THE CONSTRUCTION OF THE OBSERVERS FOR DYNAMIC SYSTEMS WITH FAST AND SLOW VARIABLES

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Abstract. The estimation problem for dynamic systems with several time scales is considered. The method of asymptotic decomposition is used to reduce dimension and to simplify the structure of the observers.

Keywords: multirate dynamic systems, estimation problem

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Introduction

The existence of the processes with essentially different velocities is a feature of the complex dynamic systems. The mathematical models of such systems are the singularly perturbed differential systems, which contain one or several small parameters at some derivatives. The problems of numerical analysis and control of such systems are very difficult due to high dimension and multirate components.
For analyzing of the dynamic systems' behavior and making control law we must measure the state vector (phase vector). In real problems the measuring of the state vector is difficult due to technical or economic reasons. Furthermore the measurement devices have a complex structure and can modify the dynamic of the control object.
It makes relevant the application of indirect state estimation methods. The most widespread approach to the estimation problem is the creation of the system which is called "observer". Any solution of such system tends to the solution of initial system. We will consider two types of observers: full order observer and lower order observer (Luenberger observer). We analyzed the features of structure of observers for linear dynamic systems with slow and fast variables. We created the algorithm of the construction of the observers for such systems, which is based on the method of asymptotic decomposition.

Observers

Let us consider a linear dynamic model

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{1}$$

where state vector $x(t) \in R^n$, input vector $u(t) \in R^m$, output vector $y \in R^l$,

$$y(t) = C(t)x(t), \tag{2}$$

C is ($l \times n$)-matrix of measurements.
The so called "full order observer" is

$$\dot{m}(t) = A(t)m(t) + B(t)u(t) + V(t)(y(t) - C(t)m(t)), \tag{3}$$

where ($m \times l$)-matrix $V(t)$ is chosen to guarantee the asymptotic convergence of the state estimation error $\Delta(t) = x(t) - m(t)$.
For autonomous systems any eigenvalue of matrix $(A - VC)$ must satisfy the inequality $\operatorname{Re} \lambda_j(A - VC) < 0$.
Let $rank(C) = l$, $((n - l) \times n)$– matrix W be such that $(n \times n)$–matrix

$$Q = \begin{pmatrix} C \\ W \end{pmatrix}$$

is nonsingular. Write matrix Q^{-1} in the form $Q^{-1} = (R \;\; D)$, where R is $(n \times l)$–matrix, D is $(n \times (n - l))$–matrix.
The so called "lower order observer" (Luenberger observer) dynamic is

$$m(t) = D\alpha(t) + (R + DV)y(t), \tag{4}$$

where

$$\dot{\alpha} = (W - VC)[AD\alpha + Bu + A(DV + R)y], \quad \alpha(0) = \alpha^0. \tag{5}$$

The state estimation error Δ satisfies the equation

$$\dot{\Delta}(t) = D(W - VC)A\Delta(t). \tag{6}$$

For autonomous systems any eigenvalue of matrix $(W - VC)AD$ must satisfy the inequality $\operatorname{Re} \lambda_j((W - VC)AD) < 0$.

Asymptotic decomposition of linear singularly perturbed systems

Let us consider a system

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2, \\ \varepsilon\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 \end{aligned} \tag{7}$$

with output vector

$$y = C_1x_1 + C_2x_2, \tag{8}$$

where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$, $y \in R^l$, ε – small positive parameter,

$$A_{ij} = A_{ij}(t, \varepsilon) = A_{ij}^{(0)}(t) + \varepsilon A_{ij}^{(1)}(t) + \dots \; ,$$

$C = (C_1 \quad C_2)$ is the matrix of measurements. Let any eigenvalue of matrix $A_{22}^{(0)}(t)$ satisfies the inequality $\operatorname{Re} \lambda_j(A_{22}^{(0)}(t) < 0$.
One of the approaches that allows to reduce the complex multirate dynamic systems is the asymptotic decomposition method is based on the theory of integral manifolds [1-9]. This method combines elements of geometric and asymptotic methods of analysis.
Using a coordinate transformation

$$x_2 = z + Lx_1, \quad x_1 = v + \varepsilon Pz, \tag{9}$$

where $L = L(t,\varepsilon)$, $P = P(t,\varepsilon)$ are matrix functions, which satisfy the equations

$$\varepsilon\dot{L} + \varepsilon L A_{11} + \varepsilon L A_{12} L = A_{21} + A_{22} L,$$

$$\varepsilon\dot{P} + P A_{22} - \varepsilon P L A_{12} = \varepsilon A_{11} P + A_{12} + \varepsilon A_{12} L P,$$

we can transform the system (7) to the block diagonal form

$$\dot{v} \quad = \quad A_1 v, \quad \varepsilon\dot{z} = A_2 z, \tag{10}$$

where $A_1 = A_1(t,\varepsilon) = A_{11} + A_{12}L, \quad A_2 = A_2(t,\varepsilon) = A_{22} - \varepsilon L A_{12}$.
It can be proved that matrix functions $L,\ P$ may be constructed with any degree of accuracy as asymptotic series in small parameter ε

$$L = L(t,\varepsilon) = L^{(0)}(t) + \varepsilon L^{(1)}(t) + \ldots,$$

$$P = P(t,\varepsilon) = P^{(0)}(t) + \varepsilon P^{(1)}(t) + \ldots,$$

where

$$\begin{aligned}
L^{(0)} &= -\left(A_{22}^{(0)}\right)^{-1} A_{21}^{(0)},\\
L^{(1)} &= -\left(A_{22}^{(0)}\right)^{-1}\left[A_{21}^{(1)} + A_{22}^{(1)} L^{(0)} - L^{(0)} A_{11}^{(0)} - L^{(0)} A_{12}^{(0)} L^{(0)} - \dot{L}^{(0)}\right],\\
P^{(0)} &= A_{12}^{(0)}\left(A_{22}^{(0)}\right)^{-1},\\
P^{(1)} &= (A_{12}^{(1)} + A_{11}^{(0)} P^{(0)} + A_{12}^{(0)} L^{(0)} P^{(0)} - P^{(0)} A_{22}^{(1)}\\
&+ P^{(0)} L^{(1)} A_{12}^{(1)} - \dot{P}^{(0)})\left(A_{22}^{(0)}\right)^{-1}.
\end{aligned}$$

The output vector y takes the form $y = \tilde{C}_1 v + \tilde{C}_2 z$, were

$$\tilde{C}_1 = C_1 + C_2 L, \quad \tilde{C}_2 = \varepsilon C_1 P + C_2 + \varepsilon C_2 L P.$$

Let

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & \varepsilon^{-1} A_2 \end{pmatrix}.$$

Let us construct the full order observer for block diagonal system (10) as (3) with $u = 0$. Let

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}.$$

Then the observer takes the form

$$\begin{pmatrix} \dot{m}_v \\ \dot{m}_z \end{pmatrix} = \begin{pmatrix} A_1 - V_1\tilde{C}_1 & -V_1\tilde{C}_2 \\ -V_2\tilde{C}_1 & A_2\varepsilon^{-1} - V_2\tilde{C}_2 \end{pmatrix}\begin{pmatrix} m_v \\ m_z \end{pmatrix} + \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} y. \tag{11}$$

Since all eigenvalues of the matrix $A_{22}^{(0)}(t)$ satisfy inequalities $\operatorname{Re}\lambda_j < 0$, we can take $V_2 = 0$.
The system (11) takes the block-triangular form

$$\begin{pmatrix} \dot{m}_v \\ \varepsilon\dot{m}_z \end{pmatrix} = \begin{pmatrix} A_1 - V_1\tilde{C}_1 & -V_1\tilde{C}_2 \\ 0 & A_2 \end{pmatrix}\begin{pmatrix} m_v \\ m_z \end{pmatrix} + \begin{pmatrix} V_1 \\ 0 \end{pmatrix} y.$$

We can choose the block V_1 from the condition of asymptotic stability of the slow subsystem, which for an autonomous system takes the form

$$\operatorname{Re}\lambda_j(A_1 - V_1\tilde{C}_1) < 0. \tag{12}$$

For the estimation of state vector of the initial system we have

$$m_1 = m_v + \varepsilon P m_z, \quad m_2 = m_z + L m_1.$$

Let us construct the full order observer for slow subsystem of the block diagonal system (10)

$$\begin{aligned} \dot{v} &= A_1 v, \\ y &= \tilde{C}_1 v \end{aligned} \tag{13}$$

as

$$\dot{n_v} = (A_1 - V_1\tilde{C}_1)n_v + V_1 y.$$

We can choose the block V_1 from the condition of the asymptotic stability which for an autonomous system has a form of the inequalities (12).
It can be proved that

$$\lim_{t\to\infty} \|n_v - m_v\| = 0.$$

As an estimation of fast variable we can use any solution of the system

$$\varepsilon \dot{m}_z = A_2 m_z.$$

For estimation of state vector of the initial system we have

$$m_1 = n_v + \varepsilon P m_z, \quad m_2 = m_z + L m_1.$$

The similar reasoning can be used for construction of the Luenberger observer. For the slow subsystem (13) we choose matrix W such that matrix $Q = \begin{pmatrix} \tilde{C}_1 \\ W \end{pmatrix}$ is nonsingular. Let $Q^{-1} = (R \;\; D)$. The estimations of state vector take the form

$$m_1 \;\; = \;\; D\alpha + (R + DV_1)y, \quad m_2 = m_z,$$

where

$$\dot{\alpha} = (W - V_1\tilde{C}_1)(A_1 D\alpha + A_1(DV_1 + R)y), \quad \alpha(0) = \alpha_0,$$

$$\varepsilon \dot{m}_z = A_2 m_z.$$

Aircraft model

Consider the model of a longitudinal motion of an aircraft, see Figure 1, [10]

$$\begin{aligned} \ddot{v} &= d_1\alpha - d_2\delta \\ \dot{\theta} &= d_3\alpha, \\ T\dot{\delta} + \delta &= K_{rm} u. \end{aligned} \tag{14}$$

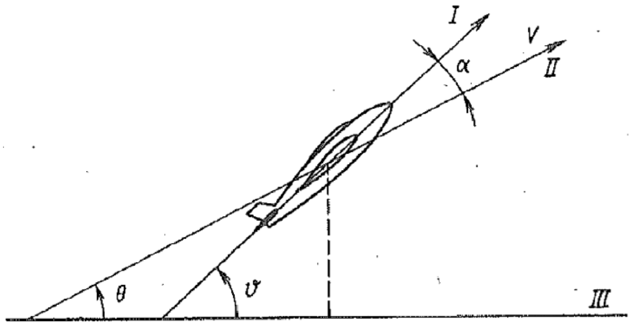

Fig. 1. Aircraft model

where ν is a pitch angle, θ is a flight path angle, $\alpha = v - \theta$ is an angle of attack, δ is a deviation of the elevator, d_i is the aerodynamic coefficients, T and K_{rm} are the characteristics of control–surface actuator.
The typical values of the parameters are $d_1 = 36$, $d_2 = 18$, $d_3 = 1.2$, $T = 0.1$.
Let $\varepsilon = T$ and

$$x_1 = \begin{pmatrix} \dot{\nu} \\ \nu \\ \theta \end{pmatrix}, \quad x_2 = \delta.$$

The system (14) takes the form

$$\begin{aligned} \dot{x_1} &= A_{11}x_1 + A_{12}x_2 \\ \varepsilon\dot{x_2} &= -x_2, \end{aligned} \tag{15}$$

where

$$A_{11} = \begin{pmatrix} 0 & -d_1 & d_1 \\ 1 & 0 & 0 \\ 0 & d_3 & -d_3 \end{pmatrix} \quad A_{12} = \begin{pmatrix} -d_2 \\ 0 \\ 0 \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, \quad A_{22} = -1.$$

Let the outputs be ν and θ, then

$$y = C\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Using the coordinate transformation

$$x_2 = z, \quad x_1 = v + \varepsilon P z,$$

where $P = P(\varepsilon)$ is the matrix function, which satisfies the equation

$$-P = \varepsilon A_{11} P + A_{12},$$

we can transform the system (15) to the block diagonal form

$$\dot{v} = A_{11}v, \quad \varepsilon\dot{z} = -z. \tag{16}$$

The matrix function $P(\varepsilon)$ may be constructed with any degree of accuracy as asymptotic series in small parameter ε

$$P = P(\varepsilon) = P^{(0)} + \varepsilon P^{(1)} + \ldots,$$

where

$$P^{(0)} = -A_{12} = \begin{pmatrix} d_1 \\ 0 \\ 0 \end{pmatrix}, \quad P^{(1)} = -A_{11}P^{(0)} = \begin{pmatrix} 0 \\ -d_2 \\ 0 \end{pmatrix}.$$

Output vector y takes the form

$$y \;=\; \tilde{C}\begin{pmatrix} v \\ z \end{pmatrix},$$

were

$$\tilde{C} = (\tilde{C}_1 \quad \tilde{C}_2), \quad \tilde{C}_1 = C_1, \quad \tilde{C}_2 = \varepsilon C_1 P + C_2,$$

$$\tilde{C}_1 = C_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{C}_2 = \begin{pmatrix} -\varepsilon^2 d_2 \\ 1 \end{pmatrix}$$

Let us construct the full order observer for the slow subsystem of the block diagonal system (16)

$$\begin{aligned} \dot{v} &= A_{11}v, \\ y &= \tilde{C}_1 v, \end{aligned}$$

in the form

$$\dot{n}_v = (A_1 - V_1\tilde{C}_1)n_v + V_1 y.$$

We can choose the block V_1 from the condition of the asymptotic stability in the form

$$V_1 = \begin{pmatrix} a & 0 \\ b & 0 \\ c & 0 \end{pmatrix}.$$

Then the estimation of the state vector v must satisfy the equation

$$\dot{n}_v = \begin{pmatrix} 0 & -d_1 & d_1 - a \\ 1 & 0 & -b \\ 0 & d_3 & -d_3 - c \end{pmatrix} n_v + \begin{pmatrix} a \\ b \\ c \end{pmatrix} y_1(t).$$

For example, let us put $a = 0$, $b = 1$, $c = 1$.
As an estimation of the fast variable z we can use any solution of the system

$$\varepsilon \dot{m}_z = -m_z.$$

For the estimation of the state vector of the initial system we have

$$m_1 = n_v + \varepsilon P m_z, \quad m_2 = m_z.$$

The Figures 2 – 5 demonstrate the dynamic of the state vector and its estimation. Similar reasoning can be used for construction the Luenberger observer.

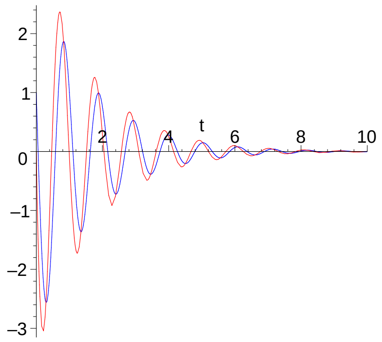

Fig. 2. $\dot{v}$, $m_{\dot{v}}$

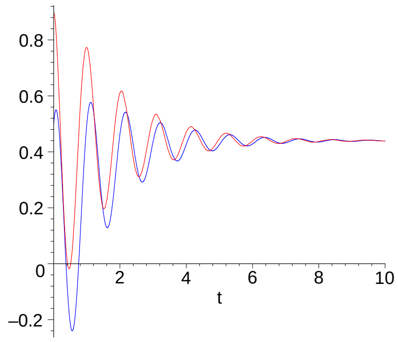

Fig. 3. v, m_v

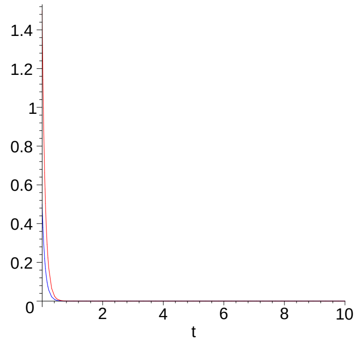

Fig. 4. δ, m_δ

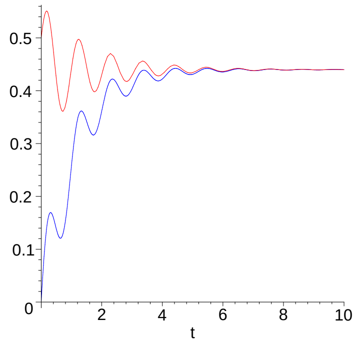

Fig. 5. θ, m_θ

For slow subsystem we notice that the second row of matrix $\tilde{C}_1$ is zero. We choose matrix W such that matrix

$$Q = \begin{pmatrix} \bar{C}_1 \\ W \end{pmatrix}, \text{ where } \bar{C}_1 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix},$$

is nonsingular. For example,

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Write matrix Q^{-1} in the form $Q^{-1} = (R \;\; D)$, where

$$R = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

The estimation of the vector v takes the form

$$m_v \quad = \quad D\alpha + (R + DV_1)y,$$

where

$$\dot{\alpha} = (W - V_1\bar{C}_1)(A_{11}D\alpha + A_{11}(DV_1 + R)y), \quad \alpha(0) = \alpha_0.$$

Let

$$V_1 = \begin{pmatrix} a \\ b \end{pmatrix}.$$

We have

$$(W - V_1\bar{C}_1)A_{11}D = \begin{pmatrix} -a & d_1 \\ -b & -d_3 \end{pmatrix}.$$

For example, let us put $a = 0$, $b > 0$.
As an estimation of the fast variable we can use any solution of the system

$$\varepsilon \dot{m}_z = -m_z.$$

For the estimation of the state vector of the initial system we have

$$m_1 = m_v + \varepsilon P m_2, \quad m_2 = m_z.$$

The Figures 6–9 demonstrate the dynamic of the state vector and its estimation.

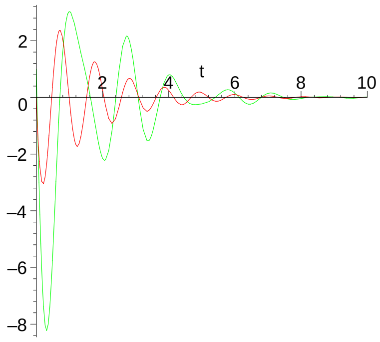

Fig. 6. $\dot{v}$, $m_{\dot{v}}$

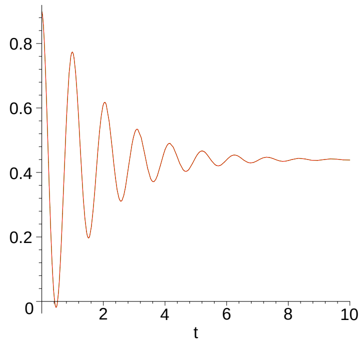

Fig. 7. v, m_v

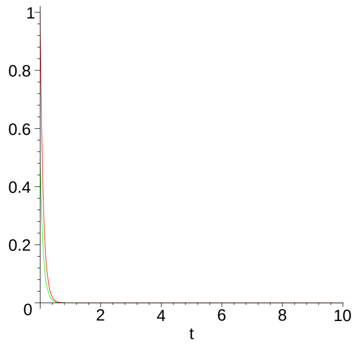

Fig. 8. δ, m_δ

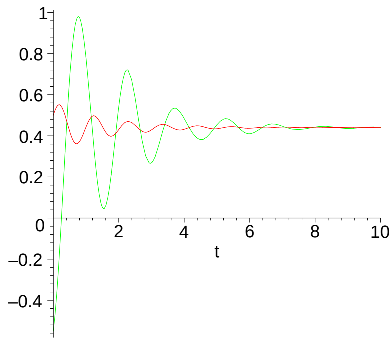

Fig. 9. θ, m_θ

Conclusion

The asymptotic decomposition method helped us to reduce the observation problems for the dynamic systems with slow and fast variables. This approach can be also used for solving the observation problems in a stochastic case.

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