\overline{\mu}^{\mathrm{h}}_{jist}\overline{x}_{jist} \to \min,$$

$$\tau^{\mathrm{w}}_s v_{ji} - \lambda^{\mathrm{w}}_{jst} - \underline{\mu}^{\mathrm{w}}_{jist} + \overline{\mu}^{\mathrm{w}}_{jist} = 0, \quad j\in J, \quad i\in I, \quad s\in S, \quad t\in T; \tag{35}$$

$$\tau^{\mathrm{h}}_s v_{ji} - \lambda^{\mathrm{h}}_{jst} - \underline{\mu}^{\mathrm{h}}_{jist} + \overline{\mu}^{\mathrm{h}}_{jist} = 0, \quad j\in J, \quad i\in I, \quad s\in S, \quad t\in T; \tag{36}$$

$$fk_{ji} + b_{ji} - \sum_{s\in R}\sum_{t\in Q_s}\sigma_{jst} - \underline{\xi}_{ji} + \overline{\xi}_{ji} = 0, \quad j\in J, \quad i\in I; \tag{37}$$

$$f\kappa_{jj'} + \beta_{jj'} - \underline{\nu}_{jj'} + \overline{\nu}_{jj'} - \sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{w}}_{jj'st} - \sum_{s\in S}\sum_{t\in T}\overline{\gamma}^{\mathrm{h}}_{jj'st} - \tag{38}$$

$$-\sum_{s\in R}\sum_{t\in Q_s}\overline{\eta}_{jj'st} = 0, \quad j\in J, \quad j'\in J, \quad j'>J, \quad s\in S, \quad t\in T;$$

$$\lambda^{\mathrm{w}}_{jst} - \lambda^{\mathrm{w}}_{j'st}(1-\pi_{j'j}) - \underline{\gamma}^{\mathrm{w}}_{jj'st} + \overline{\gamma}^{\mathrm{w}}_{jj'st} = 0, \tag{39}$$

$$j\in J, \quad j'\in J, \quad j'\neq J, \quad s\in S, \quad t\in T;$$

$$\lambda^{\mathrm{h}}_{jst} - \lambda^{\mathrm{h}}_{j'st}(1-\pi_{j'j}) - \underline{\gamma}^{\mathrm{h}}_{jj'st} + \overline{\gamma}^{\mathrm{h}}_{jj'st} = 0, \tag{40}$$

$$j\in J, \quad j'\in J, \quad j'\neq J, \quad s\in S, \quad t\in T;$$

$$\sigma_{jst} - \sigma_{j'st}(1-\pi_{j'j}) - \underline{\eta}_{jj'st} + \overline{\eta}_{jj'st} = 0, \tag{41}$$

$$j\in J, \quad j'\in J, \quad j'\neq J, \quad s\in R, \quad t\in Q_s;$$

$$\sigma_{jst} \geq 0, \quad j\in J, \quad s\in R, \quad t\in Q_s, \tag{42}$$

$$\underline{\xi}_{ji}\geq 0, \quad \overline{\xi}_{ji}\geq 0, \quad j\in J, \quad i\in I, \tag{43}$$

$$\underline{\nu}_{jj'}\geq 0, \quad \overline{\nu}_{jj'}\geq 0, \quad j\in J, \quad j'\in J, \quad j' > j \tag{44}$$

$$\underline{\mu}^{\mathrm{w}}\geq 0, \quad \overline{\mu}^{\mathrm{w}}\geq 0, \quad \underline{\mu}^{\mathrm{h}}\geq 0, \quad \overline{\mu}^{\mathrm{h}}\geq 0, \quad j\in J, \quad i\in I, \quad s\in S, \quad t\in T, \tag{45}$$

$$\underline{\gamma}^{\mathrm{w}}_{jj'st}\geq 0, \quad \overline{\gamma}^{\mathrm{w}}_{jj'st}\geq 0, \quad \underline{\gamma}^{\mathrm{h}}_{jj'st}\geq 0, \quad \overline{\gamma}^{\mathrm{h}}_{jj'st}\geq 0, \tag{46}$$

$$j\in J, \quad j'\in J, \quad j'\neq j, \quad s\in S, \quad t\in T,$$

$$\underline{\eta}_{jj'st}\geq 0, \quad \overline{\eta}_{jj'st}\geq 0, \quad j\in J, \quad j'\in J, \quad j'\neq j, \quad s\in S, \quad t\in T\ . \tag{47}$$

4 Example

To give an interpretation of the above approach we consider an illustrative example. The network consists of three nodes: $J = \{"Volga", "Center", "South"\}$. There are two periods of time: $T = \{0, 1\}$. Period 0 is a peak period. Assume that f=0.02, π=0.025, ϵ=0.1. There are connections between the lines ("Volga" – "Center", "Center" – "South", "South" – "Volga"). The node and lines parameters of the model are presented in Tab. 1–2. The values of the primal and dual variables are presented in Tab. 3–6.

Table 1. The node parameters $\underline{y}_j$, $\overline{y}_j$, k_j, b_j, v_{jt}, D_{jt}

Node, j	$\underline{y}_j$	$\overline{y}_j$	k_j	b_j	v_{j0}	v_{j1}	D_{j0}	D_{j1}
Volga	10	100	500	30	640	960	120	50
Center	12	120	200	17	1536	1600	100	90
South	20	200	350	21	1280	1408	100	45

It is shown in [3, 4] that the dual variables corresponding to every constraint in the primal problem have a certain conceptual meaning. Let us consider some of the most

Table 2. Lines parameters $\overline{a}_{jj'}$, $\kappa_{jj'}$, $\beta_{jj'}$

Connection	$\overline{a}_{jj'}$	$\kappa_{jj'}$	$\beta_{jj'}$
Volga – Center	35	200	50
Center – South	50	190	76
South – Volga	78	180	91

Table 3. The values of primal variables x_{jt}, y_j and dual variables $\underline{\mu}_{jt}$, λ_{jt}, σ_{jt}, $\overline{\xi}_j$

Node, j	x_{j0}	x_{j1}	y_j	$\overline{\mu}_{j0}$	$\overline{\mu}_{j1}$	λ_{j0}	λ_{j1}	σ_{j0}	$\overline{\xi}_j$
Volga	100	100	100	857.6	412.80	1497.60	1372.80	20.48	1250.88
Center	23.22	7.13	54.43	0	0	1536	1600	21	0
South	200	80.38	200	85.56	0	1365.56	1408	19.96	77.52

Table 4. The values of primal variables $u_{jj't}$ and dual variables $\underline{\gamma}_{jj't}$, $\overline{\gamma}_{jj't}$

Flow	$u_{jj'0}$	$u_{jj'1}$	$\underline{\gamma}_{jj'0}$	$\underline{\gamma}_{jj'1}$	$\overline{\gamma}_{jj'0}$	$\overline{\gamma}_{jj'1}$
Volga $\rightarrow$ Center	28.75	35	0	0	0	187.2
Center $\rightarrow$ Volga	0	0	75.84	261.52	0	0
Center $\rightarrow$ South	0	0	204.58	227.2	0	0
South $\rightarrow$ Center	50	50	0	0	132.04	152
South $\rightarrow$ Volga	50	0	0	69.52	94.6	0
Volga $\rightarrow$ South	0	15	166.18	0	0	0

Table 5. The values of primal variables $\widetilde{u}_{jj't}$ and dual variables $\underline{\eta}_{jj't}$

Flow	$\widetilde{u}_{jj'0}$	$\underline{\eta}_{jj'0}$	$\overline{\eta}_{jj'0}$
Volga $\rightarrow$ Center	7	0	0
Center $\rightarrow$ Volga	0	1.04	0
Center $\rightarrow$ South	0	1.54	0
South $\rightarrow$ Center	50	0	0.51
South $\rightarrow$ Volga	40	0	0
Volga $\rightarrow$ South	0	0	0

Table 6. The values of primal variables $a_{jj'}$ and dual variables $\overline{\nu}_{jj'}$

Connection	$a_{jj'}$	$\overline{\nu}_{jj'}$
Volga – Center	35	133.20
Center – South	50	204.75
South – Volga	50	0

important of them.

First, it is necessary to say about the dual variables λ_{jt}. They are nodal prices. These prices vary in time. The concept of nodal prices is well studied. In the very beginning the nodal prices were proposed and developed by Schweppe and his collaborators in [6]. For this example the equations (35)–(36) are wrote in the following form:

$$v_j - \lambda_{jt} - \underline{\mu}_{jt} + \overline{\mu}_{jt} = 0 \ . \tag{48}$$

Substitute the value of primal and the dual variables in the equation (48):
Node "Volga" at 0 and 1 hours respectively:

$$640 - 1497.60 + 857.60 = 0;$$
$$960 - 1372.80 + 412.8 = 0 \ .$$

Node "Center" at 0 and 1 hours respectively:

$$1536 - 1536 = 0;$$
$$1600 - 1600 = 0 \ .$$

Node "South" at 0 and 1 hours respectively:

$$1280 - 1365.56 + 85.56 = 0;$$
$$1408 - 1408 = 0 \ .$$

The prices in the node "Volga" at 0 and 1 hours are higher than unit variable costs. Producers get producer surplus: 857.60 at 0 hour and 412.80 at 1 hour per unit. Also producers get producer surplus in the node "South" at 0 hour and it equals 85.56 per unit. In the node "Center" at 0 and 1 hours, in the node "South" at 1 hour there is no producer surplus, the electricity hour price equals unit variable costs.
Now we consider equations (39)–(40). They are wrote in the following form:

$$\lambda_{jt} - \lambda_{j't}(1 - \pi) - \underline{\gamma}_{jj't} + \overline{\gamma}_{jj't} = 0 \ . \tag{49}$$

Substitute the value of primal and the dual variables in the equation (49):
Flow from the node "Volga" to the node "Center" at 0 and 1 hours:

$$1497.60 - 1536(1 - 0.025) = 0;$$
$$1372.80 - 1600(1 - 0.025) = 0 \ .$$

Flow from the node "South" to the node "Center" at 0 and 1 hours:

$$1365.56 - 1536(1 - 0.025) + 132.04 = 0;$$
$$1408 - 1600(1 - 0.025) + 152 = 0 \ .$$

Flow from the node "South" to the node "Volga" at 0 hours:

$$1365.56 - 1497.60(1 - 0.025) + 94.60 = 0 \ .$$

Flow from the node "Volga" to the node "South" at 1 hours:

$$1372.80 - 1408(1 - 0.025) = 0 \ .$$

We can see that the price in the node where power come is higher than the price in another node for the value of line losses. Moreover, if the line is used on maximum capacity the consumer pays for the line. When transferring power from the node "South" to the node "Center" the consumers pay 132.04 at 0 hour and 152 at 1 hour per unit for the use of line.

5 Conclusion

In conclusion it is possible to state the following:

1. The dual variables corresponding to constraints on the operating power balance of power stations (11)–(12) are electricity hour prices in the model. These electricity hour prices take into account line losses, and use of maximum line capacity.
2. More effective producers will get consumer surplus, which is defined by the dual variables corresponding to the operating power $\overline{\mu}_{jist}$. Also it is necessary to point that consumers don't compensate fixed costs of power plants.
3. The example in this paper has small dimension and it is illustrative. It is necessary to point that there was solved model of large dimension for the central part of Russia, which united five regions: the North-West, Central, Volga, South and Ural. Also there are 24 hour in a day, 4 seasons, working days and holidays, peak hours in the model. The model not inclusive in this paper because of large dimension.

References

1. Belyaev, L., Chyudinova, L., Khamisov, O., Kovalev, G., Lebedeva, L., Podkovalnikov, S., Saveliev, V.: Studies of interstate electric ties in Northeast Asia. International Journal of Global Energy Issues, vol. 17, No. 3, p. 228–249 (2002)
2. Belyaev, L., Podkovalnikov, S., Saveliev, V., Chyudinova, L: Effectiveness of interstate electric ties (in Russian). Nauka, Novosibirsk. 239 p. (2008)
3. Gakh, S.: Using the dual variables to find a solution in the market optimization model and development generating capacity (in Russian). Publishing house of IMM UB RAS. p. 81–82 (2015)
4. Gakh, S., Khamisov, O.: Dual analisys of long-run development EPS model considering development of generating powers (in Russian). Optimisation problems and economical applications: materials of VI International Conference (Omsk, 28 June – 4 July, 2015). Publishing house of OmSU. P.169 (2015)
5. Khamisov, O., Podkovalnikov, S: Modeling and study of Russian oligopolistic electricity market considering generating capacity extension. Proceedings of the PowerTech 2011 Conference, Trondheim, Norway, p. 506–512 (2011)
6. Schweppe, F., Caramanis, M., Tabors, R., Bohn, R.: Spot pricing of electricity. Kluwer Academic Publishers, Boston (1988)
7. Vasilev, F.: Optimisation methods (in Russian). Publishing house "Factorial-Press". 824 p. (2002)