The Yoneda Path to the Buddhist Monk Blend

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Abstract. Mazzola et al. propose a metaphor to describe the process by means of which an open question is solved in a creative way, likening it to the manner by which, due to the Yoneda Lemma of category theory, the internal structure of any object is completely characterised by a diagram of certain selected objects and morphisms. Building upon this metaphor, we explore how the creativity underlying conceptual blending could be characterised by diagrams of image schemas. We further illustrate this by providing a formalisation of the Buddhist Monk riddle and of the blend that solves it, together with a computational realisation that views conceptual blending as an amalgam-based process of generalisation and colimit computation.

Keywords. Conceptual blending, image schemas, Yoneda Lemma, Buddhist Monk riddle.

1. Introduction

The theory of conceptual blending (aka conceptual integration) has been proposed as a fundamental cognitive operation underlying much of everyday thought and language [7,8]. It models the way by which several mental spaces—small conceptual packets constructed as we think and talk for purposes of local understanding and action—are blended into a novel space by combining the particular elements and their relations pertaining to the initial spaces into a new whole, which eventually is more than the sum of spaces. Conceptual blending has been thoroughly used as an analytic tool for understanding the origin of ideas and concepts *a posteriori* [27], but it has also been proposed as a basis for computational models of creativity, using the theory to guide the design and implementation of algorithms for generating novel ideas and concepts [28,24,11,12,6].

According to Fauconnier and Turner's model [7,8], a conceptual blend is determined by a network of relationships between mental spaces that serve as input to the blend and generic spaces that encode the common structure underlying the input spaces. The prototypical network consists of two input spaces and one generic space, modelling the cross-space mapping between shared structure in input spaces, but more complex blends require more complex networks (see, e.g., the *megablends* reported in Chapter 8 of [8]).

Goguen has been the first to suggest a mathematical framework for conceptual blending, by applying techniques from category theory. He proposed to model blending

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as a certain kind of generalised colimit in preordered categories, with the generalised pushout of two inputs and one generic space as the prototypical case of blending [9]. Bou et al. have built upon Goguen's intuitions and proposed a uniform model, also based on category theory [3], that includes the closely related notion of *amalgam* initially proposed for reasoning about cases in Case-Based Reasoning [23]. This latter approach has also been given a computational realisation based on Answer-Set Programming [6].

Still, an important issue of conceptual blending, if it is to be used as a generative technique for concept invention and computational creativity, is that of determining the mental spaces to be blended and the generic spaces that encode their common structure, as well as the relationships between those spaces. Generic techniques such as structure mapping and anti-unification have been proposed [12], and several examples of blending have been re-created using these techniques [4,5]. In this paper, we explore an alternative approach, one that is grounded on an embodied understanding of cognition, and we shall pay attention to the image schemas that underlie the concept invention process. To this aim, we take inspiration from a metaphor for the creative process proposed by Mazzola et al. [20], and which draws from the insights of the Yoneda Lemma in category theory.

2. On the Yoneda-Based Creative Process

In order to better grasp the metaphor which sees the creative process in the light of the Yoneda Lemma, we will need to first give a very brief overview of this lemma and its implications, before moving on to present the metaphor. For this, some basic knowledge of category theory is assumed. But it is not necessary to fully comprehend the meaning of the Yoneda Lemma for the purpose of understanding this paper.

2.1. The Yoneda Lemma

The Yoneda Lemma is a technically simple result but with deep implications in category theory. It implies that, for any category $\mathscr{C}$ (i.e., a collection of mathematical objects such as sets, graphs, groups, etc., and its structure-preserving mappings), a $\mathscr{C}$-object C can be completely characterised by how it relates via structure-preserving mappings —called morphisms— to all other objects in the category; That is, an object can be understood either by understanding *all* the $\mathscr{C}$-morphisms from C to *any other* $\mathscr{C}$-object X, or by understanding *all* the $\mathscr{C}$-morphisms from *any other* $\mathscr{C}$-object X to C. In addition, $\mathscr{C}$-morphisms from C to D can be completely characterised by focusing on how all morphisms from and to C are transformed into morphisms from and to D.

A major task in category theory is to identify a subcategory $\mathscr{A}$ of $\mathscr{C}$, such that, with $\mathscr{A}$-objects only, we have enough to characterise all $\mathscr{C}$-objects. Of particular interest are subcategories $\mathscr{A}$ for which every $\mathscr{C}$-object C is (structurally isomorphic to) a colimit of a diagram of $\mathscr{A}$-objects and $\mathscr{A}$-morphisms. (The colimit is the $\mathscr{C}$-object that includes all structure —and not more— present in the $\mathscr{A}$-objects of the diagram, in a way that it respects the relationships between $\mathscr{A}$-objects as given by the $\mathscr{A}$-morphisms of the diagram.) Such a subcategory is called *dense* and the $\mathscr{A}$-diagrams whose colimits are (structurally isomorphic to) $\mathscr{C}$-objects are said to be *canonical*. Canonical diagrams externalise, as it were, the internal structure of an object, in the form of a diagram that only uses objects of the dense subcategory. Let's look at an example:

Source Domain CATEGORY THEORY		*Target Domain* CREATIVITY
To understand the object A	$\to$	Exhibiting the open question
Category $\mathscr{C}$	$\to$	Identifying the semiotic context
A	$\to$	Finding the question's critical sign or concept in the semiotic context
The uncontrolled behaviour of $\mathrm{Hom}(-,A)$	$\to$	Identifying the concept's walls
Finding a dense subcategory $\mathscr{A}$	$\to$	Opening the walls
Find the canonical colimit C for A	$\to$	Displaying its new perspectives
To understand A via the isomorphism $C \cong A$	$\to$	Evaluating the extended walls

Figure 1. The Yoneda-Based Creative Process Metaphor

Take the category **Grf** of (directed) graphs and graph homomorphisms. Let G_V be the graph consisting of a single vertex and no edges, and let G_E be the graph consisting of two vertices and a single edge from one vertex to the other.
For any graph G, the set $\mathrm{Hom}(G_V, G)$ of graph homomorphism from G_V to G is isomorphic to the set of G's vertices, and the set $\mathrm{Hom}(G_E, G)$ of graph homomorphism from G_E to G is isomorphic to the set of G's edges.
For the category **Grf**, the subcategory $\mathscr{A}$ consisting of the two graphs G_V and G_E and graph homomorphisms $s : G_V \to G_E$ (which sends the unique vertex in G_V to the vertex at the source of the unique edge in G_E) and $t : G_V \to G_E$ (which sends the unique vertex in G_V to the vertex at the target of the unique edge in G_E) is dense, and each graph can be characterised as a diagram with objects and morphisms from $\mathscr{A}$.
Let, for instance, G be the graph

$$a \xrightarrow{f} b \xrightarrow{g} c$$

The canonical diagram whose colimit is isomorphic to G has the following shape:

$$G_V \xrightarrow{s} G_E \xleftarrow{t} G_V \xrightarrow{s} G_E \xleftarrow{t} G_V$$

2.2. A Metaphor of Creativity

Mazzola et al. propose to take the insights offered by the Yoneda Lemma as a metaphor for the process by which an open question may be solved in a creative way [20]. They conjecture that, in the same manner in which a canonical diagram describes an object by making its internal structure explicit, it may well be that by externalising the inner workings of our open question we may gain new perspectives that yield a creative answer. These new perspectives are like the morphisms of the colimit of a canonical diagram, which project the constitutive elements of the diagram in a structure-preserving way onto the object under study.

Each of the metaphorical mappings that constitute this metaphor of the creative process, and which we illustrate in Figure 1 following the notational convention of [18], can also be seen as concrete steps of the process that the metaphor seeks to conceptualise [1].

3. Image Schemas

In the last decades, researchers in cognitive science have advocated for the central role that the embodied mind plays in thought and language. Their main claim is that concepts are grounded on our bodily experience with the environment. We structure concepts and reason with them according to basic skeletal patterns that recur in our sensory and motor experience, called *image schemas* [15,17].

Quoting Johnson, *"an image schema is a recurring dynamic pattern of our perceptual interaction and motor programs that gives coherence and structure to our experience."* [15]. Hampe provides the following characterisations: Image schemas are *directly meaningful* ("experiential"/"embodied"), *pre-conceptual structures*, which arise from or are grounded in human recurrent bodily movements through space, perceptual interactions, and ways of manipulating objects; are highly *schematic gestalts* that capture the structural *contours* of sensory-motor experience, integrating information from multiple modalities; exist as *continuous* and *analogue* patterns *beneath* conscious awareness, prior to and independently of other concepts; and are both *internally structured* and highly *flexible* [13].

3.1. The PATH *Image Schema*

An example of image schema is PATH. This image schema —sometimes also referred to as the SOURCE-PATH-GOAL schema— is presented in slightly different ways in the literature, but in its simplest form it consists of: a *source* or starting point, a *goal* or end-point, and a *path* or sequence of contiguous locations connecting the source with the goal. As such, PATH is a specialisation of another image schema, called the LINK schema, which consists of two entities and a link between them. Here, we have a source and a goal location that are linked by a path.

Since the PATH schema arises from our bodily experience of moving about in space, it often includes the notion of a *trajector* moving from source to goal, and the associated notion of this trajector "being on the path". Furthermore, because our experience of moving about in space is tightly linked with our perception of time, often the temporal dimension is also included into the schema, so as to take into account that the trajector starts at a certain time at the source position and ends at a later time at the goal position, being on the path at any intermediate time instance.

The internal logic and built-in inferences of this schema allow us to state, for instance, that if somebody has travelled from A to B and from B to C, then he or she has travelled from A to C. We shall see a formalisation of this schema in Section 4.

3.2. Conceptual Blends of Image Schemas

Lakoff and Johnson identify several of these image schemas and show how they ground the meaning we give to abstract concepts and situations on our bodily experience through conceptual metaphors that project the structure of image schemas upon the new domains of thought we create [18,19]. Following this view of how conceptualisations might work, it is reasonable to conjecture that the internal structure of an abstract concept can eventually be described by mainly focusing on the image schemas upon which the concept is grounded, and on the conceptual metaphors that project the structure of these image schemas onto the concept under consideration. Situating this conjecture in the context of our previous discussion on the creative process proposed by Mazzola et al., we shall explore the idea that conceptual blends can be metaphorically understood as colimits over canonical diagrams of image schemas.

4. The Buddhist Monk Riddle

We conjecture that Mazzola et al.'s metaphor of the creative process might be useful as a guide to find out which diagrams are relevant as a basis for conceptual blending. To explore this conjecture we shall apply the steps of this metaphor to exhibit and solve the riddle of the Buddhist Monk, that was firstly discussed by Koestler to illustrate creativity [16]. It goes as follows:

> One morning, exactly at sunrise, a Buddhist monk began to climb a tall mountain. The narrow path, no more than a foot or two wide, spiralled around the mountain to a glittering temple at the summit. The monk ascended the path at varying rates of speed, stopping many times along the way to rest and to eat the dried fruit he carried with him. He reached the temple shortly before sunset. After several days of fasting and meditation, he began his journey back along the same path, starting at sunrise and again walking at variable speeds with many pauses along the way. His average speed descending was, of course, greater than his average climbing speed. Prove that there is a single spot along the path the monk will occupy on both trips at precisely the same time of day. [16, p. 183–184]

Koestler further states how he experienced his mental process of solving the riddle by superimposing the image of the ascending and descending monk as two figures that must meet at some point at some time [16, p. 184].

The Buddhist Monk riddle was later picked up by Fauconnier and Turner to describe their theory of conceptual blending and its constitutive elements [8]. The superimposition that Koestler mentions can be seen as an example of conceptual blending, where the mental image of the monk walking up the mountain is overlaid with the image of the monk walking down the same mountain on the same path. The composition of those images is then completed and elaborated with our experience that two objects that approach each other on the same path will necessarily meet at some point.

4.1. The Creative Process to Solve the Riddle

Our objective is to illustrate a step-by-step process by which to solve the riddle, by looking it at as a creative process—as modelled by Mazzola et al.—using their metaphor based on the Yoneda Lemma, together with the view of conceptual blends as amalgams, i.e., as a process of diagram generalisation and colimit computation. Let us, hence, follow the steps suggested by Mazzola et al. as they apply to the riddle of the Buddhist Monk.

Exhibiting the open question: Our open question is, obviously, the riddle. Metaphorically speaking, this would be the object A that we seek to understand, and that we will need to formalise it in some way. We will use the Common Algebraic Specification Language (CASL) [2], a general-purpose specification language based on first-order logic, which is supported by the tool set HETS [22]. This will allow us to represent conceptual integration networks as diagrams of CASL theories and to compute blends by colimit computation over these diagrams. Let us take, for instance, the BUDDHIST_MONK_SIGNATURE specification for expressing the Buddhist Monk riddle as

```
spec BUDDHIST_MONK_SIGNATURE =
then sorts Journey, Spot, MountainPath, Person,
      Time, Day, TimeOfDay
     ops  ascent, descent : Journey;
          foot, summit : Spot;
          narrow_path : MountainPath;
          monk : Person;
          startAscent, endAscent : Time;
          startDescent, endDescent : Time;
          day1, day2 : Day;
          sunrise, sunset : TimeOfDay;
          occupies : Journey × Person × Time
                                        → Spot
          timeOfDay : Time → TimeOfDay
     preds on : Spot × MountainPath
end
```

```
∃ s : Spot; t1, t2 : Time
 • on(s, narrow_path)
   ∧ occupies(ascent, monk, t1) = s
   ∧ occupies(descent, monk, t2) = s
   ∧ timeOfDay(t1) = timeOfDay(t2)
```

Figure 2. Signature and open question of the Buddhist Monk riddle.

shown in Figure 2 on the left. One way to formalise the open question of our riddle, given this signature, is shown in Figure 2 on the right.

Identifying the semiotic context: Since we use CASL to express the open question, and also the input and generic spaces relevant to solve the riddle, the category $\mathscr{C}$ we would be dealing with is the category of CASL theories and CASL theory morphisms [21].

Finding the question's critical sign or concept in the semiotic context. Identifying the concept's walls: We maintain that the *critical sign* or *concept* to solve the riddle is the relationship between time intervals and paths, because the open question is about positions on a path with respect to time instances as this path is traversed; so image schemas for these concepts are going to play a relevant role in order to solve the riddle. Following our metaphor, since we cannot work with the entire set $\mathrm{Hom}(-,A)$, we will need to focus solely on the relevant image schemas.

Opening the walls: Our claim is that the image schemas could play the role, in this metaphor, of the dense subcategory $\mathscr{A}$ as described in Section 2.1. We are not claiming that image schemas form a dense subcategory for the category $\mathscr{C}$ of CASL theories and CASL theory morphisms. This would be unfounded. But, still, we can draw some insights from The Yoneda-Based Creative Process Metaphor, and we conjecture that, if concepts—concrete and abstract—are eventually grounded on image schemas, then there might exist a sufficiently interesting collection of CASL specifications which should be fully described in terms of image schemas, in the same way categorical objects are fully described by objects from a dense subcategory.

For the particular example of the Buddhist Monk riddle, Figure 3 shows a specification of the image schemas that we found relevant. We made the following modelling decision. For each image schema, there is a generic specification of the general structure and logic of the schema (OBJECT, LINK and PATH on the left-hand side of Figure 3), and also an additional specification extending the generic one that introduces constants for the particular constitutive elements of the schema (AN_OBJECT, A_LINK and A_PATH on the right-hand side of Figure 3). This way we are able to have, in our diagrams, several distinct occurrences of an image schema, whose constitutive elements are kept separate,

```
spec OBJECT =
then sort  Object
end
```

```
spec AN_OBJECT = OBJECT
then ops   o : Object
end
```

```
spec LINK =
then sorts Link, Entity
     pred  linked : Entity × Link
     ∀ l : Link • ∃!2 e : Entity • linked(e,l)
end
```

```
spec A_LINK = LINK
then ops   e1, e2 : Entity
           l : Link
     • e1 ≠ e2
     • linked(e1,l)
     • linked(e2,l)
end
```

```
spec PATH = LINK with Entity ↦ Location
and  OBJECT with Object ↦ Trajector
and  INSTANT
then sorts Path
     ops   source, goal : Path → Location
           route : Path → Link
           trajector : Path → Trajector
           start, finish : Path → Instant
           position : Path × Trajector × Instant → Location
     preds on : Location × Link
     ∀p : Path
     • source(p) ≠ goal(p)
     • linked(source(p),route(p))
     • linked(goal(p),route(p))
     • position(p, trajector(p), start(p)) = source(p)
     • position(p, trajector(p), finish(p)) = goal(p)
     • ∀x : Instant
       • (start(p) < x ∧ x < finish(p))
         ⇒ on(position(p,trajector(p),x),route(p))
end
```

```
spec A_PATH = PATH
then ops   p : Path
           s,g : Location
           r : Link
           t : Trajector
           b,e : Instant
     • source(p) = s
     • goal(p) = g
     • route(p) = r
     • trajector(p) = t
     • start(p) = b
     • finish(p) = e
end
```

Figure 3. CASL specifications of image schemas. Quantification $\exists^{!2}$ in LINK stands for *there are exactly two*.

but which share the general structure and logic of the schema they are occurrences of. Additionally, we will need some rudimentary theory of time and time intervals as shown in Figure 4.

Displaying its new perspectives: Our objective is to describe the riddle, not as a stand-alone CASL theory, but by means of a diagram of image schemas, externalising, as it were, its internal structure. This externalisation is captured by a diagram—the canonical diagram, metaphorically speaking. Figure 5 shows the image-schematic structure of the riddle, with morphisms depicted in solid lines and specified in Figure 6. By computing the colimit of this diagram we should obtain a complete description of the riddle. In particular, we are interested in the CASL theory that is isomorphic to the theory at the apex of the colimit and that uses the signature BUDDHIST_MONK_SIGNATURE of the open question. This colimit is shown in Figure 5 with the morphisms depicted in dashed lines and specified in Figure 7. These morphisms constitute the Yoneda perspectives of the Buddhist Monk riddle. We can see the entire diagram as the conceptual integration network that represents a conceptual blend of several input spaces linked by several generic spaces in a bipartite graph.

```
spec INSTANT =
then sorts Instant
     preds __ < __ : Instant × Instant
                [irrefl., trans., total]
end
```

```
spec TIME =
then sorts Time, Day, TimeOfDay
     preds __ < __ : TimeOfDay × TimeOfDay
     ops   day : Time → Day
           timeOfDay : Time → TimeOfDay
end
```

```
spec AN_INTERVAL = INSTANT
then ops beginning, ending : Instant
     • beginning < ending
end
```

```
spec A_TOD_INTERVAL = TIME
then ops beginTime, endTime : Time
         beginDay, endDay : Day
         beginTOD, endTOD : TimeOfDay
     • beginTOD < endTOD
     • day(beginTime) = beginDay
     • timeOfDay(beginTime) = beginTOD
     • day(endTime) = endDay
     • timeOfDay(endTime) = endTOD
end
```

Figure 4. CASL specifications of a rudimentary theory of time instants and intervals

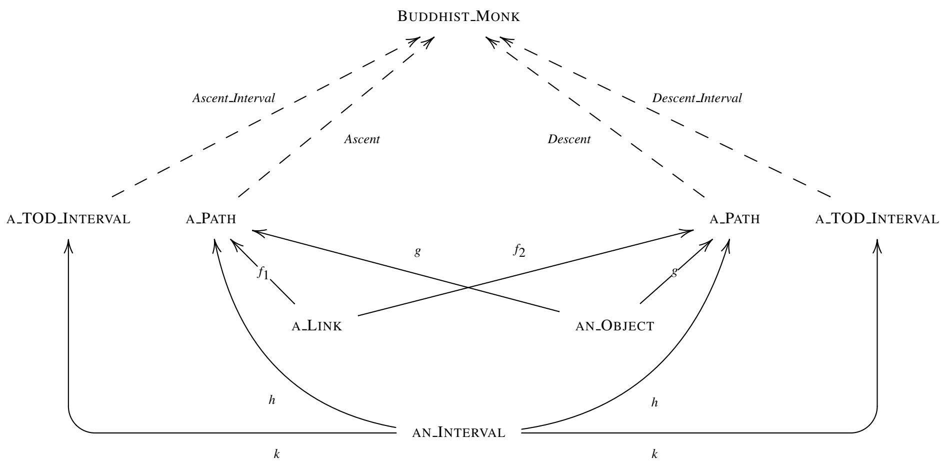

Figure 5. "Canonical" diagram (solid arrows) and colimit (dashed arrows) of the Buddhist Monk riddle

Evaluating the extended walls: The colimit theory computed given the diagram discussed so far does not help us to answer our open question. It does, however, make the image-schematic structure of the problem explicit for us to explore a way to solve the riddle. Here is where our intuition deviates slightly from Mazzola et al.'s, since our idea is to realise this exploration using the technique of conceptual blending as amalgam computation: to explore several generalisations of input and generic spaces (i.e., generalisation of the canonical diagram) until the colimit computation generates a blend that allows us to answer the open question.

view f_1 : A_LINK **to** A_PATH =
  $l \mapsto r$, $e_1 \mapsto s$, $e_2 \mapsto g$

view f_2 : A_LINK **to** A_PATH =
  $l \mapsto r$, $e_1 \mapsto g$, $e_2 \mapsto s$

view g : AN_OBJECT **to** A_PATH =
  $o \mapsto t$

view h : AN_INTERVAL **to** A_PATH =
  beginning $\mapsto b$, *ending* $\mapsto e$

view k : AN_INTERVAL **to** A_TOD_INTERVAL =
  Instant $\mapsto$ *TimeOfDay*,
  beginning $\mapsto$ *beginTOD*, *ending* $\mapsto$ *endTOD*

Figure 6. CASL signature morphisms of the "canonical" diagram

view *Ascent* : A_PATH **to** BUDDHIST_MONK =
  Path $\mapsto$ *Journey*, *Link* $\mapsto$ *MountainPath*, *Trajector* $\mapsto$ *Person*,
  $p \mapsto$ *ascent*, $s \mapsto$ *foot*, $g \mapsto$ *summit*, $r \mapsto$ *narrow_path*, $t \mapsto$ *monk*,
  $b \mapsto$ *sunrise*, $e \mapsto$ *sunset*

view *Ascent_Interval* : A_TOD_INTERVAL **to** BUDDHIST_MONK =
  beginTime $\mapsto$ *startAscent*, *endTime* $\mapsto$ *endAscent*,
  beginDay $\mapsto$ *day1*, *endDay* $\mapsto$ *day1*, *beginTOD* $\mapsto$ *sunrise*, *endTOD* $\mapsto$ *sunset*

view *Descent* : A_PATH **to** BUDDHIST_MONK =
  Path $\mapsto$ *Journey*, *Link* $\mapsto$ *MountainPath*, *Trajector* $\mapsto$ *Person*,
  $p \mapsto$ *descent*, $s \mapsto$ *summit*, $g \mapsto$ *foot*, $r \mapsto$ *narrow_path*, $t \mapsto$ *monk*,
  $b \mapsto$ *sunrise*, $e \mapsto$ *sunset*

view *Descent_Interval* : A_TOD_INTERVAL **to** BUDDHIST_MONK =
  beginTime $\mapsto$ *startDescent*, *endTime* $\mapsto$ *endDescent*,
  beginDay $\mapsto$ *day2*, *endDay* $\mapsto$ *day2*, *beginTOD* $\mapsto$ *sunrise*, *endTOD* $\mapsto$ *sunset*

Figure 7. Morphisms into the apex of the colimit

5. Solving the Riddle

In order to solve the riddle by "evaluating the extended walls" we will follow a general process that consists of the repeated application of (i) diagram generalisation, (ii) colimit computation, (iii) image-schematic completion and (iv) reasoning at a distance, until we reach a solution. Steps (i) and (ii) constitute the amalgam-based technique for computing conceptual blends described in [6], and we will not go further into the details in this paper. Step (iii) is how we imagine that blend completion could be realised. For the Buddhist Monk riddle in particular, we think it is cognitively more plausible that completion will draw from our sensory-motor experience that we have coded in form of image schemas than by including an abstract specification of the Intermediate Value Theorem, as done by Goguen in [10]. The final step (iv) is the one that allows us to answer the riddle in its original formulation, namely for one monk ascending and descending on different days, using the blend that involves two monks journeying on the same day. Let us see these steps in more detail for our example.

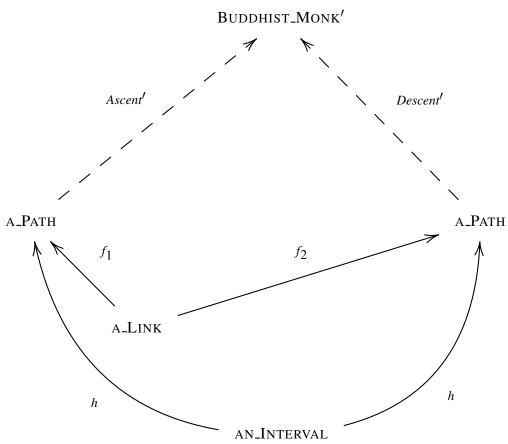

Figure 8. Generalised diagram (solid arrows) and colimit (dashed arrows) of the Buddhist Monk riddle

5.1. Diagram Generalisation

Two generalisations will be required to generate the right blend to be able to solve the Buddhist Monk riddle. The first generalisation forgets the calendrical time of the journey and focuses only on times of day as start and finishing times of the ascent and descent journeys, thus generalising the two occurrences of A_TOD_INTERVAL in our diagram to AN_INTERVAL, along morphism *k* (see Figure 6). This amounts to remove these two occurrences and those of morphism *k*. The second generalisation is to forget that the trajector—the monk— of both journeys, the ascent and the descent, are the same entity, by removing the AN_OBJECT image schema and the occurrences of morphism *g* that were responsible for the identification of trajectors in the apex of the colimit.

5.2. Colimit Computation

After these two generalisations our new colimit diagram looks as shown in Figure 8. Again, we are interested in the CASL theory that is isomorphic to the theory at the apex of the colimit and that uses the signature BUDDHIST_MONK_SIGNATURE of the open question. (To be precise, our signature will need two separate symbols for monks standing for two separate trajectors.)

We claim that the colimit specification over this generalised diagram allows us to solve the riddle, not immediately, but by completing it in the appropriate way. In [10] it was conjectured that such completion could be realised by an appropriate "meeting space" that specifies a suitable version of the Intermediate Value Theorem, and which is drawn into the colimit (see also [26]). We think that such meeting space, as suggested by Goguen, is too abstract a specification to play such direct role in solving the riddle. Instead, we conjecture that the generated colimit is completed with image-schematic structure of an extension of the PATH schema, namely the one that has two trajectors.

spec TWO_TRAJECTORS_PATH = PATH
then $\forall p_1, p_2 : Path; t_1, t_2 : Time; \forall x, y : Location$
- $t_1 \neq t_2 \wedge$
  $position(p1, trajector(p1), t_1) = x \wedge position(p1, trajector(p1), t_2) = y \wedge$
  $position(p2, trajector(p2), t_1) = y \wedge position(p2, trajector(p2), t_2) = x$
  $\Rightarrow \exists\, s : Location; t : Time$
  - $t_1 < t \wedge t < t_2 \wedge$
    $position(p1, trajector(p1), t) = s \wedge position(p2, trajector(p2), t) = s$

end

spec A_TWO_TRAJECTORS_PATH = TWO_TRAJECTORS_PATH
then ops $p_1 , p_2 : Path$
$s_1, s_2, g_1, g_2 : Location$
$r : Link$
$t_1, t_2 : Trajector$
$b, e : Time$

• $source(p_1) = s_1$	• $source(p_2) = s_2$
• $goal(p_1) = g_1$	• $goal(p_2) = g_2$
• $route(p_1) = r$	• $route(p_2) = r$
• $trajector(p_1) = t_1$	• $trajector(p_2) = t_2$
• $start(p_1) = b$	• $start(p_2) = b$
• $finish(p_1) = e$	• $finish(p_2) = e$

end

Figure 9. Extension of the PATH image schema with an additional trajector moving at the same time interval on the same route.

5.3. Image-Schematic Completion

Following Hedblom, Kutz and Neuhaus [14], our repository of images schemas (playing the role of dense subcategory in The Yoneda-Based Creative Process Metaphor) could include several variants of a same core image schema organised in a graph of theories, such as that suggested for PATH. The blending completion process would consist of detecting image-schema variants that could be partially mapped into the newly created space at the apex of the colimit diagram of Figure 8. In this case A_TWO_TRAJECTORS_PATH (see Figure 9) can be partially mapped onto BUDDHIST_MONK'. The specification of this image schema includes an additional axiom specifying the inherent logic of this extension; and a particular occurrence of this schema (A_TWO_TRAJECTORS_PATH) consists of two source-path-goal structures (with their respective source, target, and trajector) that share the route and the time interval in which trajectors move. By bringing into BUDDHIST_MONK' the remaining structure and logic of A_TWO_TRAJECTORS_PATH we will be able to deduce the open question from the resulting specification; not directly, but by moving the reasoning along the morphism induced by the generalisation of the "canonical" diagram, as we will see next.

5.4. Reasoning at a Distance

A diagram generalisation as the one described for the Buddhist Monk riddle is formally captured by a generalisation morphisms between diagrams. So, if diagram $\mathscr{D}$ is generalised to $\mathscr{D}'$, there will be a morphism (a monomorphism to be precise) from $\mathscr{D}'$ to $\mathscr{D}$.

This generalisation morphism induces a morphism f from the apex of the colimit of $\mathscr{D}$' to the apex of the colimit of $\mathscr{D}$. This morphism allows us to use the theory at the apex of the colimit of the generalised diagram to reason with the entities of the apex of the colimit of the original diagram. This is done as follows.

Let $\mathscr{D}$ be a diagram of CASL theories and let $\mathscr{D}$' a generalised diagram. Let B and B' be the CASL theories at the apexes of the colimits of $\mathscr{D}$ and $\mathscr{D}'$, respectively, and let $f : B' \to B$ be the morphisms induced by the diagram generalisation. Let φ be a sentence in the signature of B. We will say that $B \vdash_f \varphi$ when there exists a sentence φ', such that $\varphi = f(\varphi')$ and $B' \vdash \varphi'$.

For our Buddhist Monk riddle, let $\mathscr{D}$ be the base diagram of Figure 5, and $\mathscr{D}'$ that of Figure 8. The morphisms f : BUDDHIST_MONK$'$ → BUDDHIST_MONK will be the identity on all signature elements, except for the following cases:

$$f(monk_1) = monk \qquad f(t_1) = timeOfDay(t_1)$$
$$f(monk_2) = monk \qquad f(t_2) = timeOfDay(t_2)$$

Let φ be the open question as formalised in the step *Exhibiting the open question* given above. In order to see if this open question holds, we will need to check if there exists a φ' in the signature of BUDDHIST_MONK$'$ that holds for BUDDHIST_MONK$'$. Indeed such sentence exists, and it is:

$$\varphi' = \boxed{\begin{array}{l} \exists\, s : Spot;\, t_1, t_2 : Time \\ \bullet\ on(s, narrow\_path) \\ \quad \wedge\ occupies(ascent, monk_1, t_1) = s \wedge occupies(descent, monk_2, t_2) = s \wedge t_1 = t_2 \end{array}}$$

6. Implementation

The HEterogeneous ToolSet (HETS) [22] is a reasoning engine that supports the colimit computation of CASL theories and is capable of translating CASL to various input languages understood by theorem provers. The HETS GUI allows us to graphically represent the entire theory modelling the riddle (see Figure 8). The diagram shows that after computing the colimit BUDDHIST_MONK', this is completed with the A_TWO_TRAJECTORS_PATH instantiated schema and further elaborated by bringing in the open question, which is then proven to follow from BUDDHIST_MONK'.

In the current implementation, blend completion is realised as the union ('`and`' in CASL) of the A_TWO_TRAJECTORS_PATH and BUDDHIST_MONK' theories. It is worthy noticing that blend completion can also be seen as a process of conceptual blending, by which a generated blend is further blended with the image-schematic structure it is completed with. In Figure 8, for instance, A_TWO_TRAJECTORS_PATH could be blended with BUDDHIST_MONK' to obtain BUDDHIST_MONK_COMPLETION theory. This will amount to create another base diagram, whose inputs are BUDDHIST_MONK' and A_TWO_TRAJECTORS_PATH. This is left as future work. The current implementation is available at `https://ontohub.org/yoneda-path/monk-gen.casl`.

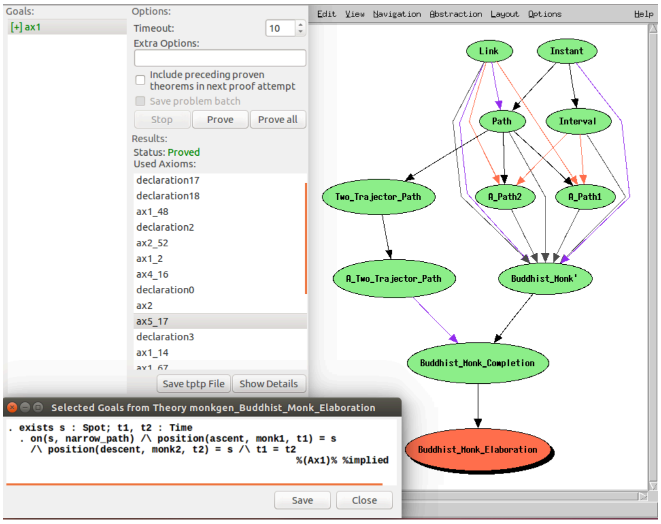

Figure 10. Generalised diagram and proof of the open question displayed in the HETS GUI.

7. Conclusion

We claimed that Mazzola et al.'s metaphor for the creative process can be useful to make explicit the external structure of the concept or idea we want to creatively explore. This metaphor likens the creative process to the task of finding a canonical diagram that externalises the structure of a categorical object. Besides Mazzola et al.'s model, there are other proposals for modeling cerativity in general, and concept invention in particular, such as [25], and it is worthwile to consider these as well. In this paper, however, we have explored the idea of applying Mazzola et al.'s metaphor to solve the Buddhist Monk riddle, by making explicit the image-schematic structure underlying the conceptual blends required to solve the riddle. Moreover, we have modelled the blend completion process in terms of incorporating additional relevant image-schematic structure that was initially not present in the problem specification, but which became relevant once the conceptual blends were realised. In our approach, the blending process itself is modelled as a amalgam-based process of generalisation and colimit computation.

As future work, we intend to further explore our approach in other domains, validating the hypothesis that a relevant collection of image schemas should be sufficient to model diagrams such that, via generalisation and colimit computation, yield the expected blends. Moreover, we surmise that for complex situations we will have not a blend but a web of blends, e.g., situations where one (or both) input mental spaces are recursively blended. Such a web of blends is called Hyper-Blending Web in [27]. We intend to explore the span of the hypothesis that the input concepts in such a web of blends are image schemas and their specialisations, while the blend concepts are created by generalisation and colimit computation of image schemas and previous blends in the web.

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