SIMULATION OF OPTICAL SIGNALS PROPAGATION IN A RANDOM MEDIA

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Abstract. The operator of the optical beam propagation through turbulent environment using the Fresnel approximation is being considered. The correlation function of random field describing inhomogeneous medium is given in the form of Gaussian function. The process of random field modeling using the Fourier transform is demonstrated. A selective correlation function is calculated, the deviation from the preset one is defined. The intensity distributions after propagation of optical beams in free space and in a random medium are given. As the input beam such optical distributions as Hermite - Gauss modes, rectangular pulse, and vortex beams were considered.

Keywords: turbulent media, random field, Fourier transform, Fresnel transform, Gaussian modes, vortex beams.

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1 Introduction

Distribution of the optical signal in a free space can be subject to distortions associated with the turbulence of the medium [1, 2]. Due to the properties of turbulence random changes in atmospheric refractive index can cause distortion of the laser radiation intensity. Classical methods for describing wave propagation through the turbulent atmosphere are based on the Rytov method applications [3] and the method of parabolic equations [4], connection between them has been demonstrated previously [5].

With the help of these methods propagation of optical signals from partially coherent source has been investigated [6, 7], and also features of propagation of different laser beams in a turbulent medium have been studied, including Gaussian beams of higher orders [8], hollow beams [9], diffraction-free beams, such as Gauss-Bessel and Eerie beams [10, 11], as well as cosine beams [12]. Wherein it was found that the higher-order Gaussian beams, including vortex bundles [13], as well as various spatially structured bundles become broader under the influence of turbulence in a lesser de-

gree than the fundamental Gaussian beam. In addition, the spread of beams with an inhomogeneous polarization in a turbulent medium was studied [14, 15].
The most convenient and effective means of forming random beams with specified properties are the methods of diffraction optics [16-20].
In this paper modeling of random optical beams through random medium with a given correlation function in the form of the Gaussian function is being considered. The comparative calculations of the propagation of Gaussian modes (Hermite-Gaussian and Laguerre-Gaussian) through the random medium are being performed.

2 Passing through the random medium

The distribution of a laser beam in a random medium can be described on the basis of the integral expressing the extended Huygens-Fresnel principle [8, 21]:

$$E\left(u,v,z,t\right)=-\frac{ik}{2\pi z}\exp(ikz)\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_0(x,y)\exp\left\{\frac{ik}{2z}\left[\left(x-u\right)^2+\left(y-v\right)^2\right]+\Psi(x,y,u,v,z)-i\omega t\right\}\mathrm{d}\,x\,\mathrm{d}\,y, \tag{1}$$

where $E_0(x,y)$ is a field in the input plane (at z = 0), $E\left(u,v,z,t\right)$ is a field at a distance z from the input plane, $\Psi(x,y,u,v,z)$ is a random part in the propagation operator, related to atmospheric turbulence, ω is the frequency of laser oscillation, t is time.
Equation (1) corresponds to the Rytov method [21], and the function $\Psi(x,y,u,v,z)$ describes accidental deviations of the phase function of a spherical wave, propagating from the initial to the output plane.
Note that in this way the complex phase can be implemented also in other integral transforms, for example, into fractional Fourier transform [22, 23]which describes propagation of optical signal in parabolic fiber [24, 25].
Consider the one-dimensional case for simplicity. Let the correlation function of the random field has the shape of a Gaussian function:

$$R\left\{\exp\left[\psi\left(x_1,u_1,0\right)\right],\exp\left[\psi\left(x_2,u_2,z\right)\right]\right\}=$$
$$=A\exp\left[-\frac{1}{\sigma_x^2}\left(x_1-x_2\right)^2\right]=R_{\exp[\psi]}\left(\left|x_1-x_2\right|,z\right), \tag{2}$$

where $A>0$ and $\sigma_x>0$. Note that this correlation function depends on the module of difference between the coordinates x_1 and x_2, but not on each one of them individually. Moreover, it does not depend on the coordinates in the output plane u_1 and u_2, but it depends only on the distance z to it.

Let us assume that the average intensity of the beam passing through a turbulent atmosphere is maintained, and for this we`ll define the average amplitude of the random field equal to one:

$$\left\langle \exp\left[\psi\left(x,u,z\right)\right]\right\rangle = 1 . \tag{3}$$

3 Generation of random field

Without loss of generality let us consider a random field with zero mathematical expectation, ie, a field of the type:

$$U\left(x,u,z\right) = \exp\left[\psi\left(x,u,z\right)\right] - \left\langle \exp\left[\psi\left(x,u,z\right)\right]\right\rangle , \tag{4}$$

since after its modeling it is easy to get the desired selection:

$$\exp\left[\psi\left(x,u,z\right)\right] = U\left(x,u,z\right) + \left\langle \exp\left[\psi\left(x,u,z\right)\right]\right\rangle . \tag{5}$$

Wherein the correlation function remains unchanged:

$$R_{\exp[\psi]}\left(x_1 - x_2, z\right) = R_U\left(x_1 - x_2, z\right) = R_U(x,z) . \tag{6}$$

Let us drop all the variables, except x, and assume that the required random field $U(x)$ can be obtained through the passage of a complex-valued white noise $\xi(x)$ with a unit dispersion and a correlation function equal to the Dirac delta function $\delta(x)$, through a linear filter:

$$U(x) = h(x) * \xi(x) , \tag{7}$$

where the asterisk * denotes convolution operation, $h(x)$ is a determined function (pulse characteristic). Then the generation problem reduces to finding $h(x)$ function.
By definition, the correlation function $U(x)$ is equal to:

$$\begin{aligned}
R_U(x) &= \left\langle U(x+x')U^*(x')\right\rangle = \\
&= \left\langle \int_{-\infty}^{\infty} h(\alpha)\xi(x+x'-\alpha)d\alpha \int_{-\infty}^{\infty} h^*(\beta)\xi^*(x'-\beta)d\beta \right\rangle = \\
&= \iint_{\mathbb{R}^2} h(\alpha)h^*(\beta)\left\langle \xi(x+x'-\alpha)\xi^*(x'-\beta)\right\rangle d\alpha d\beta = \\
&= \iint_{\mathbb{R}^2} h(\alpha)h^*(\beta)R_\xi(x-\alpha+\beta)d\alpha d\beta = \\
&= \iint_{\mathbb{R}^2} h(\alpha)h^*(\beta)\delta(x-\alpha+\beta)d\alpha d\beta = \\
&= \int_{-\infty}^{\infty} h(x+\beta)h^*(\beta)d\beta.
\end{aligned} \tag{8}$$

Thus, we find that the correlation function $R_U(x)$ can be defined as autocorrelation of the function $h(x)$:

$$R_U(x) = h(x) \otimes h(x) , \tag{9}$$

where the $\otimes$ symbol denotes the operation of mutual correlation. Using the Fourier transformation $\Im$, we get:

$$\Im\left[R_U(x)\right] = \left|\Im\left[h(x)\right]\right|^2 , \tag{10}$$

from where

$$\Im\left[h(x)\right] = \sqrt{\Im\left[R_U(x)\right]} . \tag{11}$$

The last formula makes it possible to find the $h(x)$ function, using the inverse Fourier transform. However, for calculation according to formula (7) the easiest way is to re-use the Fourier transform to avoid fold operation:

$$\begin{aligned} U(x) &= \Im^{-1}\left[\Im\left[h(x')\right]\Im\left[\xi(x')\right]\right] = \\ &= \Im^{-1}\left[\sqrt{\Im\left[R_U(x')\right]}\Im\left[\xi(x')\right]\right]. \end{aligned} \tag{12}$$

Modeling of $U(x)$ random field can be realized by the formula (12) using a fast Fourier transform algorithm. Note that the Fourier transform from white noise will also be white noise.

4 Modeling of the one-dimensional signals propagation

In the modeling process the following parameters were used: the wavelength $\lambda = 2\pi / k = 633nm$, the input region width $\left[-a;a\right] = \left[-60mm;60mm\right]$, $A = 1$, $\sigma_x = 1.5m$. Fresnel transformation formulas (1) and random field generations (12) were implemented using fast Fourier transform.

Input optical distribution passes consecutively 6 times through the Fresnel transformation (1) with $z = 10m$; thus, the field extends over 60 meters.

Note that the passage of 60 meters using only one transformation does not change the overall picture, as the random field has a correlation function that depends on the propagation distance.

Figures 1a, 1b are examples of generating a random field in the region $\left[-250mm;250mm\right]$.

The view of the correlation function and of the selective correlation function is shown in Figure 2.

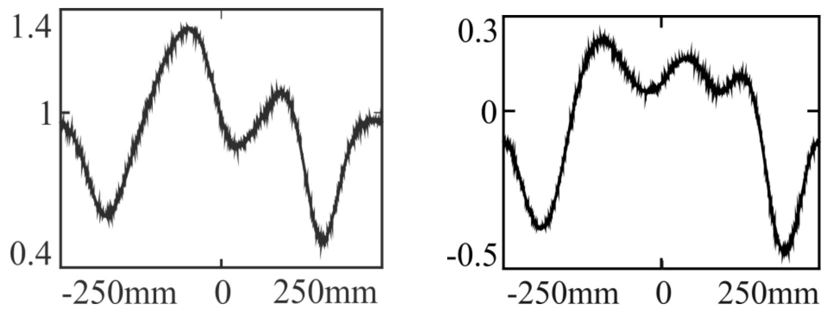

Fig. 1. Example of a random field: amplitude and phase

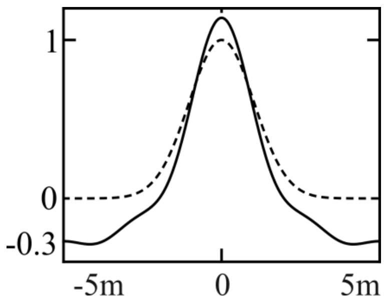

Fig. 2. Sample correlation function (solid line) compared with a given (dotted line)

As the input beam Gaussian function $f_1(x)$ ((Hermite-Gaussian mode of zero order), Hermite-Gaussian mode of the fourth order $f_2(x)$, and rectangular function $f_3(x)$:

$$\begin{aligned} &f_1(x) = \exp\left(-x^2 / 2\sigma_f^2\right), \\ &f_2(x) = \exp\left(-x^2 / 2\sigma_f^2\right) H_4\left(x / \sigma_f\right), \\ &f_3(x) = \operatorname{rect}\left(x / 2\gamma\right) = \begin{cases} 1, & |x| \le \gamma, \\ 0, & |x| > \gamma, \end{cases} \end{aligned} \quad (13)$$

where $\sigma_f = 1mm$, $\gamma = 30mm$, $H_4\left(x / \sigma_f\right)$ is the fourth Hermite polynomial.

The results are shown in Figures 3, 4, and 5, respectively, in comparison with the propagation of the field in free space without irregularities.

5 Modeling of the singular Gaussian beams propagation

In analogy to the one-dimensional case we can consider the distribution of optical beams in two-dimensions variant. The difference will consist only in the fact that the corresponding one-dimensional transformations are replaced by the two-dimensional ones.

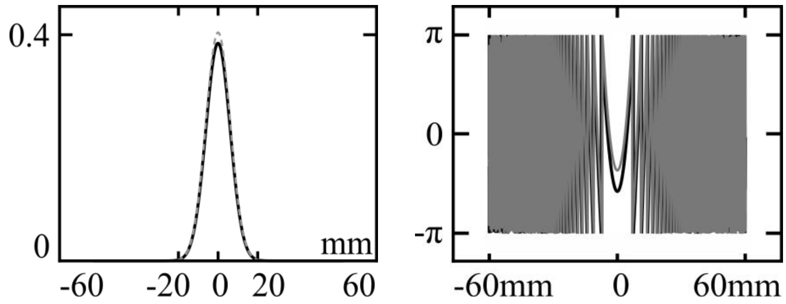

Fig. 3. Propagation of a Gaussian beam in free space (gray graph) and in a turbulent medium (black graph): the amplitude and phase

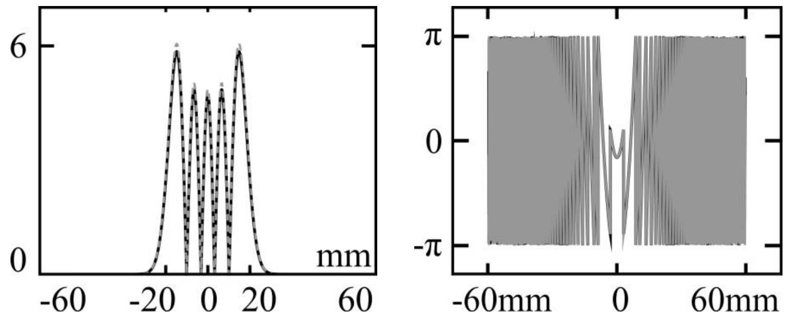

Fig. 4. Propagation of Hermite-Gauss mode in free space (gray graph) and in a turbulent medium (black graph): the amplitude and phase

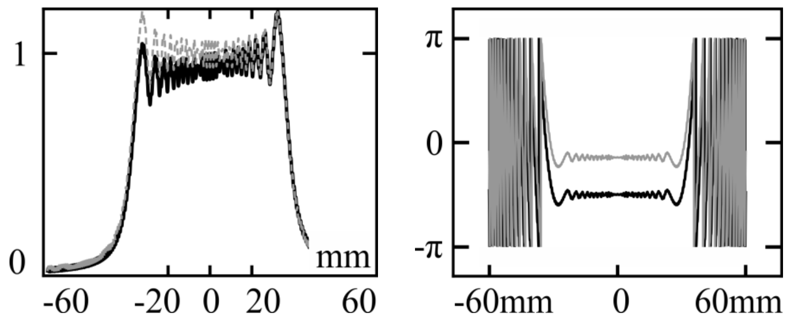

Fig. 5. Rectangular signal propagation in free space (gray graph) and in a turbulent medium (black graph): the amplitude and phase

Figure 6 shows an example of generating a random two-dimensional field with correlation function in the form of Gaussian function. Figure 7 shows an image of a sample correlation function.

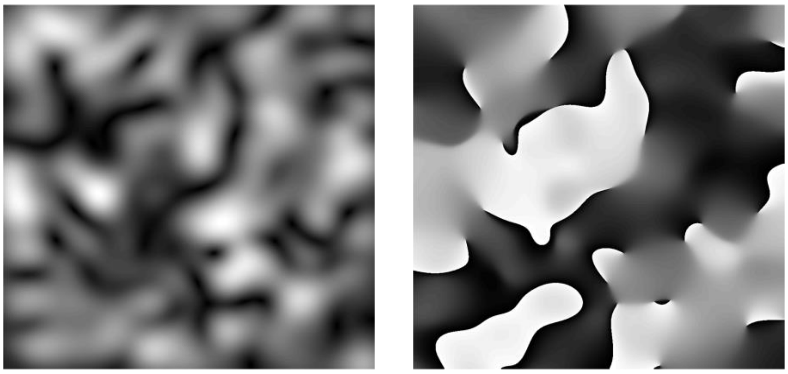

Fig. 6. Generation of a random two-dimensional field: amplitude and phase

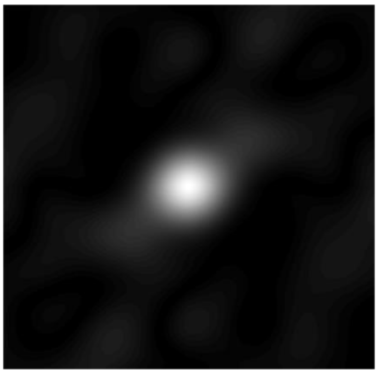

Fig. 7. Sample correlation function of the two-dimensional field

Hermite-Gauss mode is selected as an example of the optical beam propagation through random field [26-28]. The result is shown in Figure 8.

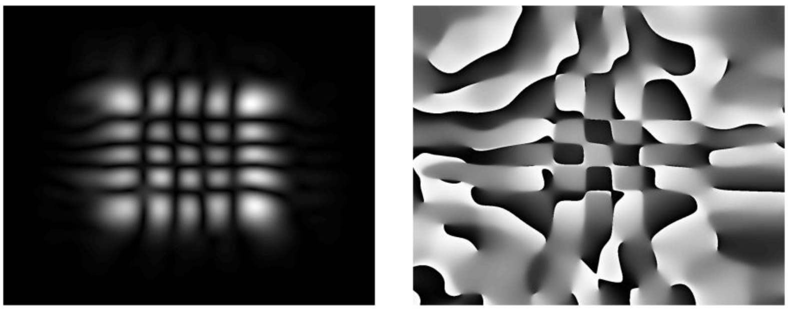

Fig. 8. Hermite-Gauss mode after passing through the random field: amplitude and phase

In addition to the extended Huygens-Fresnel principle (1), there is a different approach for modeling random medium, based on the scheme with screens with a random phase [29, 30].

Table 1 shows the results of comparative modeling of propagation of the fundamental Gaussian beam and of vortex Laguerre-Gaussian laser beams [26, 31, 32] in a random medium. A lot of thin phase screens with a random distribution, separated by a free space, were used for modeling. Thus, the beam periodically acquires random phase changes in the thin screen, passes part of the way in free space. During the modeling the screens with uniform random phase noise in the range of $[0, \pi]$ were located every 1.5 km of the passage way of 15 km.

As follows from the given results of the modeling, at a distance of 15 km the sizes of the Gaussian and of the vortex beams become virtually identical, although originally the Gaussian beam was more compact. Note that the vortex phase structure of the Laguerre-Gaussian beam remains sufficiently expressed despite the significant noise and the distance covered.

The stability of the vortex beam to turbulent impact was noted in the work [13], where it was stated that the vortex beam can "split, deviate, wander" outside the area of the detector, but will never disappear. In this work it was shown that a vortex beam of the fifth order is stored in the turbulent medium for over 2 kilometers, and is then splits into first order vortices that are lasting longer than 10 kilometers.

6 Conclusion

The operator, based on extended Huygens-Fresnel principle, of optical beam propagation through turbulence media was considered. The generation of a random field describing the inhomogeneous media with a given correlation function is implemented by linear filtering of white noise in the spectral domain. Deviation between the calculated correlation function and the given correlation function showed good performance of this algorithm. The computations of intensity distributions of Gaussian beams of high order were performed in the free space and random media.

Less broadening of the vortex beam in comparison with fundamental beam was shown based on comparative modeling of fundamental Gaussian beam and vortex laser beam propagation in random media.

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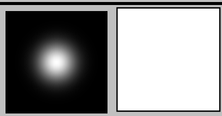
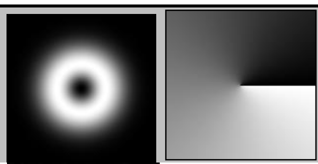
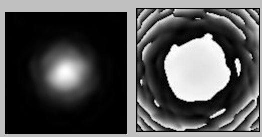
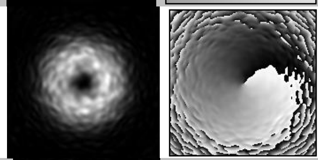
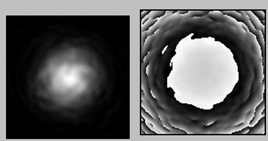
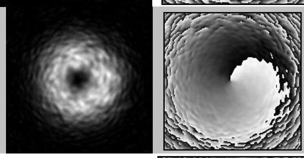
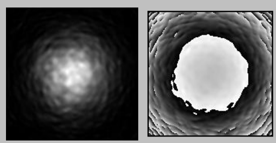
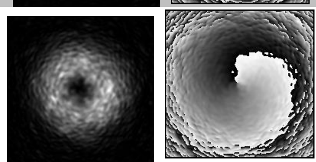
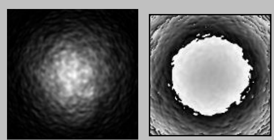
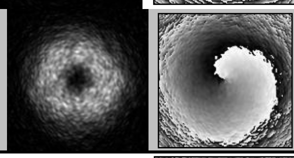
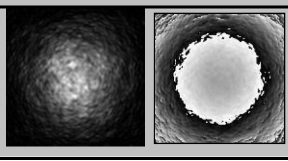
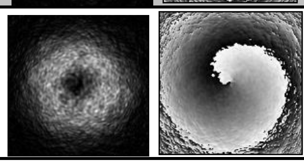
Table 1. Results of modeling of laser beam propagation with periodical phase noising of complex beam distribution

Intensity and phase distributions at a distance z	Gaussian beam	Vortex Laguerre-Gaussian beam
z = 0		
z = 3 km		
z = 6 km		
z = 9 km		
z = 12 km		
z = 15 km		

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