# Reversing Transitions in Bounded Petri Nets*


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**Abstract.** Reversible computation deals with mechanisms for undoing the effects of actions executed by a dynamic system. This paper is concerned with reversibility in the context of Petri nets which are a general formal model of concurrent systems. A key construction we investigate amounts to adding 'reverse' versions of selected net transitions. Such a static modification can severely impact on the behaviour of the system, e.g., the problem of establishing whether the modified net has the same states as the original one is undecidable. We therefore concentrate on nets with finite state spaces and show, in particular, that every transition in such nets can be reversed using a suitable set of new transitions.



**Keywords:** Petri net, reversibility, reversible computation


## 1 Introduction

Reversible computation deals with (typically local) mechanisms for undoing the effects of actions executed by a dynamic system. Such an approach has been applied, in particular, to various kinds of process calculi and event structures (see, e.g., [3–6, 8, 11, 12, 10]), and to a category theory based setting [7].


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This paper is concerned with reversibility in the context of Petri nets which are a general formal model of concurrent systems. A key construction we investigate amounts to adding ‘reverse’ versions of selected net transitions, e.g., a ‘straightforward’ reverse simply changes the directions of arcs adjacent to a transition being reversed. As shown in [2], such a static modification can severely impact on the behaviour of the system, e.g., the problem of establishing whether the modified net has the same states as the original one is undecidable.

We therefore concentrate in this paper on Petri nets with finite state spaces, more precisely bounded Place/Transition-nets (PT-nets). The state spaces of such nets can be represented by finite labelled transition systems (flts's) which are a convenient tool for specifying different variants of reversibility. One can therefore aim at synthesising a PT-net with ‘reversed’ behaviour given by an flts.

In this paper we show that it is, in general, impossible to reverse a transition using its straightforward reverse. What is more, the situation does not change if we relax the notion of a reverse by only requiring that the effect of its execution is opposite to that of the original transition. We therefore relax the requirement further, by allowing several reverses for a single transition. This leads to our main result that every transition in a bounded PT-net can be reversed using a suitable set of new transitions.

## 2 Preliminaries

**Transition systems**
A *finite labelled transition system* (or, simply, *flts*) is a tuple $TS = (S, T, \to, s_0)$ with a finite set of *states* $S$, a finite set of *labels* $T$, a finite set of *arcs* $\to \subseteq (S \times T \times S)$, and an *initial state* $s_0 \in S$.[4] A label $t$ is *fireable* at $s \in S$, denoted by $s[t\rangle$, if $(s, t, s') \in \to$, for some $s' \in S$. A state $s'$ is *reachable* from $s$ through the execution of $\sigma \in T^*$, denoted by $s[\sigma\rangle s'$, if there is a directed path from $s$ to $s'$ whose arcs are labelled consecutively by $\sigma$. The set of states reachable from $s$ is denoted by $[s\rangle$. A sequence $\sigma \in T^*$ is *fireable*, from a state $s$, denoted by $s[\sigma\rangle$, if there is some state $s'$ such that $s[\sigma\rangle s'$.

Let $t^\bullet_{TS} = \{s \in S \mid (s', t, s) \in \to,\ \text{for some}\ s' \in S\}$ and ${}^\bullet t_{TS} = \{s \in S \mid (s, t, s') \in \to,\ \text{for some}\ s' \in S\}$ be respectively the sets of all states having an incoming arc labeled with $t$, and an outgoing arc labeled with $t$. The set of all arcs labelled by $t$ is denoted by $\overrightarrow{t}$. We assume that each $\overrightarrow{t}$ is nonempty.

Two flts's, $TS_1 = (S_1, T, \to_1, s_{0_1})$ and $TS_2 = (S_2, T, \to_2, s_{0_2})$, are *isomorphic* if there is a bijection $\zeta \colon S_1 \to S_2$ with $\zeta(s_{0_1}) = s_{0_2}$ and $(s, t, s') \in \to_1 \Leftrightarrow (\zeta(s), t, \zeta(s')) \in \to_2$, for all $s, s' \in S_1$.

**Petri nets**
A *Place/Transition Petri net* (or, simply, *net*) is a tuple $N = (P, T, F, M_0)$,

[4] An flts may be considered as a finite automaton with no accepting states.

where $P$ is a finite set of *places*, $T$ is a finite set of *transitions* (or actions), $F$ is the *flow function* $F\colon ((P \times T) \cup (T \times P)) \to \mathbb{N}$ specifying the arc weights, and $M_0$ is the *initial marking* (where a marking is a mapping $M\colon P \to \mathbb{N}$, indicating the number of tokens in each place). A transition $t \in T$ is *enabled* at a marking $M$, denoted by $M[t\rangle$, if $M(p) \geq F(p,t)$, for all $p \in P$. The *effect* of a transition $t$ on a place $p$ is $\mathit{eff}_p(t) = F(t,p) - F(p,t)$. The firing of $t$ at marking $M$ leads to $M'$, denoted by $M[t\rangle M'$, if $M[t\rangle$ and $M'(p) = M(p) + \mathit{eff}_p(t)$ for every $p \in P$.

The notions of enabledness and firing, $M[\sigma\rangle$ and $M[\sigma\rangle M'$, are extended in the usual way to sequences $\sigma \in T^*$, and $[M\rangle$ denotes the set of all markings reachable from $M$. We assume that each transition is enabled in at least one reachable marking. There is a partial order relation $<$ on the markings of a Petri net defined so that $M \leq M'$ if $M(p) \leq M'(p)$, for every place $p \in P$. It is easy to observe that transition enabledness is *monotonic*, which means that if a transition $t$ is enabled at a marking $M$ and $M \leq M'$, then $t$ is also enabled at $M'$.

A Petri net $N = (P, T, F, M_0)$ net is *bounded* if $[M_0\rangle$ is finite, and its *reachability graph* is then defined as an flts

$$RG(N) = ([M_0\rangle, T, \{(M, t, M') \mid M, M' \in [M_0\rangle \wedge M[t\rangle M'\}, M_0).$$

If a labelled transition system $TS$ is isomorphic to the reachability graph of a Petri net $N$, then we say that $N$ *solves* $TS$, and $TS$ is *synthesisable* to $N$.

**Definition 1 (transition reverse).** *A* (strict) reverse *of a transition $t \in T$ in a net $N = (P, T, F, M_0)$ is a new transition $\underline{t}$ such that $F(p, \underline{t}) = F(t, p)$ and $F(\underline{t}, p) = F(p, t)$. An* effect-reverse *of a transition $t \in T$ is a new transition $\overline{t}$ such that $\mathit{eff}_p(\overline{t}) = -\mathit{eff}_p(t)$, for all places $p \in P$.*

To improve readability, we depict newly created reverses and adjacent arcs by dashed (or dotted) lines. Clearly, for a given transition $t$, its strict reverse $\underline{t}$ is unique and, at the same time, it is an effect-reverse of $t$. However, an effect-reverse $\overline{t}$ is not necessarily a strict reverse (see Figure 1).

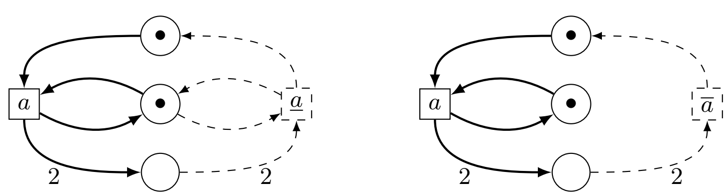


**Fig. 1.** A transition $a$ and its (strict) reverse $\underline{a}$ (lhs), and an effect-reverse $\overline{a}$, which is not a strict reverse (rhs).

## (Un)solvable words

A word $w = t_1 t_2 \ldots t_n$ of length $n \in \mathbb{N}$ uniquely corresponds to a labelled transition system $TS(w) = (\{0, \ldots, n\}, T, \{(i-1, t_i, i) \mid 0 < i \leq n \wedge t_i \in T\}, 0)$.

We say that a net $N$ solves a word $w$ if it solves $TS(w)$. A word $w$ is then called *solvable*, and otherwise *unsolvable*.

If a word $w$ is solvable, then so are all its factors (where a *factor* $w'$ satisfies $w = vw'u$, for some $v$ and $u$). Thus, the unsolvability of any proper factor of $w$ entails the unsolvability of $w$. For this reason, the notion of a *minimal unsolvable word*, defined as an unsolvable word with all proper factors being solvable, is well-defined (see [1] for details).

The *mirror image* $w^R$ of a word $w$ is $w$ written from right to left.

## 3 Solvability of flts's with reverses

We now define reverses for labelled transition systems, and investigate how they affect the solvability of the resulting flts's. We first introduce the notions of reduction and extension of an flts.

**Definition 2 (flts reduction and extension).** *Let $TS = (S, T, \rightarrow, s_0)$ be a solvable flts.*

- *The* reduction *of $TS$ by deleting $t \in T$ is an flts $TS^{[-t]} = (S', T \setminus \{t\}, \rightarrow', s_0)$ such that:*
  - *$S' \subseteq S$ are all the states reachable in $TS$ without using $\overrightarrow{t}$;*
  - *$(s_1, a, s_2) \in \rightarrow'$ if $(s_1, a, s_2) \in \rightarrow$, for all $a \neq t$ and $s_1, s_2 \in S'$.*
- *The* extension *of $TS$ by reversing $t \in T$ is an flts $TS^{[+\bar{t}]} = (S, T \cup \{\bar{t}\}, \rightarrow', s_0)$ such that, for all $s_1, s_2 \in S$:*
  - *$(s_1, a, s_2) \in \rightarrow'$ if $(s_1, a, s_2) \in \rightarrow$, for all $a \in T$;*
  - *$(s_1, \bar{t}, s_2) \in \rightarrow'$ if $(s_2, t, s_1) \in \rightarrow$.*

These above notions can be extended to finite sets of transitions, by setting $TS^{[-t_1,t_2\ldots t_n]} = TS^{[-t_1][-t_2]\ldots[-t_n]}$ and $TS^{[+\overline{t_1,t_2\ldots t_n}]} = TS^{[+\overline{t_1}][+\overline{t_2}]\ldots[+\overline{t_n}]}$.

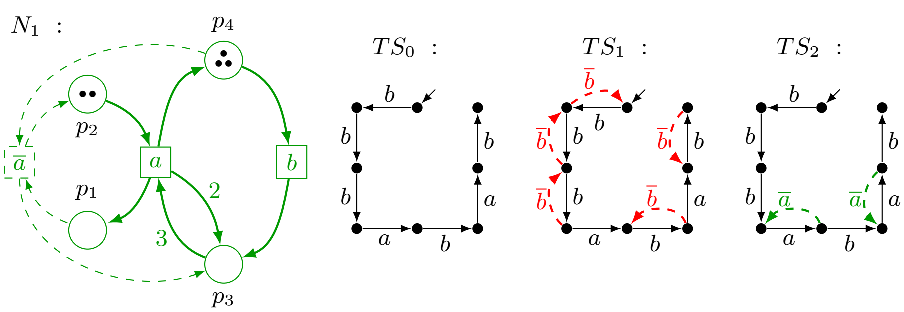


**Fig. 2.** $TS_0$ and $TS_2 = TS_0^{[+\bar{a}]}$ are solvable by net $N_1$ (respectively without and with the dashed part), while $TS_1 = TS_0^{[+\bar{b}]}$ is unsolvable.

Consider a word $w = bbbabab$ which, in Figure 2, corresponds to a solvable flts $TS_0$. If we add a reverse of transition $a$, we obtain $TS_2$ which is solvable by $N_1$. We will later show that reversing transition $b$ leads to an unsolvable flts $TS_1$.

The $\overline{a}$ in Figure 2 is an effect-reverse but not a strict reverse of $a$. We will now show that if a label $a$ can be effect-reversed, i.e., $TS^{[+\overline{a}]}$ is solvable, then there exists a solution in which transition $\overline{a}$ is a strict reverse of $a$.

**Proposition 1.** *Let $TS = (S, T, \rightarrow, s_0)$ be a solvable flts and $a \in T$. If $TS^{[+\overline{a}]}$ is solvable then there exists its solution such that $\overline{a}$ is a strict reverse of $a$.*

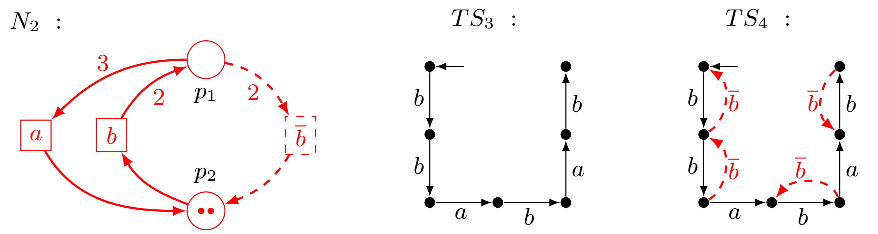


**Fig. 3.** Adding a reverse $\overline{b}$ in $TS_4 = TS_3^{[+\overline{b}]}$ does not violate solvability.

Consider $N_2$ of Figure 3 without the dashed part. It solves the word *bbabab*, and so its reachability graph is isomorphic to $TS_3$. Unlike the case with the reverse of $b$ in $TS_1$, $TS_4$ obtained from $TS_3$ by adding a reverse for transition $b$ is solvable by $N_2$ with dashed part. Note that, in $N_2$, $\overline{b}$ is a strict reverse of $b$.
Similarly, we may reverse $a$ in $TS_3$, obtaining $TS_5$ of Figure 4. This flts is solvable by the net $N_3$ with the dashed part.

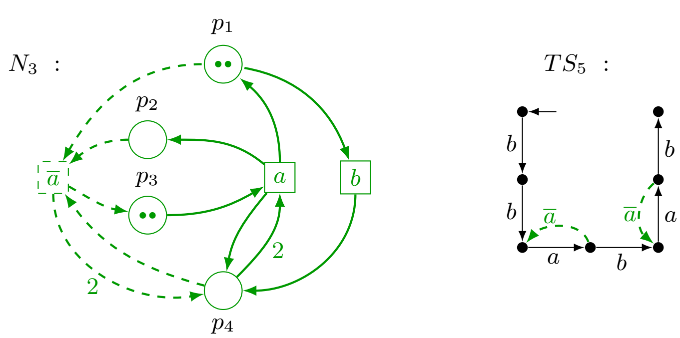


**Fig. 4.** $TS_5 = TS_3^{[+\overline{a}]}$ is solvable (e.g. by $N_3$).

The next result states that for a given flts and two of its transitions, if adding a reverse for each of them separately yields solvable flts's, then the flts with both reverses is also solvable.

**Proposition 2.** *Let* $TS = (S, T, \to, s_0)$ *be a solvable flts and* $a \neq b \in T$*. If both* $TS^{[+\overline{a}]}$ *and* $TS^{[+\overline{b}]}$ *are solvable, then so is* $TS^{[+\overline{a,b}]}$*.*

For $TS = TS_3$ of Figure 3, by Proposition 2, starting from the solutions for $TS_4 = TS^{[+\overline{a}]}$ and $TS_5 = TS^{[+\overline{b}]}$, we can construct a solution $N_4$ for $TS_6 = TS^{[+\overline{a,b}]}$ depicted in Figure 5.

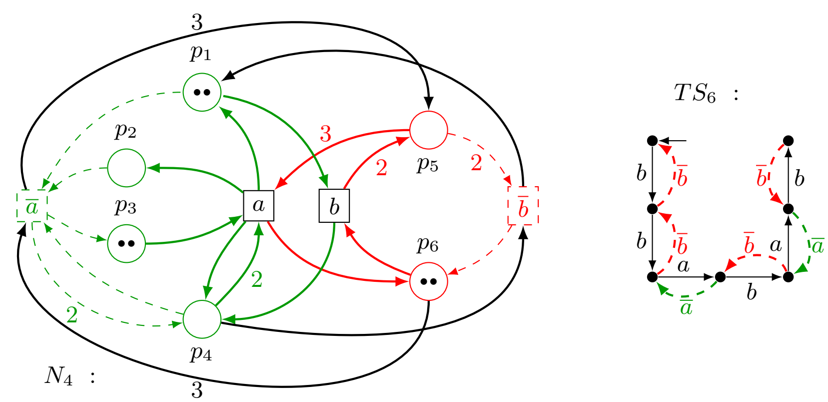


**Fig. 5.** $N_4$ solving $TS_6 = TS_3^{[+\overline{a,b}]}$ derived by synchronising the transitions of $N_2$ and $N_3$.

We end this section looking at the solvability of words over a two-letter alphabet.

**Proposition 3.** *Let* $w \in \{a, b\}^*$ *be a minimal unsolvable word. Then* $TS(w^R)$ *is solvable.*

Due to Propositions 2 and 3, reversing of both transitions in the mirror image $w^R$ of some minimal unsolvable word $w$ over $\{a, b\}$ yields solvability of $w$, which is a contradiction. Hence, the following corollary holds

**Corollary 1.** *Let* $w \in \{a, b\}^*$ *be a minimal unsolvable word and* $TS = TS(w^R)$*. Then* $TS^{[+\overline{a}]}$ *or* $TS^{[+\overline{b}]}$ *is unsolvable.*

The above result explains why $b$ in $TS_1$ of Figure 2 cannot be reversed. All we need to observe is that $w = bbbabab$ is the mirror image of a minimal unsolvable word $bababbb$, and then recall that $a$ can be reversed in $TS_1$.

# 4 Splitting reverses

In this section we discuss the possibility of "splitting" reverses. More specifically, we investigate flts's in which more than one reverse to a given transition can exist.

Consider $N_5$ of Figure 6, together with its reachability graph $TS_7$. First, we observe that $\mathit{eff}_{\overline{b}_1}(p) = \mathit{eff}_{\overline{b}_2}(p) = -\mathit{eff}_{b}(p)$, for every place $p$. Hence, transitions $\overline{b}_1$ and $\overline{b}_2$ are both effect-reverses for $b$. We have already seen that it is impossible to synthesise an flts with just one reverse of $b$ (i.e., $TS_1$ of Figure 2), but the behaviour of $N_5$ is exactly what one might indeed want to obtain. The only difference is that $N_5$ has more than one reverse for $b$. In what follows, we show that every action of a bounded net can be reversed using finitely many effect-reverses.

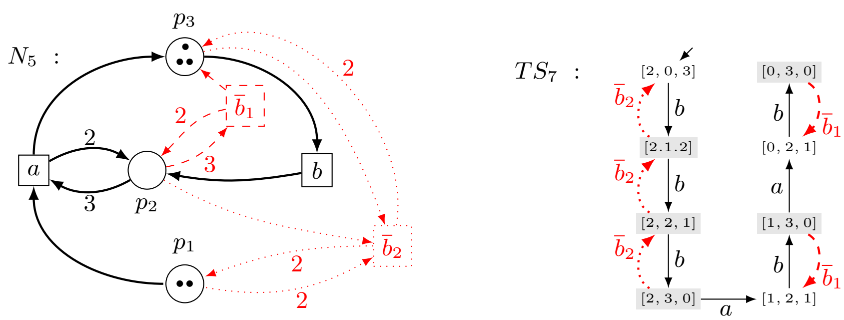


**Fig. 6.** Splitting reverses in $TS_7$ results in solvability.

**Definition 3 (splitting reverse).** *Let $TS = (S, T, \rightarrow, s_0)$ be a solvable flts. The* extension *of $TS$ by a set $\overline{T}$ of reverses of $t \in T$ is an flts $TS^{[+t\phi]} = (S, T', \rightarrow', s_0)$ such that:*

- *$\phi : \overrightarrow{t} \rightarrow 2^{\overline{T}} \setminus \{\emptyset\}$ is a mapping specifying all possible ways in which each of $t$-labelled arcs can be reversed;*
- *$T' = T \cup \overline{T}$;*
- *$(s_1, a, s_2) \in \rightarrow'$ if $(s_1, a, s_2) \in \rightarrow$, for any $a \in T$;*
- *$(s_1, t', s_2) \in \rightarrow'$ if $(s_2, t, s_1) \in \rightarrow$ and $t' \in \phi((s_1, t, s_2))$.*

We also extend the above notion in the usual way to $TS^{[+t_1\phi_1, t_2\phi_2, \ldots, t_n\phi_n]}$.

**Lemma 1.** *Let $N = (P, T, F, M_0)$ be a bounded net, $TS = ([M_0\rangle, T, \rightarrow, M_0)$ be its reachability graph, and $t \notin T$ be a new transition symbol. If a reachable marking $M$ is $\leq$-maximal in $[M_0\rangle$ and $M' \in [M_0\rangle$, then*

$$TS' = ([M_0\rangle, T \cup \{t\}, \rightarrow \cup \{(M, t, M')\}, M_0)$$

*is a solvable flts.*

*Proof.* Let $N' = (P, T \cup \{t\}, F', M_0)$, where:

$$\begin{array}{lll} F'(p,a) &= F(p,a) & \text{for all } p \in P \text{ and } a \in T \\ F'(a,p) &= F(a,p) & \text{for all } p \in P \text{ and } a \in T \\ F'(p,t) &= M(p) & \text{for every } p \in P \\ F'(t,p) &= M'(p) & \text{for every } p \in P. \end{array}$$

We then obtain that:

(1) $t$ is not enabled at any marking $M'' \neq M$ reachable in $N$. Indeed, suppose that there exists such a marking $M''$. Then, by the definition of enabledness, $M''(p) \geq F'(p,t) = M(p)$, for every $p \in P$. Hence $M'' \geq M$, which contradicts the $\leq$-maximality of $M$.
(2) $M[t\rangle M'$. This follows directly from the definition of $F'$.

We then observe that, by (1) and (2), the sets of reachable markings of the nets $N$ and $N'$ are equal, and $RG(N') = TS'$. □

Lemma 1 states that to a given solvable flts (with a solution $N = (P, T, F, M_0)$) one can always add a new edge $(s, t_{(s,s')}, s')$, obtaining another solvable flts, provided that $s$ is a state corresponding to some marking $M$, which is $\leq$-maximal in $[M_0\rangle$, and $t_{(s,s')}$ denotes the label of the edge from $s$ to $s'$, such that $t_{(s,s')} \notin T$. We will use this fact to prove the following theorem

**Theorem 1.** *Let $TS = (S, T, \rightarrow, s_0)$ be a solvable flts. Then, for every $t \in T$, there exists a finite set $\overline{T}$ and a function $\phi : \overrightarrow{t} \rightarrow 2^{\overline{T}} \setminus \{\emptyset\}$ such that $TS^{[+t\phi]}$ is solvable.*

*Proof.* Let $N = (P, T, F, M_0)$ be a net solving $TS$. As $TS$ is finite, $N$ is bounded, and so we can calculate a common bound $n$ on the tokens in the reachable markings for all the places, $n = \max(M(p) \mid M \in [M_0\rangle, p \in P)$.

We extend $N$ to $N' = (P \cup P', T, F', M_0')$ by adding complement places [9] $P' = \{p' \mid p \in P\}$ in such a way that, for all $M \in [M_0\rangle$ and $p \in P$, we define $M'$, such that $M'(p) = M(p)$ and $M'(p') = n - M(p)$. This can be done by inserting in the initial marking $n - M_0(p)$ tokens into each $p' \in P'$, and setting $F'(p', a) = F(a, p)$ as well as $F'(a, p') = F(p, a)$, for all $p' \in P'$ and $a \in T$.

Since, for distict markings $M_1, M_2 \in [M_0'\rangle$, there exists a place $p \in P$ (in which they differ) such that $M_1(p) > M_2(p)$ and $M_1(p') < M_2(p')$, or $M_2(p) > M_1(p)$ and $M_2(p') < M_1(p')$, all distinct markings reachable in $N'$ are $\leq$-incomparable. Hence all markings reachable in $N'$ are $\leq$-maximal in $[M_0'\rangle$. By the construction, the reachability graph of $N'$ is isomorphic to $TS$.

We then construct $TS'$ by adding to $TS$ a set $\overline{T}$ of $|\overrightarrow{t}|$ new transitions in such a way that, for every $(p, t, q) \in \rightarrow$, we also add $(q, t_{(q,p)}, p) \in \rightarrow$. We then define a function $\phi : \overrightarrow{t} \rightarrow 2^{\overline{T}} \setminus \{\emptyset\}$ in such a way that $\phi((p, t, q)) = \{t_{(q,p)}\}$.

Finally, by repeatedly using Lemma 1 for the net $N'$, we obtain that $TS' = TS^{[+t\phi]}$ is solvable. □

The construction described in the proof of Theorem 1 will in most cases lead to a substantial enlargement of the net, as the size of places is doubled, and the number of newly created transitions is bounded by the size of the reachability graph of the initial net. However, as illustrated by the example depicted in Figure 6, there may also exist solutions that are much smaller. Hence, there is a room for improvement of the suggested constructive technique.

## 5 Infeasibility for reversing

To draw attention to another important issue, which becomes relevant during the analysis of flts's from the viewpoint of reversibility of transitions, let us consider the following example.

Suppose that one attempted to introduce a reverse for $a$ in $TS_8$ of Figure 7, which can be solved by $N_6$. Although there exists a (strict) reverse $\overline{a}$ in $N_6$, depicted in Figure 7, the meaning of $\overline{a}$ may be confusing. We cannot regard it as an undoing of the executing of action $a$, since $N_6$ can fire $bc\overline{a}$ where $a$ does not occur at all. What is more, we can keep repeating the firing of $bc\overline{a}$ indefinitely, without executing $a$ even once. To address this situation, we introduce the notion of *infeasibility for reversing.*

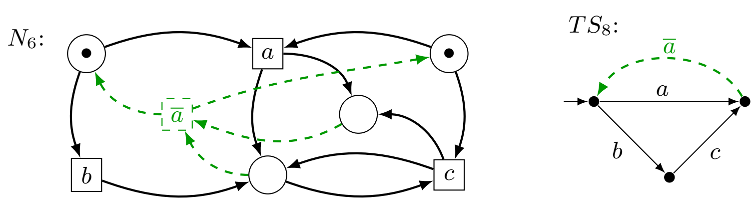


**Fig. 7.** $TS^{[+\overline{a}]}$ allows execution of $\overline{a}$ without executing of $a$.

**Definition 4.** *Let $TS = (S, T, \rightarrow, s_0)$ be an flts. Then $a \in T$ is* infeasible (for reversing)*, if $TS^{[+\overline{a}]}$ has a path starting from $s_0$ with more occurrences of $\overline{a}$ than $a$. Otherwise, $a$ is* feasible (for reversing)*.*

There is a straightforward necessary condition for being feasible for reversing.

**Proposition 4.** *Let $TS = (S, T, \rightarrow, s_0)$ be an flts and $t \in T$. If $TS^{[-t]}$ has a path from ${}^\bullet t_{TS} \cup \{s_0\}$ to $t^\bullet_{TS}$ then $t$ is infeasible for reversing.*

In general, the reversed implication does not hold. Take, for example, $TS_{10} = TS_9^{[+\overline{a}]}$ of Figure 8. It has a path labelled $acd\overline{a}\overline{a}$, with more $\overline{a}$'s than $a$'s, implying the infeasibility for reversing of transition $a$ in $TS_9$. However, the reduction of $TS_9$ by deleting $a$, namely $TS_9^{[-a]}$ has no path starting from ${}^\bullet a_{TS_9} \cup \{s_0\}$ to $a^\bullet_{TS}$. Note that $TS_{10}$ and $TS_9$ are both solvable (see $N_7$ of Figure 8 with or without dashed arcs, respectively). We will now show that one can always

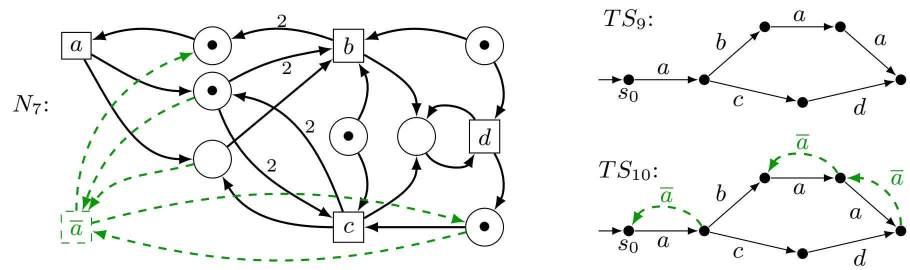


**Fig. 8.** $a$ is infeasible for reversing in $TS_9$, even though $TS_9^{[-a]}$ has no path from ${}^\bullet a_{TS_9} \cup \{s_0\}$ to $a^\bullet_{TS_9}$.

establish whether a label of an flts is (in)feasible for reversing. To this end, formulate the following decision problem:

**Feasibility for Reversing Problem**
*Instance:* An flts $TS = (S, T, \to, s_0)$ and $t \in T$.
*Question:* Is $t$ feasible for reversing in $TS$?

**Proposition 5.** *The Feasibility for Reversing Problem is decidable.*

*Proof (Sketch of the algorithm.).*

The following algorithm reduces the problem of checking the feasibility of a transition for reversing to the problem of finding shortest paths in a weighted digraph.

**Input:** An flts $TS = (S, T, \to, s_0)$ and $t \in T$.
**Output:** YES if $t$ is feasible for reversing in $TS$; otherwise NO.

**Procedure:**

1. Compute a weighted graph $G = (V, E, w)$ on the basis of the extension $TS^{[+\overline{t}]} = (S, T \cup \{\overline{t}\}, \to', s_0)$ of $TS$, in the following way (for all $s, s' \in S$, $a \in T \cup \{\overline{t}\}$):
   - $V = S$;
   - $(s, s') \in E$ if $(s, a, s') \in \to'$;
   - $w((s, s')) = \begin{cases} 1 & \text{if } (s, t, s') \in \to' \\ -1 & \text{if } (s, \overline{t}, s') \in \to' \\ 0 & \text{otherwise.} \end{cases}$

2. Use, e.g., Bellman-Ford algorithm, to search for a state $s_{wit}$, such that the distance between $s_0$ and $s_{wit}$ is negative.
3. If $s_{wit}$ exists, return NO and otherwise YES. □

For a transition system consisting of $n$ states the preprocessing phase (step 1) can be done in time $O(n^2)$. The computation of step 2 can be performed in time $O(n^3)$ (basing on Bellman-Ford algorithm). Therefore the overall complexity of the algorithm is $O(n^3)$.

## 6 Concluding remarks

In this paper, we have investigated reversibility of transitions in bounded nets. In particular, we have shown that each transition in such nets can be reversed using a suitable set of new transitions, but not necessarily a single reverse transition. In future, we plan to investigate ways in which the generation of sets of reverses could be optimised.

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## References

1. Kamila Barylska, Eike Best, Evgeny Erofeev, Łukasz Mikulski, and Marcin Piątkowski. On binary words being Petri net solvable. *Proceedings of the International Workshop on Algorithms & Theories for the Analysis of Event Data, ATAED 2015*, 1371:1–15, 2015.
2. Kamila Barylska, Maciej Koutny, Łukasz Mikulski, and Marcin Piątkowski. Reversible computation vs. reversibility in Petri nets. *Reversible Computation - 8th International Conference, RC 2016, Proceedings*, 9720:105–118, 2016.
3. Gérard Berry and Gérard Boudol. The chemical abstract machine. *Theoretical Computer Science*, 96(1):217–248, 1992.
4. Luca Cardelli and Cosimo Laneve. Reversible structures. In François Fages, editor, *Proceedings of 9th International Computational Methods in Systems Biology (CMSB'11)*, pages 131–140. ACM, 2011.
5. Vincent Danos and Jean Krivine. Reversible communicating systems. In *Proceedings of 15th International Conference on Concurrency Theory (CONCUR'04)*, volume 3170 of *Lecture Notes in Computer Science (LNCS)*, pages 292–307. Springer-Verlag (New York), 2004.
6. Vincent Danos and Jean Krivine. Transactions in RCCS. In *Proceedings of 16th International Conference on Concurrency Theory (CONCUR'05)*, volume 3653 of *Lecture Notes in Computer Science (LNCS)*, pages 398–412. Springer-Verlag (New York), 2005.

7. Vincent Danos, Jean Krivine, and Pawel Sobocinski. General reversibility. *Electronic Notes Theoretical Computer Science*, 175(3):75–86, 2007.
8. Ivan Lanese, Claudio Antares Mezzina, and Jean-Bernard Stefani. Reversing higher-order Pi. In *Proceedings of 21th International Conference on Concurrency Theory (CONCUR'10)*, volume 6269 of *Lecture Notes in Computer Science*, pages 478–493. Springer, 2010.
9. Tadao Murata. Petri nets: Properties, analysis and applications. *Proceedings of the IEEE*, 77:541 – 580, 1989.
10. Iain Phillips and Irek Ulidowski. Reversing algebraic process calculi. *Journal of Logic and Algebraic Programming*, 73(1-2):70–96, 2007.
11. Iain Phillips and Irek Ulidowski. Reversibility and asymmetric conflict in event structures. *Journal of Logic and Algebraic Methods in Programming*, 84(6):781–805, 2015.
12. Iain Phillips, Irek Ulidowski, and Shoji Yuen. A reversible process calculus and the modelling of the ERK signalling pathway. In *Proceedings of 4th Workshop on Reversible Computation (RC'12)*, volume 7581 of *Lecture Notes in Computer Science*, pages 218–232. Springer, 2012.