GRID and Quanputers

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A general method of hamiltonization of the dynamical systems is described. A class of discrete-time invertable dynamical systems is extended by a corresponding linear subsystem - quanputers defined. Grid computing is considered as an evolution of a discrete-time dynamical system that permits updating up to the quanputer (sub)systems.

Keywords: grid computing, dynamical systems, quanputers

In this talk, we consider a general method of **hamiltonization** of the dynamical systems. In the case of the discrete dynamical systems, we define a family of time-invertible dynamical systems and their linear extensions - quanputers, which contains contemporary models of quantum computers. Then we describe GRID as a discrete dynamical system and suggest some contemporary and future modifications of GRID according to the **quanputer technologies.**

Let us consider a general dynamical system described by the following system of the ordinary differential equations:

$$\dot{x}_n = v_n(x), \ 1 \leq n \leq N, \tag{1}$$

where $\dot{x}_n$ stands for the total derivative with respect to the time parameter t.

When the number of the degrees of freedom is even, and

$$v_n(x) = \varepsilon_{nm} \frac{\partial H_0}{\partial x_m}, \ 1 \leq n, m \leq 2M, \tag{2}$$

the system (1) is Hamiltonian one and can be put in the form

$$\dot{x}_n = \{x_n, H_0\}_0, \tag{3}$$

where the Poisson bracket is defined as

$$\{A, B\}_0 = \varepsilon_{nm} \frac{\partial A}{\partial x_n} \frac{\partial B}{\partial x_m} = A \frac{\overleftarrow{\partial}}{\partial x_n} \varepsilon_{nm} \frac{\overrightarrow{\partial}}{\partial x_m} B, \tag{4}$$

and summation rule under repeated indices has been used.

Let us consider the following Lagrangian:

$$L = (\dot{x}_n - v_n(x))\psi_n \tag{5}$$

and a corresponding equations of motion

$$\dot{x}_n = v_n(x), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n} \psi_m. \tag{6}$$

The system (6) extends the general system (1) by linear equation for the variables ψ. The extended system can be put in the Hamiltonian form [Makhaldiani, Voskresenskaya, 1997]

$$\dot{x}_n = \{x_n, H_1\}_1, \dot{\psi}_n = \{\psi_n, H_1\}_1, \tag{7}$$

where first level (order) Hamiltonian is

$$H_1 = v_n(x)\psi_n \tag{8}$$

and (first level) bracket is defined as

$$\{A, B\}_1 = A(\frac{\overleftarrow{\partial}}{\partial x_n} \frac{\overrightarrow{\partial}}{\partial \psi_n} - \frac{\overleftarrow{\partial}}{\partial \psi_n} \frac{\overrightarrow{\partial}}{\partial x_n})B. \tag{9}$$

Note that when the Grassmann grading [Berezin, 1987] of the conjugated variables x_n and ψ_n are different, the bracket (9) is known as Buttin bracket [Buttin, 1996].

In the Faddeev-Jackiw formalism [Faddeev, Jackiw, 1988] for the Hamiltonian treatment of systems defined by first-order Lagrangians, i.e. by a Lagrangian of the form

$$L = f_n(x)\dot{x}_n - H(x), \tag{10}$$

motion equations

$$f_{mn}\dot{x}_n = \frac{\partial H}{\partial x_m}, \tag{11}$$

for the regular structure function f_{mn}, can be put in the explicit hamiltonian (Poisson; Dirac) form

$$\dot{x}_n = f^{-1}_{nm}\frac{\partial H}{\partial x_m} = \{x_n, x_m\}\frac{\partial H}{\partial x_m} = \{x_n, H\}, \tag{12}$$

where the fundamental Poisson (Dirac) bracket is

$$\{x_n, x_m\} = f^{-1}_{nm}, \;\; f_{mn} = \partial_m f_n - \partial_n f_m. \tag{13}$$

The system (6) is an important example of the first order regular hamiltonian systems. Indeed, in the new variables,

$$y^1_n = x_n, y^2_n = \psi_n, \tag{14}$$

lagrangian (5) takes the following first order form:

$$\begin{aligned} L &= (\dot{x}_n - v_n(x))\psi_n \Rightarrow \frac{1}{2}(\dot{x}_n\psi_n - \dot{\psi}_n x_n) - v_n(x)\psi_n = \frac{1}{2}y^a_n\varepsilon^{ab}\dot{y}^b_n - H(y) \\ &= f^a_n(y)\dot{y}^a_n - H(y),\, f^a_n = \frac{1}{2}y^b_n\varepsilon^{ba}, H = v_n(y^1)y^2_n,\, f^{ab}_{nm} = \frac{\partial f^b_m}{\partial y^a_n} - \frac{\partial f^a_n}{\partial y^b_m} = \varepsilon^{ab}\delta_{nm}; \end{aligned} \tag{15}$$

corresponding motion equations and the fundamental Poisson bracket are

$$\dot{y}^a_n = \varepsilon_{ab}\delta_{nm}\frac{\partial H}{\partial y^b_m} = \{y^a_n, H\}, \{y^a_n, y^b_m\} = \varepsilon_{ab}\delta_{nm}. \tag{16}$$

Computers are physical devices and their behavior is determined by physical laws. The Quantum Computation [Nielsen, Chuang, 2000], Quantum Computing, Quanputing [Makhaldiani, 2007], is a new interdisciplinary field of research, which benefits from the contributions of physicists, computer scientists, mathematicians, chemists and engineers.

The contemporary digital computer and its logical elements can be considered as a spatial type of discrete dynamical systems [Makhaldiani, 2001]

$$S_n(k+1) = \Phi_n(S(k)), \tag{17}$$

where

$$S_n(k), \;\; 1 \le n \le N(k), \tag{18}$$

is the state vector of the system at the discrete time step k. Vector S may describe the state and Φ transition rule of some Cellular Automata [Toffoli, Margolus, 1987].The systems of the type (17) appears in applied mathematics as an explicit finite difference scheme approximation of the equations of the physics [Samarskii, Gulin, 1989].

Definition: *We assume that the system (17) is time-reversible if we can define the reverse dynamical system*

$$S_n(k) = \Phi^{-1}_n(S(k+1)). \tag{19}$$

In this case the following matrix

$$M_{nm} = \frac{\partial \Phi_n(S(k))}{\partial S_m(k)}, \tag{20}$$

is regular, i.e. has an inverse. If the matrix is not regular, this is the case, for example, when $N(k+1) \neq N(k)$, we have an irreversible dynamical system (usual digital computers and/or corresponding irreversible gates).

Let us consider an extension of the dynamical system (17) given by the following action function:

$$A = \sum_{kn} l_n(k)(S_n(k+1) - \Phi_n(S(k))) \tag{21}$$

and corresponding motion equations

$$S_n(k+1) = \Phi_n(S(k)) = \frac{\partial H}{\partial l_n(k)},\ l_n(k-1) = l_m(k)\frac{\partial \Phi_m(S(k))}{\partial S_n(k)} = l_m(k)M_{mn}(S(k)) = \frac{\partial H}{\partial S_n(k)}, \tag{22}$$

where

$$H = \sum_{kn} l_n(k)\Phi_n(S(k)), \tag{23}$$

is discrete Hamiltonian. In the regular case, we put the system (22) in an explicit form

$$S_n(k+1) = \Phi_n(S(k)),\ l_n(k+1) = l_m(k)M^{-1}_{mn}(S(k+1)). \tag{24}$$

From this system it is obvious that, when the initial value $l_n(k_0)$ is given, the evolution of the vector $l(k)$ is defined by evolution of the state vector $S(k)$. The equation of motion for $l_n(k)$ - Elenka is linear and has an important property that a linear superpositions of the solutions are also solutions. **Statement:** *Any time-reversible dynamical system (e.g. a time-reversible computer) can be extended by corresponding linear dynamical system (quantum - like processor) which is controlled by the dynamical system and has a huge computational power,* [Makhaldiani, 2001; Makhaldiani, 2007; Makhaldiani, 2011; Makhaldiani, 2016].

Nowadays there are several big collaborations in science, e.g. LHC. Scientific value of LHC depends on three components, the highest quality of accelerator, highest quality of detectors and distributed data processing. The first two components need good mathematical and physical modeling. The third component and the collaboration as a social structure are not under (anther) the control by scientific methods and corresponding modeling. By definition, scientific collaborations (SC) have a main scientific aim: to obtain answer to the important scientific question(s) and maybe gain extra scientific bonus: new important questions and discoveries. SC is a more open information system than e.g. finance or military systems. So, it is possible to describe and optimize SC by scientific methods. Profit from scientific modeling of SC maybe also for other information systems and social structures.

As an example of GRID, we take LHC Computing Grid. The LHC Computing Grid (LCG), is an international collaborative project that consists of a grid-based computer network infrastructure incorporating over 170 computing centers in 36 countries. It was designed by CERN to handle the prodigious volume of data produced by Large Hadron Collider (LHC) experiments. The Large Hadron Collider at CERN was designed to prove or disprove the existence of the Higgs boson, an important but elusive piece of knowledge that had been sought by particle physicists for over 40 years. A very powerful particle accelerator was needed, because Higgs bosons might not be seen in lower energy experiments, and because vast numbers of collisions would need to be studied. Such a collider would also produce unprecedented quantities of collision data requiring analysis. Therefore, advanced computing facilities were needed to process the data. A design report was published in 2005. It was announced to be ready for data on 3 October 2008. It incorporates both private fiber optic cable links and existing high-speed portions of the public Internet. At the end of 2010, the Grid consisted of some 200,000 processing cores and 150 petabytes of disk space, distributed across 34 countries. The data stream from the detectors provides approximately 300 GByte/s of data, which after filtering for "interesting events results in a data stream of about 300 MByte/s. The CERN computer center, considered "Tier 0"of the LHC Computing Grid, has a dedicated 10 Gbit/s connection to the counting room. The project was expected to generate 27 TB of raw data per day, plus 10 TB of "event summary data which represents the output of calculations done by the CPU farm at the CERN data center. This data is sent out from CERN to eleven Tier 1 academic institutions in Europe, Asia, and North America, via dedicated 10 Gbit/s links. This is called the LHC Optical Private Network. More than 150 Tier 2 institutions are connected to the Tier 1 institutions by general-purpose national research and education networks. The data produced by the LHC on all of its distributed computing grid is expected to add up to 10-15 PB of data each year. In total, the four main detectors at the LHC produced 13 petabytes of data in 2010. The Tier 1 institutions receive specific subsets of the raw data, for which they serve as a backup repository for CERN. They also perform reprocessing when recalibration is necessary. In 2015, CERN switched away from Scientific Linux to CentOS. Distributed computing resources for analysis by end-user physicists are provided by the Open Science Grid, Enabling Grids for E-sciencE, and LHC@home projects, http://wlcg.web.cern.ch/. Update of the Computing Models of the WLCG and the LHC Experiments: http://cds.cern.ch/record/1695401/files/LCG-TDR-002.pdf

The idea of computations on quanputers is in finding the needed (value of the) state (wave function $\psi(t,x)$) from the initial, easy constructible, state ($\psi(0,x)$) which is superposition of different states, including interesting one, with the same weight. During the computation the weight of the interesting state is growing until the value when we can guess the solution of the problem and then test it, which is much more easier then to find it.

Let us consider the following nonlinear evolution equation

$$iV_t = \Delta V - \frac{1}{2}V^2 + J, \tag{25}$$

extended Lagrangian and Hamiltonian

$$L = \int dx^D (iV_t - \Delta V + \frac{1}{2}V^2 - J)\psi,\ H = \int dx^D (\Delta V - \frac{1}{2}V^2 + J)\psi \tag{26}$$

and corresponding Hamiltonian motion equations

$$iV_t = \Delta V - \frac{1}{2}V^2 + J = \{V, H\},\ i\psi_t = -\Delta\psi + V\psi = \{\psi, H\},\ \{V(t,x), \psi(t,y)\} = \delta^D(x-y) \tag{27}$$

The solution of the problem is given in the form

$$|T) = U(T)|0),\ \psi(t,x) =< x|t),\ U(T) = Pexp(-i\int_0^T dtH(t)) \tag{28}$$

Under the programming of the quanputer we understand construction of the potential V, or the corresponding Hamiltonian. For the given potential, we calculate corresponding source J. The discrete version of the system can be put in the form

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),\ \Psi_m(n-1) = A_{mk}(S(n))\Psi_k(n),\ A_{mk}(S(n)) = \frac{\partial\Phi_k(S(n))}{\partial S_m(n)} \tag{29}$$

or, in the regular case, when the matrix A is regular, we obtain explicit form of the corresponding discrete dynamics

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),\ \Psi_m(n+1) = A^{-1}_{mk}(S(n+1))\Psi_k(n), \tag{30}$$

Now the state vector $S(n)$ and wave vector $\Psi_m(n)$ may correspond not only to the discrete values of the potential $V(n,m) = S_m(n)$, and wave function $\psi(n,m) = \Psi_m(n)$ but also any representation of the computing process from theoretical to experimental realization on a quanputer, including algorithm of solution, higher level programm realization of the algorithm.

Today, without big efforts, we can modify (some) GRID elements in time-invertible form. After development of the quanputer technologies, we can modify (some) GRID elements in quanputer forms.

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