Investigating the Effect of Uniform Random Distribution of Nodes in Wireless Sensor Networks using an Epidemic Worm Model

ChukwuNonso H. Nwokoye Nnamdi Azikiwe University, Awka, Nigeria. explode2kg@yahoo.com

Ikechukwu Umeh Nnamdi Azikiwe University, Awka, Nigeria. ikumeh1@gmail.com Virginia E. Ejiofor Nnamdi Azikiwe University, Awka, Nigeria. virguche2004@yahoo.com Rita Orji University of Waterloo, Canada. purity.rita@gmail.com

Njideka N. Mbeledogu Nnamdi Azikiwe University, Awka, Nigeria. njidembeledogu@yahoo.com

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1. INTRODUCTION

In recent times, Wireless Sensor Networks (WSNs) has enjoyed considerable use in civilian applications for precision farming [17] and the provision of smart and quality healthcare. In the military, WSNs are used to monitor rebel activities and to detect enemy movements etc. It consists of large number of communicating devices which are randomly deployed in unreachable territories without an engineered or predetermined position for the nodes [9], [14]. These territories are basically unfriendly and unguarded.

The sensor nodes are distributed in a sensor field where they are wirelessly connected to the sink. Although they have minimal battery capacity they are able to monitor and collect data about a given area. The data and information can be territorial parameters like pressure, position/condition of objects/humans, humidity, temperature etc. The collected data and information are sent back to the local sink through transmission between neighboring nodes. This transmission is basically done in a "multihop infrastructureless" manner. Subsequently, analyses are performed on the collated data accessed by a remote user through the internet for suitable decision making.

The constrained nature of sensor resources that gives rise to frail protective potential makes them suitable prey for self-replicating malevolent codes (such as worms) that spread without human involvement. In addition, these worms often tamper with the confidentiality, integrity and availability measures of neighboring sensor nodes due to its distributed nature.

With the proliferation of the use of network technologies, increasing efforts have been focused on developing appropriate cyber protection structure in order to secure both stationary and moving information. Wireless Sensor Network research believes that achieving this objective is overly expedient. As a result, several continuous (and discrete) equation-based models that characterize, investigate and aid better comprehension of the behavioral tendencies of worm variants have been developed. Predictions of worm behavior are largely dependent on the presuppositions of the model characterization and analysis.

ABSTRACT

The emergence of malicious codes that attack Wireless Sensor Networks (WSN) made it necessary to direct research attention to security. These attacks arising from worms pose devastating threats to networks which can lead to substantial losses or damages. However, recent models developed for the purpose of understanding worm transmission patterns and ensuring its containment did not account for the effect of uniform random deployment of sensor nodes on the Exposed and the Vaccinated compartments. Therefore, in this paper we present a modified Susceptible-Exposed-Infectious-Recovered-Susceptible with Vaccination (SEIRS-V) model for worm propagation dynamics in sensor networks. Our model applies the expression for uniform distribution deployment of sensor nodes so as to study the effect of distribution density and transmission range on the characterized compartments. Furthermore, we presented solutions for the equilibrium points, the reproduction number and proof of stability. Finally, we employed numerical methods to solve and simulate with real values the developed system of differential equations.

CCS Concepts

Computer systems organizations \rightarrow Embedded and cyberphysical systems \rightarrow Sensor networks. • Security and privacy \rightarrow Intrusion/anomaly detection and malware mitigation \rightarrow Malware and its mitigation. • Mathematics of computing \rightarrow Mathematical analysis \rightarrow Differential equations \rightarrow Ordinary differential equations. • Computing Methodologies \rightarrow modeling and simulation \rightarrow Model development and analysis, Simulation support systems.

Keywords

Wireless Sensor Network; Worm; Epidemic Model; Uniform

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2. RELATED WORKS

It is unarguably true that propagation of malicious agents in cyberspace is similar to the spread of epidemic in the biological world. Therefore, modeling and analysis enhance optimized containment by providing better understanding of the factors that aid faster propagation of malicious codes in networks. The Susceptible-Infectious-Recovered (SIR) model by [5], [4] and [6] initiated the journey into developing mathematical models for worm/virus propagation. Building on the SIR model, other extensions such as Susceptible-Exposed-Infectious-Recovered (SIER) [13], [12], Susceptible-Exposed-Infectious-Recovered-Vaccinated (SER-V) [9] etc., were developed to address several concerns arising in a real world network environment.

In this paper we modify the Susceptible-Exposed-Infectious-Recovered-Susceptible with a Vaccination compartment (SEIRS-V) epidemic model of [9] by applying [15] expression for uniform random distribution of sensor nodes with the aim of investigating the effect of both distribution density (σ) and transmission range (r_0^2) on the characterized compartments. We discovered that though [15] represented distribution density and transmission range their SIR model did not include analyses for the Exposed and the Vaccinated compartments. On the other hand, though [9] included these two compartments their analyses did not involve distribution density and transmission range. Wang and Yang [16] also applied Tang and Mark's formulation but their model used the Susceptible-Infectious (SI) compartments for their analyses; and didn't discuss the Exposed and Vaccinated compartments. Considering the argument by [3] that standard incidence "presents a more reasonable and practical scenario of contact than the simple mass action incidence", we would modify the above model using the former.

In epidemiology, the Exposed class contains nodes that are infected but not infectious. These nodes which are in a latent phase possess different infectivity rate when compared to the Infectious nodes. Common symptom for nodes in this latent stage is slow data transmission speed [9]. On the hand, vaccination (or immunization) is a known countermeasure in epidemiology. It is aimed at fortifying a fraction of the total sensor node population prior to the outset of an epidemic. This study is necessary since there is a strong likelihood that sensor nodes can exist in the latent stage and that network managers can employ vaccination strategies to ensure security. In addition, the analyses of distribution density and transmission range using worm models can positively impact sensor deployment activities for institutions.

3. METHODOLOGY

In this study, we basically perform modeling and simulation. Specifically, we employ a widely applied method for investigating network epidemic [10], [9] and [11] etc. This method is called modeling and analysis of dynamical systems. Here, the WSN is treated like a dynamical system and equilibrium positions are studied. The methodology starts with; a. model formulation (and optionally drawing the schematic

diagram); b. finding the equilibrium states (for the worm-free and the endemic states); and c. deriving the Reproduction number. Subsequent stages include; d. proof of stability and e. simulations experiments using software such as MatLab, Maple etc.

During model formulation, the analyst presents the equations that represent a real world phenomenon (in this case worm propagation in WSNs). Some authors also present a conceptual (or schematic) diagram at this point. Next, the analyst will derive the equilibrium points by equating the model (or system of differential equation) to zero. The Reproduction number is then derived to establish a threshold for disease/infection extinction in the network. Furthermore, stability analyses (using several renowned methods in literature) and simulation experiments are performed. The simulations experiments are more like sensitivity analyses. They are done by first solving the proposed model using a numerical method and applying real values for the simulation. Depending on the modeler's intention the method can take different turns for analyses. But stages such as model formulation and simulation experiments are significantly part of this methodology.

3.1 The Modified SEIRS-V Model

We characterize worm attack in wireless sensor network using the Susceptible-Exposed-Infectious-Recovered-Susceptible with a Vaccination compartment (SEIRS-V). Our assumptions include addition of nodes in the network and removal (i.e. death) of nodes as a result of worm attack or due to hardware/software failure. All sensor nodes are susceptible to potential of worm attack and with time (probably) get infected (i.e. forming nodes in the Infectious compartment). Some sensor nodes before becoming infectious exists in the Exposed stage where the worm is latent and the nodes cannot transmit the infection. A symptom of this stage is slower data transmission for affected nodes. However, due to the existence of several worm variants in cyberspace the sensor nodes never acquire a permanent immunity i.e. they become susceptible to worm infection with time

The total population N(t) represents the nodes in the Wireless Sensor Network which is subdivided into Susceptible, Exposed (latent), Infectious (contagious), Recovered (temporarily immune), Vaccinated (immunized) denoted by S(t), E(t), I(t), R(t) and V(t). This implies that S(t) + E(t) + I(t) + R(t) + V (t) = N(t).

Specifically, the sensor nodes are uniformly and randomly deployed with a distribution density of σ and a transmission range of r_0^2 , this implies that the effective contact with an infected node for transfer of infection is in the order of $\sigma \pi r_0^2$. Other parameters include λ which is the inclusion rate of nodes into the sensor network population, β is the Infectivity contact rate, τ is the mortality or the death rate of nodes due to hardware or software failure, ω is the crashing rate due to attack of malicious objects (in this case worm), θ is the rate at which exposed nodes become infectious, ν is the recovery rate, φ is the rate at which recovered nodes become susceptible to infection, ρ is the rate of transmission from the Vaccinated compartment to the Susceptible compartment.



Figure 1. Schematic diagram for the flow of worms in sensor network

The schematic diagram for the dynamical transmission of worms in a Wireless Sensor Network given our assumption is depicted in Fig. 1. The system of differential equation (1) is adapted from [9] but modified to capture distribution density and transmission range.

The modified SEIRS-V model is represented using the following system of differential equations;

$$\frac{ds}{dt} = \lambda N - \beta SI \frac{\delta n r_0^2}{N} - \tau S - \rho S + \varphi R + \xi V$$

$$\frac{dE}{dt} = \beta SI \frac{\sigma n r_0^2}{N} - (\tau + \theta) E$$

$$\frac{dI}{dt} = \theta E - (\tau + \omega + \nu) I \qquad (1)$$

$$\frac{dR}{dt} = \nu I - (\tau + \varphi) R$$

$$\frac{dv}{dt} = \rho S - (\tau + \xi) V$$

3.2 Solutions of Equilibrium Points

We equate the modified system of differential equations (1) to zero i.e.

 $\frac{dS}{dt} = 0; \frac{dE}{dt} = 0; \frac{dI}{dt} = 0; \frac{dR}{dt} = 0; \frac{dV}{dt} = 0$; to obtain two solutions which are the Worm-free equilibrium and the Endemic equilibrium points. The Worm-free equilibrium describes the absence of worms while the Endemic equilibrium describes the presence of worms in the Wireless Sensor Network using formulated mathematical model.

The solutions of equilibrium points are Worm-free equilibrium $E_0^F = (S_0^*, E_0^*, I_0^*, R_0^*, V_0^*)$ i.e.

$$S_0^* = N \frac{\lambda(\xi + \tau)}{\tau(\xi + \rho + \tau)}$$

$$E_0^* = 0; I_0^* = 0; R_0^* = 0, \qquad (2)$$

$$V_0^* = N \frac{\lambda \rho}{\tau(\xi + \rho + \tau)}$$

and Endemic equilibrium
$$E_{1}^{E} = (S_{1}^{*}, E_{1}^{*}, I_{1}^{*}, R_{1}^{*}, V_{1}^{*})$$
 i.e.
 $S_{1}^{*} = N \frac{(\theta + \tau)(\nu + \tau + \omega)}{\beta \theta \sigma \pi r_{0}^{2}}$
 $E_{1}^{*} = N \frac{(\tau + \varphi)(\nu + \tau + \omega)(\lambda - \frac{\tau(\theta + \tau)(\xi + \rho + \tau)(\nu + \tau + \omega)}{\beta \theta(\xi + \tau)\sigma \pi r_{0}^{2}})}{\theta \tau(\nu + \tau + \varphi) + \theta(\tau + \varphi)\omega + \tau(\tau + \varphi)(\nu + \tau + \omega)}$
 $I_{1}^{*} = N \frac{(\tau + \varphi)(\theta \lambda - \frac{\tau(\theta + \tau)(\xi + \rho + \tau)(\nu + \tau + \omega)}{\beta(\xi + \tau)\sigma \pi r_{0}^{2}})}{\theta \tau(\nu + \tau + \varphi) + \theta(\tau + \varphi)\omega + \tau(\tau + \varphi)(\nu + \tau + \omega)}$
 $R_{1}^{*} = N \frac{\nu(\theta \lambda - \frac{\tau(\theta + \tau)(\xi + \rho + \tau)(\nu + \tau + \omega)}{\beta(\xi + \tau)\sigma \pi r_{0}^{2}}}{\theta \tau(\nu + \tau + \varphi) + \theta(\tau + \varphi)\omega + \tau(\tau + \varphi)(\nu + \tau + \omega)}$
(3)

A cursory look at the symbolic solutions of the endemic equilibrium in [9] shows the differences. It is observed here that the expression for uniform distribution deployment formed part of the solutions; this is absent in the solutions of [9].

3.3 The Basic Reproduction Number

The Reproduction number commonly denoted as R_0 is a threshold quantity defined as "the expected number of secondary cases produced in a completely susceptible population, by a typical infective individual" [1] or the spectral radius which can also be referred to as the "dominant eigenvalue of the matrix G = \mathbf{FV}^{-1} " [2], [9]. Some authors also refer to it as the inverse of the susceptible (S_1^*) at the endemic equilibrium [10]. The Reproduction number (R_0) is $\frac{\beta\theta\sigma\pi r_0^2}{(\theta+\tau)(\nu+\tau+\omega)}$. (4)

In this study, the Reproduction number is different from what was obtained in [15] and [9]. Our Reproduction number can be used to determine the possible contained/endemic dynamics of worm propagation in WSNs considering distribution density and transmission range.

3.4 Stability of the Worm-free Equilibrium point

We show the proof of local asymptotic stability at the Wormfree Equilibrium using the jacobian method. This is done by showing that "the eigen-values of the jacobian matrix all have negative real parts" [9] or that the "characteristic equation of the jacobian matrix" derived from the system of equations has negative roots [8].

Theorem: The worm-free equilibrium is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof: Using \mathbf{E}_{0}^{F} - the characteristic equation of system (1) at worm-free equilibrium is

$$\det \begin{vmatrix} -(\tau+\rho)-x & 0 & \frac{-\beta\sigma\pi r_0^2\lambda(\xi+\tau)}{\tau(\xi+\tau)} & \varphi & \xi \\ 0 & -(\tau+\theta)-x & \frac{\beta\sigma\pi r_0^2\lambda(\xi+\tau)}{\tau(\xi+\tau)} & 0 & 0 \\ 0 & \theta & -(\tau+w+)-x & 0 & 0 \\ 0 & 0 & v & -(\tau+\varphi)-x & 0 \\ \rho & 0 & 0 & 0 & -(\tau+\xi)-x \end{vmatrix} = 0 (5)$$

which equates to; $-(x + \tau)(x + \xi + \rho + \tau)(x + \tau + \varphi)\left((x + \theta + \tau)(x + \nu + \tau + \omega) - S_0^*\theta\beta\sigma\pi r_0^2\right) = 0.$ (6) The roots of the characteristic equation all have negative real parts i.e. $-\tau$, $-\xi - \rho - \tau$, $-\tau - \varphi$, $\frac{1}{2}\left(-\theta - \nu - 2\tau - \omega - \sqrt{(-\theta + \nu + \omega)^2 + 4S_0^*\theta\beta\sigma\pi r_0^2}\right)$, $\frac{1}{2}(-\theta - \nu - 2\tau - \omega + \sqrt{(-\theta + \nu + \omega)^2 + 4S_0^*\theta\beta\sigma\pi r_0^2})$;

Therefore the worm free equilibrium is locally asymptotically stable.

4. NUMERICAL RESULTS AND DISCUSSION

We solved the system of differential equation using a numerical method i.e. Runge-Kutta Fehlberg method of order 4 and 5. Subsequently we performed simulation experiments using this following initial values for the Wireless Sensor network: S=100; E=3; I=1; R=0; V=0. Other values used for the simulation include $\lambda = 0.33$; $\beta = 0.1$; $\tau = 0.003$; $\omega = 0.07$; $\theta = 0.25$; $\nu = 0.4$; $\varphi = 0.3$; $\rho = 0.3$; $\xi = 0.06$; adapted from the time history of [9]. We compared our results with the results of similar models in literature for the purposes of verification and validation.

Figure 2 shows the time history of the compartments used for the analyses. Note that the transient response of Figure 2 simulated with values for distribution density and transmission range differs from the time history of [9]. For the sake of clarity and ambiguity reduction we prepared the simulation experiment of Figure 2 using the same colors used in Figure 3. It is evident that our model showed increase in both the Exposed and Infected compartments and a reduction in both the Susceptible and Vaccinated compartments.



Figure 2. Dynamical behaviour of the system for different compartments of the modified model



Figure 3. Dynamical behaviour of the system for different compartments of the equivalent model Source: [9]

Figure 4 shows the dynamical behavior of the Exposed compartment with respect to changes in the distribution density and transmission range. At 0.5 and 2.0 for density and range respectively depicted with red, there was still an increase in the number of Exposed sensor nodes even though the range was kept constant (like in first response depicted with green). The effect of both density and range was again evident in the third response (0.7 and 2.5 for density and range respectively) depicted with blue when compared with the first response where 0.3 and 2.0 for density and range respectively.



Figure 4. Dynamical behaviour of Exposed Compartment versus Time w.r.t. to σ and r_o^2

Figure 5 shows the dynamical behavior of the Infectious compartment with respect to changes in the distribution density and transmission range. At 0.5 and 2.0 for density and range respectively depicted with red, there was still an increase in the number of Infectious sensor nodes even though the range was kept constant (like in the first response depicted with green).

The effect of both density and range was again evident in the third response (i.e. 0.7 and 2.5 for density and range respectively) depicted with blue when compared with the first response where 0.3 and 2.0 for density and range respectively.



Figure 5. Dynamical behaviour of Infectious Compartment versus Time w.r.t. to σ and r_a^2

Figure 6 shows the dynamical behavior of the Infectious compartment plotted against the Exposed compartment with respect to changes in the distribution density and transmission range. This figure showed how both the Exposed and Infectious compartments increased with increase in density and transmission range. The increase was observed when density was kept constant (at 0.5) and when range was kept constant (at 0.2). The difference between the Exposed and Infectious sensor nodes is in line with the real world because even though both have contacted the infection, only the Infectious sensor nodes can transmit the infection to susceptible nodes.



Figure 6. Dynamical behaviour of Infectious Compartment versus Exposed Compartment w.r.t. to σ and r_{ρ}^2

Figure 7 shows the dynamical behavior of the Susceptible compartment plotted against the Vaccinated compartment with respect to changes in the distribution density and transmission range. This figure showed a decrease in both the Susceptible and the Vaccinated compartments. The decrease was observed when density was kept constant (at 0.5) and when range was kept constant (at 0.2); this is clearly visible when compared to Figure 7a.



Figure 7. Dynamical behaviour of Susceptible Compartment versus Vaccinated Compartment w.r.t. to σ and r_{ρ}^2



Figure 7a. Dynamical behaviour of Susceptible Compartment versus Vaccinated Compartment of the equivalent model as adapted from [9]

From our simulation experiments we noticed that at transmission range of 1 and density of 0.3 depicted in Figure 8 the dynamical responses were close to Figure 3 of [9] for the Exposed and Vaccinated compartments. Increasing the range to 2 (as evident in Figure 2) visibly changed the behavior of the compartments (most especially Exposed and the Vaccinated). The Exposed nodes moved from above 30 nodes to above sixty nodes while Vaccinated nodes reduced to slightly above 20 nodes from above 40 nodes.



Figure 8. Dynamical behaviour of Infectious Compartment versus Exposed Compartment w.r.t. to σ =0.3 and r_o^2 =1

5. CONCLUSION AND FUTURE DIRECTION

In this study, we discovered that the increase in density and transmission range increased the Exposed and Infectious compartments and decreased and Vaccinated compartments. This study is consistent with [15] and [16] that employed similar expression for uniform random distribution i.e. the increase in the number of Infectious sensor nodes with the increase in both the node density and communication range.

Due to the effect of uniform random distribution on Vaccinated sensor nodes, it is only wise that the rate at which nodes are immunized is increased as density and range increase in order to reduce high susceptibility to infection. In future we would focus our analyses on how to achieve increased vaccination rate for vulnerable sensor nodes (to ensure reduced susceptibility to infection) in the light of increased density and increased communication/transmission range.

Furthermore, we would also perform analysis and simulation experiments to observe the effect of uniform random distribution on Quarantine models. In addition, pursuit of other mathematical objectives such as extending analyses to the global stability at the endemic equilibrium using the geometrical approach of [7] etc. can ensue; since the symbolic solutions at the endemic equilibrium have been provided by this study. To creatively protect the interchange of data and information the generalized form of the analytical model (that characterizes other worm variants) will be integrated into the cyberspace defense structure of organization(s) that use Wireless Sensor Networks for monitoring rebel activities, detecting enemy movements, and providing smart healthcare etc.

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