Modeling the Non-Expected Choice: A Weighted Utility Logit*

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Abstract

This work derives and simulates two choice models applying the weighted utility theory, a generalization of the expected utility theory. It shows one set of assumptions, which justify the practice of including the mean and the variance of a risky alternative into a linear utility function of the choice model. A Monte Carlo simulation provides empirical evidence on the robustness of the models.

1 Introduction

Allais paradox shows that our choices commonly violate the axioms of von Neumann-Morgenstein expected utility theory. But we still commonly apply the expected utility theory when we model our choices. One possible remedy to this discrepancy is to build a choice model that uses one generalization of the expected utility theory, the weighted utility theory.

This paper presents two binomial logit models, which assume that the decision maker has weighed utility preferences. The models have been written into a context of a transportation problem, but naturally they can be applied to any choice between two risky alternatives.

Axiomatically weighted utility differs from expected utility by a weaker version of the independence axiom. Weighted utility was first axiomatized by Chew and MacCrimmon, [1979]. Chew [1982] proved that weighted utility behavior cannot be derived from expected utility by transforming the risky variables. Further axiomatic work has been continued by Chew [1983], Fishburn [1981, 1983] and Nakamura [1984, 1985]. Fishburn [1988] contains an informative presentation of the weighted utility theory.

The descriptive strength of weighted utility has been tested in empirical laboratory experiments [Chew and Waller, 1986; Camerer 1989; Conlisk 1989]. I do not know of any choice models where weighted utility is applied.

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2 Utility Functionals of the Logit Models

Following the tradition of logit models I formulate a utility function that is separable in attributes. The simplest utility function has one sure attribute and one risky attribute. In a transportation context these can be monetary cost of travel and travel time, respectively. In the case of discrete distribution of the risky alternative, the utility functional is:

$$V(\cdot) = b_0 - b_1 \sum_i \frac{p(t_i)\, w(t_i)\, U(t_i)}{\sum_m p(t_m)\, w(t_m)} - b_2 c$$

where $p(t_i)$ denotes the probability of possible travel time outcome t_i, $w(t_i)$ the weight the decision maker places on the outcome t_i, $U(t_i)$ the utility of the outcome t_i, and c the sure monetary cost.

An exponential works well as the weight function.

$$w(t_i) = \exp(\alpha t_i)$$

If $\alpha = 0$, the weight function gets a value one throughout the domain and reduces the weighted utility expression to an expected utility. If $\alpha > 0$, the traveler emphasizes the potential of longer travel times. Correspondingly, if $\alpha < 0$, the traveler behaves as if he would consider the shorter travel times as "more weighty" than what expected utility would warrant.

For the model with continuous distribution of the risky attribute the assumptions are: $t \sim N(\mu,\sigma^2)$, $U = -b_1 t$, and $w(t) = \exp(\alpha t)$. With these assumptions the utility functional is:

$$V(\cdot) = b_0 - b_1 \frac{\frac{1}{\sqrt{2\pi\sigma}} \int t \cdot \exp(\alpha t) \cdot \exp\left(\frac{-(t-\mu^2)}{2\sigma^2}\right) dt}{\frac{1}{\sqrt{2\pi\sigma}} \int \exp(\alpha t) \cdot \exp\left(\frac{-(t-\mu^2)}{2\sigma^2}\right) dt} - b_2 c.$$

This form has the welcome property that it simplifies to

$$V(\cdot) = b_0 - b_1(\mu + \alpha\sigma^2) - b_2 c\ .$$

This is a welcome find because it justifies the commonly practiced ad hoc inclusion of the risky attribute's variance as a fully separate explanatory variable in addition to the mean in the utility expression of an estimated choice model. On the other hand, it demonstrates that this common practice is not compatible with the expected utility theory. A demonstration of this property in a 3-outcome space is available from the author by request.

2.1 Parameter restrictions

It is customary to require that a utility function exhibits risk aversion and monotonicity.

Risk aversion is defined to mean that the utility of the expected outcome is preferred to the utility of a gamble. Assuming two arbitrary outcomes, the requirement of risk aversion simplifies to a requirement that the ratio of weight functions of the outcomes cannot equal to one, that is, α should not equal zero. This requirement reflects the fact that this particular formulation of weighted utility reduces to expected utility only in the case of risk neutrality.

Monotonicity of utility function in outcomes generalizes into a requirement that the utility functional exhibits first order stochastic dominance (FSD). For the discrete model it is possible to arbitrarily define the range of outcomes as [L_1, L2] and thus the range for $V[\{p(t_i)\}]$ as [$-b_1L_2$,$-b_1L_1$]. The definitions lead to two conditions for FSD: $\alpha < 1/(L_2-L_1)$ and $\alpha > 0$. If the risky attribute has an infinite range of outcomes, the decision maker violates monotonicity if she is risk averse, that is, if her $\alpha \neq 0$.

3 Monte Carlo Simulations

The Monte Carlo simulations consisted of rounds of first creating the true choices according to three models: a continuous risky attribute, a discrete risky attribute, and a sure attribute, and later taking the created choice data as given and estimating the three models on each data set.

The weighted utility formulations worked well. In all the simulation runs the continuous model specification gave more consistent results than the discrete one, which should be expected due to the simpler functional form. The true b-parameters were more consistently retrieved in both specifications than α. When true value of α was set to strongly violate FSD, only the continuous model specification was able to converge reliably and retrieve the correct values. But when true α was set to 0.15, which still moderately violated FSD, the discrete model formulation converged each time and the mean of the 50 parameter estimates (0.1864) was within two standard deviations of the true value of 0.15. When the true α-value was set to not violate FSD, the weighted utility models retrieved the true parameters very well. The same held when the true behavior was created by mean value utility, that is to say the true α had a value zero.

When the true behavior was generated by weighted preferences, but was estimated by mean value utility model, the estimated parameters were consistently downward biased towards a point where their proportions stay true. This demonstration is something that should be taken into account in the interpretations of models where the ratio of parameters is assumed to not contain a risk premium for the unreliable attribute, like in the value-of-time estimation. If the true preferences driving the choices comply with weighted utility, the parameters estimated from a mean value utility model will produce estimates that include a risk premium.

4 Conclusions

The model simulations demonstrated that the weighted utility logit models give reliable estimates in a wide range of true weighted utility risk preferences. Especially the discrete version of the model poses possibilities for situations where the decision maker tends to succumb to Allais paradox and bases his decisions on a small number of perceived possible realizations of the risky alternative.

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