Twin Graphs for Designing Resilient Optical Backbone Networks

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Abstract

In this work, we investigate the use of twin graphs as an alternative to model optical backbone networks, as recently proposed in the literature [PCS13]. Twin graphs are suitable for resilient and cost-effective optical networks, because of the following property: any single node failure causes no impact on the pairwise hopcounts in the remaining network; and no other graphs with fewer links satisfy this property [FP97].

In recent works, a special family of 2-connected graphs called twin graphs has been proposed to model optical backbone topologies due to interesting properties with respect to fault tolerance, resilience, cost (in number of links) and scalability [PCS13]. Twin graphs belong to the class of 2-geodetically-connected graphs, which means that each of them provides at least two node-disjoint geodesics (i.e., shortest paths with respect to the number of links), for all non-adjacent node pairs. Moreover, no other graphs with fewer links satisfy this property [FP97].

Thus, in topologies modeled as twin graphs, the survivable routing through shortest paths ensures that both the working and the backup paths are geodesics for any pair of non-adjacent nodes. So, compared to other 2-connected topologies, such as rings, twin graphs can tolerate single failures more efficiently, without drastic changes in network performance (see Fig. 1). In addition to their topological characteristics, the feasibility of twin graphs as optical backbone network topologies should be assessed by considering their wavelength requirements [BB97]. The minimum number of wavelengths required to support a given traffic demand corresponds to the solution of the Routing and Wavelength Assignment (RWA) problem [MG02], which is one of the most important problems in the optical network design.

In this work, we analyze the modelling of optical backbone networks as twin graphs in comparison with real-world optical backbone networks, in order to verify the advantages and disadvantages of this new model

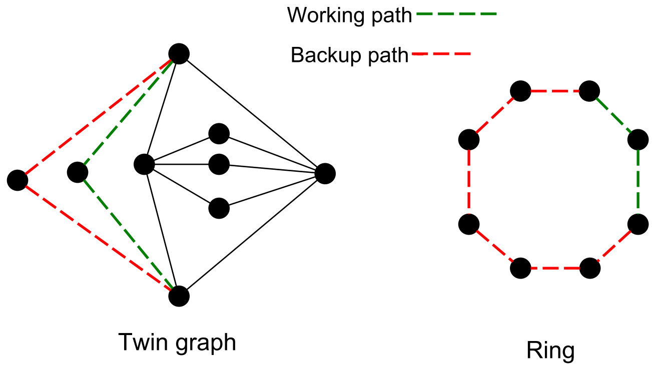

Figure 1: After any single node removal in a twin topology, there is always a backup path which has the same length of the working path. In a ring topology, the backup path can be much larger than the working path, since the length of both these paths have always to sum up the length of the ring.

compared to existing networks. For this purpose, we have considered two sets of networks: the set of all twin graphs with 9 up to 17 nodes, which totalizes 742 graphs, and a set of real-world optical backbone networks (reported in Pavan et al. [PMRP10]) with number of nodes also in the range from 9 up to 17. These sets of networks and their number of nodes (n) and average degree are presented in Tables 1 and 2, respectively.

Table 1: Set of twin graphs (all twin graphs from 9 to 17 nodes).

n	Amount by n	Average degree
9	5	3.11
10	9	3.20
11	13	3.27
12	23	3.33
13	35	3.38
14	63	3.43
15	102	3.47
16	182	3.50
17	310	3.53

Table 2: Set of real-world networks with 9 up to 17 nodes.

Network	n	Average degree
VIA Network	9	2.67
Bren	10	2.20
RNP	10	2.40
CESNET	12	3.17
vBNS	12	2.83
Italy	14	4.14
NSFNET	14	3.00
Austria	15	2.93
Mzima	15	2.53
ARNES	17	2.35
Germany	17	3.06
Spain	17	3.29

For each network in these sets, we have computed the following topological characteristics, which correspond to metrics from graph theory: number of nodes, maximum degree, minimum degree, average degree, link density, diameter, average distance, link connectivity, node connectivity, algebraic connectivity, average link betweenness, maximum link betweenness, and minimum link betweenness. All these computations where performed using the IGRAPH package available for the software R. The definition of maximum link betweenness is given as follows.

Let $G = G(V, E)$ be a graph with $u, v \in V$. Denote by $\sigma_{uv}(e)$ the number of geodesics from u to v that go through a link $e \in E$ and σ_{uv} the total number of geodesics from u to v. Then, the betweenness of link B^e, is given by [GN02]:

$$B^e = \sum_{u \neq v} \frac{\sigma_{uv}(e)}{\sigma_{uv}} \tag{1}$$

and the maximum link betweenness is $\max_{e \in E} B^e$.

Furthermore, we have computed the number of wavelengths for each network in these sets. This computation was carried out using the methodology present in Cousineau et al. [CPC+12]. These results are shown in Fig. 2.

The results were analyzed as function of the graph theory metrics in order to infer correlation between these data. Among all metrics considered, the maximum link betweenness showed the best linear correlation with the number of wavelengths. For this reason, and also for lack of space, we only present here the results obtained for the number of nodes and the maximum link betweenness, which are shown in Fig. 3.

Assuming that the number of wavelengths is a way to evaluate the network cost, twin graphs appeared as a viable alternative to model networks that have cost at least as good as real-world networks. As shown in Fig. 2, twin graphs tend to require fewer wavelengths than real-world networks with same order.

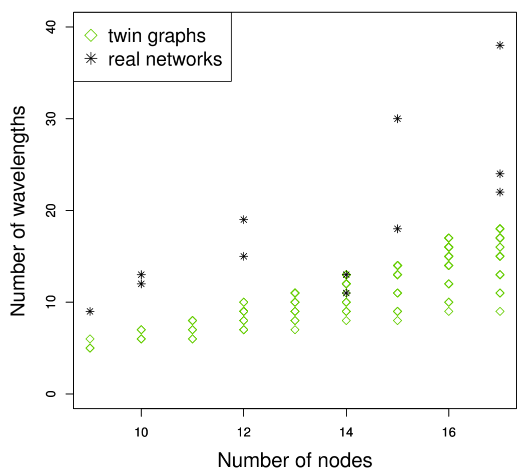

Figure 2: Number of wavelengths given the number of nodes for twin graphs and real-world networks.

For twin graphs the linear correlation coefficient (ρ) between the number of wavelengths and the maximum link betweenness was 99.5%, confirming the positive association suggested by Fig. 3. This result highlights a strong influence of the maximum congestion on a physical topology link with the number of wavelengths. Real-world networks also showed similar behavior, with $\rho = 98.3\%$.

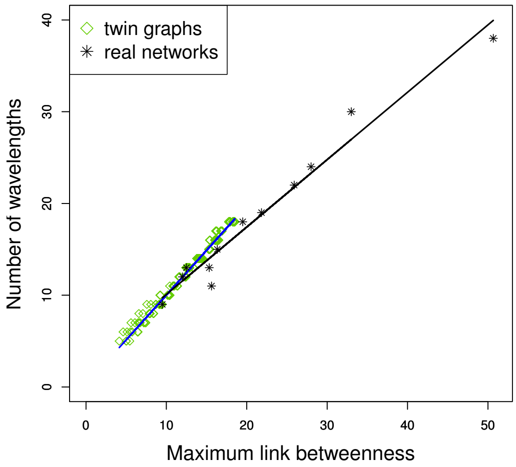

Figure 3: Number of wavelengths given the maximum link betweenness for twin graphs and real-world networks.

In summary, our studies have pointed out some advantages of using twin graphs as a model for designing optical backbone networks. This graph class has proved to be efficient in terms of resilience and cost, including the use of wavelengths. The correlation between the maximum link betweenness and the number of wavelengths observed in twin graphs can be exploited in algorithms for solving the RWA problem. For future work, we are working on lower and upper bounds for the number of wavelengths based on the maximum link betweenness. We will also explore the behavior of twin graphs in the presence of multiple failures.

References

[BB97] Stefano Baroni and Polina Bayvel. Wavelength requirements in arbitrarily connected wavelength-routed optical networks. *Journal of lightwave technology*, 15(2):242–251, 1997.

[CPC+12] M. Cousineau, S. Perron, G. Caporossi, M.H.M Paiva, and M.E.V. Segatto. RWA problem with geodesics in realistic OTN topologies. *Optical Switching and Networking*, pages 74–80, 2012.

[FP97] A. Farley and A. Proskurowski. Minimum self-repairing graphs. *Graphs and Combinatorics*, 13:345–351, 1997.

[GN02] M. Girvan and M. E. J. Newman. Community structure in social and biological networks. *Proceedings of the National Academy of Sciences*, 99(12):7821–7826, 2002.

[MG02] C.S.R. Murthy and M. Gurusamy. *WDM optical networks: concepts, design, and algorithms.* Prentice Hall, New Jersey, 2002.

[PCS13] M.H.M Paiva, G. Caporossi, and M.E.V. Segatto. Twin graphs for OTN physical topology design. *Les Cahiers du GERAD*, 48:1–12, 2013.

[PMRP10] C. Pavan, R.M. Morais, R.F. Rocha, and A.N. Pinto. Generating realistic optical transport network topologies. *IEEE/OSA Journal of Optical Communications and Networking*, 2(1):80–90, 2010.