# DIFFRACTION OF THE GAUSSIAN BEAM ON LAYERED LENS AND SIMILAR A CONICAL AND DIFFRACTION AXICONS

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**Abstract.** In this paper we consider the possibility of replacing the diffraction axicon and the conical axicon on the gradient lens with a linear variation of the refractive index. Analytically and numerically using the finite-difference time-domain method we performed a comparative study of the Gaussian beam diffraction on diffraction mikro-axicon, conical axicon and gradient microlens consisting of subwavelength layers. The parameters under consideration the types of elements estimated in the depth of focus and a transverse dimension of beam.

**Keywords:** diffraction optics, subwavelength structures, laser beams, diffraction axicon, layered lens, conical axicon, FDTD.

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# Introduction

Environments with light propagates in curved paths are the subject of gradient optics (GRIN - GRadient INdex) [1]. The flat surfaces of gradient lenses make them very useful for collimating light from the end of single mode fiber and focusing of the collimated beam to another single mode fiber [2]. Thus, light beams passing through the gradient lens can be the use for better focusing [3-5].

When transmitting information over optical fibers easier connection between the fibers do using gradient elements [6, 7], usually, such components are in some way analogue of a lens [8, 9], which forms a short focus. Typically used two gradient elements with a sufficiently precise mutual agreement: one at the output, which scatters the laser beams and one at the entrance, which collects the laser beam [10, 11].

One advantage of using the axicon is the formation of an extended focus [12, 13], including subwavelength lateral size [14, 15]. The advantage of using a diffraction axicon before the conical axicon is in the relative simplicity of manufacturing, and in the possibility of achieving, for this element of high numerical aperture values, inac-

cessible to the conical axicon due to total internal reflection [16-18]. An extended focus [19] can be used to alleviate the requirements for alignment of the optical fiber connection.

For connections required flat edge [20], and diffraction axicon has it. In this pa-per, we consider particularly focusing Gaussian beams by using gradient optical ele-ments [21, 22] and similar a conical and diffraction axicons. For the numerical simu-lation of diffraction considered laser beams used finite-difference time-domain method (FDTD) using high-performance computing [23].

### **Diffraction of the Gaussian beam**

Under the linear change of the refractive index  $n(r) = n_0(1 - \alpha r)$  the phase difference is analogous to a conical axicon:

$$\Delta \varphi_{lin}(r) = -kn_0 L\alpha r , \qquad (1)$$

where  $n_0$  – the refractive index in the center,  $k = 2\pi/\lambda$ ,  $\lambda$  is the wavelength, L – lens thickness,  $\alpha$  – parameter governing the rate of change of the refractive index. Let's define parameters of a conical axicon creating the same phase difference [14]:

$$\Delta \varphi_{\alpha}(r) = -k(n_{\alpha} - 1) \cdot r \cdot ctg\beta, \qquad (2)$$

In according to equations (1) and (2), we selected axicon angle:

$$\beta = \operatorname{arctg}\left(\frac{n_{ax} - 1}{\alpha L n_0}\right),\tag{3}$$

where nax – the refractive index of axicon material,  $\beta$  – a half of angle at the axicon tip (Fig1(c)). H – axicon height:

$$H = \frac{R}{tg\beta},\tag{4}$$

Let us consider the diffraction axicon (Fig.1 (b)). The phase difference between the central ray and a ray extending from the center at a distance is equal:

$$\Delta \phi_{\rm mb} = -\mathbf{k} \cdot \mathbf{N} \mathbf{A} \cdot \mathbf{r} \,, \tag{5}$$

where NA – the numerical aperture of the axicon, r - radius of the axicon. Then, the numerical aperture of axicon is

$$NA = n_0 L\alpha , \qquad (6)$$

where n0 = 3,47 – the value of the central layer for the considered layered lens. Period of axicon d is changed to the following law:

Savelyev DA. Diffraction of the...

$$d = \frac{\lambda}{NA},$$
(7)

Height axicon considered on the basis of the phase shift on  $\pi$ :

$$h = \frac{\pi}{k(n_{ax} - 1)} = \frac{\lambda}{2(n_{ax} - 1)} \approx 0,21\lambda,$$
(8)

when the refractive index  $n_{ax} = 3,47$ .



Fig. 1. The transverse structure (scheme) of the matched linearly layered lens (a), diffractive axicon (b) and the conical axicon (c)

Simulation parameters: the wavelength  $\lambda = 1.55$  microns, the size of the computational domain x, y,z  $\in$  [-4,5 $\lambda$ ; 4,5 $\lambda$ ]. The thickness of the absorbing layer PML ~ 0.65 $\lambda$  (1 micron), the sampling step of space –  $\lambda/31$ , the sampling step of time –  $\lambda/(62c)$ , where c is the velocity of light. As the input laser radiation with the circular polarization we use the fundamental Gaussian mode. In the case of layered lens we use the linearly changes of refractive index of lens: from n=3.47 in center to n=1.34 at the lens edge. Let's denote a lens width on propagation axis of the laser beam as L. Numerical simulation was made using the computational cluster with the power of

775 GFlops. The cluster's characteristics are the following: 116 cores, computing nodes – 7 dual servers HP ProLiant 2xBL220c, RAM volume 112 Gbit.

We consider the half width at half intensity (FWHM) and depth of field (DOF). Fix a lens width  $L = 1,55\lambda$  with refractive index n = 3,47. The numerical results studies for the axicon and the layered lenses with a corresponding  $\alpha$  are given in Table 1.

Туре  $\alpha = 0.13$  $\alpha = 0,12$  $\alpha = 0,11$  $\alpha = 0,1$ of elem ent Layered lens  $DOF = 3,2\lambda$  $DOF = 2,85\lambda$  $DOF = 3,2\lambda$  $DOF = 3,4\lambda$ FWHM =  $0,67\lambda$ FWHM =  $0,68\lambda$ FWHM =  $0,67\lambda$ FWHM =  $0,68\lambda$ **Diffraction axicon**  $DOF = 2,55\lambda$  $DOF = 3,23\lambda$  $DOF = 3.77\lambda$  $DOF = 4.36\lambda$ FWHM =  $0,71\lambda$ FWHM =  $0,75\lambda$ FWHM =  $0,77\lambda$ FWHM =  $0.84\lambda$ **Conical axicon**  $DOF = 1.36\lambda$  $DOF = 1.77\lambda$  $DOF = 2.17\lambda$  $DOF = 2.81\lambda$ **FWHM** =  $0.75\lambda$ FWHM =  $0.76\lambda$ FWHM =  $0.78\lambda$ FWHM =  $0.76\lambda$ 



Reducing the parameter  $\alpha$  for a layered lens increases the length of the light segment with a substantially constant radius of the light spot. A separate case with  $\alpha = 0.12$ , where the observed change in the overall diffraction patterns and reducing the depth of focus.

For diffraction axicon situation is as follows: reduction in  $\alpha$  (that means reducing the numerical aperture) also leads to an increase in the length of the light segment. No cases like the case  $\alpha = 0.12$  for a layered lens. And also we see expected focal spot size increases.

For the conical axicon also decrease  $\alpha$  (which is equivalent to an increase of the angle  $\beta$ ) leads to elongation of the light segment. But in this case also seems certain number

 $\alpha = 0.11$ , where the focal spot is minimal. Reduction of  $\alpha$  leads to a broadening of the beam.



**Fig. 2.** Diffraction of Gaussian beam on a layered lens with changing L ( $\alpha = 0.11$ ), the intensity: L = 1.55 $\lambda$  (black line), L = 1.75 $\lambda$  (gray line)



Table 2. Results of numerical simulation when changing the height of the layered lens

When comparing rows of Table 1 it should be noted that the use of a layered lens provides a more narrow size of the focal spot, and when the value of  $\alpha$  is higher than 0.12, and more extended focal light segment. Let us consider in more detail the layered lens effect in changing its length along the axis of propagation of the laser beam for two cases mentioned earlier:  $\alpha = 0.11$  and  $\alpha = 0.12$  (Table 2). We consider the FWHM at the point of maximum intensity.

Table 2 shows that the increase in length of the lens leads to a reduction of the focal length for DOF. Increasing the lens length of an increase of the numerical aperture, only makes sense to a certain value (Figure 2). Table 2 shows that the increase in the

length of more than  $1.55\lambda$  lens reduces the depth of focus at a constant value of FWHM.

Consider the change in the height of the diffraction axicon in case  $\alpha = 0.12$ , i.e. at a numerical aperture of NA = 0.64. We varied the refractive index n. The height of the respective axicon considered on the basis of the phase shift at  $\pi$  by the formula (8). Numerical simulation result is shown in Table 3.

 $\label{eq:Table 3. Diffraction of Gaussian beam on diffraction axicon with a change of the refractive index n$ 



Decrease in the refractive index and simultaneously increase axicon relief leads to a reduction of the lengths of light segment. However, after a certain limiting value (n = 1.68) DOF begins to increase again. It is also worth noting the reduction in the size of the focal spot with a decrease in the index of refraction of the axicon. However, it should be noted that after reaching a limiting value of the refractive index (in this case, when n = 1.68) of the focal spot size is stabilized and becomes comparable to the previously discussed case of layered lenses.

## Conclusion

Analytically and numerically using the finite-difference time-domain method we performed a comparative study of the diffraction of Gaussian beam by diffraction microaxicon and conical axicon, and gradient micro-lens consisting of sub-wavelength layers. The parameters under consideration the types of elements estimated on the depth of focus and a transverse dimension of beam.

Studies have shown that layered lens with linear variation of the refractive index has an advantage over diffraction axicon with the same numerical aperture, as it allows to form a narrower focal lengths. Increasing the numerical aperture of the axicon reduces the focal spot formed by them, but it is accompanied by a reduction of the light segment lengths. With a value of more than  $\alpha = 0.12$  (numerical aperture of more than 0.64) was obtained more extended light length segment for a layered lens.

By reducing the thickness of the layered lens is extended light segment and increases its width in the plane of maximum intensity along the propagation axis. After a certain point, in our case  $1.55\lambda$ , there is the stabilization of transverse dimension with shortening the length of a segment.

Studies on the reduction of the refractive index of the diffraction axicon show that after reaching a limiting value of the refractive index (in this case, when n = 1.68) the focal spot size is stabilized and not decreases.

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