Some properties of Janssen's Fuzzy Argumentation Frameworks

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Abstract

The majority of approaches to abstract argumentation with fuzziness yield some number as part of the computation process, which is then compared to some threshold in order to determine which arguments should appear in an extension. However, identifying such a threshold is difficult, and a more natural approach therefore involves representing the extension as a fuzzy set of arguments. Such an approach was taken by Janssen's fuzzy argumentation frameworks, and in this paper we examine this framework, clarifying some of its definitions and adding some auxiliary notions which make the system more understandable and simplify the computation of extensions. Finally, we consider a specialization of the system by instantiating it with the Gödel t-norm *min*, and demonstrate that Janssen's framework is different from some fuzzy systems, such as the one proposed by de Costa Pereira et al.

Keywords: Argumentation Framework; fuzzy arguments; relabeling

1 Introduction

Given a set of arguments and attacks between them, Dung's seminal abstract argumentation semantics [Dun95] seek to identify which subsets of arguments are in some sense consistent. Many extensions to Dung's original formalism have been proposed, containing for example different types of preference relations [Amg02]. More recently, researchers have begun investigating quantitative extensions to Dung's theory. For example, weighted argumentation frameworks [Cos12, Dun11, Mar08] associate weights with attacks, and then compute extensions based on an *inconsistency budget*. Other numerical approaches consider probability [Hun13, Li12] and fuzziness [Bis10, Don14, Gab15, Gra12, Kac10, Mar08, Tam14].

Such fuzzy arguments are useful in a variety of contexts, such as when differing degrees of membership are present. Here, rather than determining the degree of membership of a set within an extension in a fuzzy way, it makes more sense to consider whether a fuzzy set is within an extension. One work which adopts this approach is that of Janssen's [Jan08].

We begin this paper by introducing Janssen's framework and augment it with additional definitions which allow us to better characterize some of its properties. Following this, in Section 3, we consider a special instance of Janssen's framework in which the truth lattice is the unit interval [0,1] with natural order, and in which the t-norm used is the Gödel t-norm. Finally, we provide a short comparison between Janssen's framework and that of another fuzzy framework [Per11], following which we conclude.

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2 Janssen's fuzzy argumentation frameworks

Janssen's framework (hereafter referred to as JAF) is based on Dung's argumentation framework (AF). The latter consists of a tuple $(Args, Atts)$, where $Args$ is a set of arguments, and $Atts$ is a set of attacks.

To define a JAF, the truth values of arguments and attacks are drawn from a complete lattice $\mathcal{L}$, with a partial order $\geq_{\mathcal{L}}$, greatest element 1 and smallest element 0, together with a negation operator $\neg$, and $\wedge$, a t-norm on $\mathcal{L}$. Given these basic definitions, the implication operator $\rightsquigarrow$ can be defined either as the residual of the t-norm, or through a combination of the negation and t-conorm.

JAFs utilise fuzzy sets, defined through a function $\mathcal{A} : Args \to \mathcal{L}$, determining membership level of each argument $A \in Args$. We then refer to a *crisp set* as a set $S = \{A, \ldots\}$, and refer to its associated fuzzy set $\mathcal{A}(S)$. Here, $S' = \{A \in S \colon \mathcal{A}(A) \neq 0\}$ is the support of this fuzzy set. A fuzzy point is a fuzzy set with the support being a single element A, generally denoted as (A, a).

JAFs can be defined by associating an element of $\mathcal{L}$ with each attack in an AF.

Definition 1 (Definition 3 in [Jan08])**.** *A JAF is a tuple $(Args, \twoheadrightarrow)$, where Args is a crisp set of arguments and $\twoheadrightarrow\colon Args \times Args \to \mathcal{L}$ is a fuzzy relation over Args.*

$\twoheadrightarrow$ is extended to represent the degrees to which fuzzy sets of arguments attack each other as follows.

For B an argument, and $\mathcal{A}, \mathcal{B}$ fuzzy sets of arguments the degree to which $\mathcal{A}$ attacks B is defined as

$$\mathcal{A} \twoheadrightarrow B = \sup_{A \in Args} (\mathcal{A}(A) \wedge (A \twoheadrightarrow B)),$$

and the degree to which B attacks $\mathcal{A}$ is defined as

$$B \twoheadrightarrow \mathcal{A} = \sup_{A \in Args} (\mathcal{A}(A) \wedge (B \twoheadrightarrow A)).$$

Furthermore, the degree to which $\mathcal{A}$ attacks $\mathcal{B}$ is given as

$$\mathcal{A} \twoheadrightarrow \mathcal{B} = \sup_{B \in Args} (\mathcal{B}(B) \wedge (\mathcal{A} \twoheadrightarrow B)).$$

It is not difficult to prove that

Proposition 1. $\mathcal{A} \twoheadrightarrow \mathcal{B} = \sup_{B \in Args}\{\mathcal{B}(B) \wedge (\mathcal{A} \twoheadrightarrow B)\} = \sup_{A \in Args}\{\mathcal{A}(A) \wedge A \twoheadrightarrow \mathcal{B}\} = \sup_{A,B \in Args}\{\mathcal{A}(A) \wedge \mathcal{B}(B) \wedge (A \twoheadrightarrow B)\}$.

Definition 2 (Definition 4 in [Jan08])**.** *Let $(Args, \twoheadrightarrow)$ be a JAF. Then*

A fuzzy set $\mathcal{E}$ over Args is x-conflict-free, $x \in \mathcal{L}$, iff

$$(\neg(\mathcal{E} \twoheadrightarrow \mathcal{E})) \geq_{\mathcal{L}} x.$$

A fuzzy set $\mathcal{E}$ over Args is y-admissible, if it defends itself well enough against all attacks, i.e.,

$$\inf_{B \in Args} ((B \twoheadrightarrow \mathcal{E}) \rightsquigarrow (\mathcal{E} \twoheadrightarrow B)) \geq_{\mathcal{L}} y.$$

A y-preferred extension, $y \in \mathcal{L}$, is maximal (w.r.t set inclusion over fuzzy sets) y-admissible extension.

A z-stable extension, $z \in \mathcal{L}$, is a fuzzy set $\mathcal{E}$, that sufficiently attacks all external arguments, i.e.

$$\inf_{B \in Args} (\neg\mathcal{E}(B) \rightsquigarrow (\mathcal{E} \twoheadrightarrow B)) \geq_{\mathcal{L}} z.$$

While Janssen does not define complete or grounded extensions, this definition was still — to our knowledge — the first to define extensions in the form of fuzzy sets. However, much of his terminology, such as sufficient attack, defends, and so on is somewhat unclear, and we investigate these definitions in more detail below.

First, we consider what *external arguments* — as found in the definition of z-stable extensions — mean. In general, external elements of a fuzzy set $\mathcal{A}$ are those fuzzy points (B, b) — where B is an element of the language, and b a member of the truth lattice — which are not in the fuzzy set. However, within the definition of the z-stable extension, no reference is made to the element b. Therefore, it appears as if the external elements are in fact crisp elements of the support set of the fuzzy set. In order to avoid this mathematical misunderstanding, we therefore introduce the notion of a z-sufficient attack as follows.

Definition 3. *Let $(Args, \twoheadrightarrow)$ be a JAF. For each fuzzy set $\mathcal{A} \subset Args$ and each crisp argument $A \in Args$, we say that $\mathcal{A}$ z-sufficiently attacks A, if*

$$\neg\mathcal{A}(A) \rightsquigarrow (\mathcal{A} \twoheadrightarrow A) \geq_{\mathcal{L}} z.$$

With this definition in hand, we can express the z-stable extension as follows: $\mathcal{A}$ is a z-stable extension, if it z-sufficiently attacks each crisp argument in $Args$.

Another important notion is that of defence, which we obtain from the definition of y-admissible extensions.

Definition 4. *Let $\mathcal{A}$ and $\mathcal{C}$ be fuzzy sets of arguments. We say that $\mathcal{A}$ y-defends $\mathcal{C}$, if for each argument $B \in Args$,*

$$((B \twoheadrightarrow \mathcal{C}) \rightsquigarrow (\mathcal{A} \twoheadrightarrow B)) \geq_{\mathcal{L}} y.$$

Then the y-admissible extensions are those fuzzy sets of arguments which y-defend themselves.

We now turn our attention to how the extensions within the JAF are computed. From Proposition 1, we can obtain the following property.

Theorem 1. *$\mathcal{E}$ is x-conflict-free in a JAF, if for any arguments $A, B \in Args$,*

$$\neg(\mathcal{E}(A) \wedge \mathcal{E}(B) \wedge (A \twoheadrightarrow B)) \geq x.$$

Note that — according to this theorem — there is no relationship between the notions of x-conflict-freeness and z-sufficient attacks. Importantly, the former concept is not required to compute the y-admissible or z-stable extensions, which require only y-defence and z-sufficient attacks to be characterised.

Theorem 2. *$\mathcal{E}$ is y-admissible in JAF, if it y-defends itself, i.e., for each argument $A \in Args$,*

$$((A \twoheadrightarrow \mathcal{E}) \rightsquigarrow (\mathcal{E} \twoheadrightarrow A)) \geq_{\mathcal{L}} y.$$

$\mathcal{A}$ is a z-stable extension, if it z-sufficiently attacks each crisp argument in Args, i.e., for all $A \in Args$,

$$\neg\mathcal{A}(A) \rightsquigarrow (\mathcal{A} \twoheadrightarrow A) \geq_{\mathcal{L}} z.$$

As in standard argumentation approaches, y-preferred extensions are maximal y-admissible extensions, and can therefore be computed after the latter are obtained. Critically however — and unlike in standard abstract argumentation — the lack of requirement on (x-)conflict-freeness means that $Args$ itself is the unique y-preferred extension if it is y-admissible. The same argument can be applied to the z-stable extension.

Example 1. *Let's consider the FAF($\{A\}$,$\{((A,A),1)\}$). It is not difficult to check that the fuzzy set $\{(A,1)\}$ is y-admissible, y-preferred and z-stable, for all $y, z \in [0, 1]$, but not x-conflict-free, for $x > 0$.*

It is possible to add a requirement for x-conflict-freeness to the above definitions. However, characterising the extension in cases where $x \neq y$ or $x \neq z$ for the y-preferred and z-stable extensions respectively requires two values (x, y or x, z) to be considered, making analysis complex. We therefore leave such considerations for future work.

While compact, the formulae of Theorems 1 and 2 are still difficult to analyse as the operators within them ($\wedge, \neg, \rightsquigarrow$) have multiple definitions within the literature. To examine the properties of JAFs in more detail requires us to restrict these operators, and in the next section we do this by considering the Gödel t-norm, the residual implication operator, and a simple negation operator.

3 A special case of JAFs

In this part, we specialize JAFs with the following.

- Args is a finite set.
- The truth lattice $\mathcal{L} = [0, 1]$, with the natural order $\leq_{\mathcal{L}} = \leq$ on [0,1].
- The Gödel t-norm $\wedge = \min$.
- $\neg a = 1 - a$.

- The residual implication, i.e. if $a > b \in [0,1]$, then $a \rightsquigarrow b = b$; otherwise $a \rightsquigarrow b = 1$.

Then the equation in Proposition 1 will be

$$\mathcal{A} \twoheadrightarrow \mathcal{B} = \sup_{A,B \in Args} \min\{\mathcal{A}(A), \mathcal{B}(B), A \twoheadrightarrow B\}. \tag{1}$$

And similar equations can be obtained for $\mathcal{A} \twoheadrightarrow B$ and $B \twoheadrightarrow \mathcal{A}$.

3.1 z-sufficient attacks

Within our specialization, we obtain the following for z-sufficient attacks.

Theorem 3. *A fuzzy set $\mathcal{A}$ z-sufficiently attacks an argument B, iff $\exists A \in Args$, s.t.*

$$\max\{1-z, \mathcal{A}(B)\} + \min\{\mathcal{A}(A), A \twoheadrightarrow B\} \geq 1.$$

Particularly, $\mathcal{A}$ 1-sufficiently attacks B, iff $\exists A \in Args$, s.t.

$$\mathcal{A}(B) + \min\{\mathcal{A}(A), A \twoheadrightarrow B\} \geq 1.$$

Proof. By Definition 3, a fuzzy set $\mathcal{A}$ z-sufficiently attacks an argument B, if $\neg\mathcal{A}(B) \rightsquigarrow (\mathcal{A} \twoheadrightarrow B) \geq z$, i.e.

$$(1 - \mathcal{A}(B)) \rightsquigarrow \sup_{A \in Args} \min\{\mathcal{A}(A), A \twoheadrightarrow B)\} \geq z.$$

By residual property, we have

$$\min\{z, 1 - \mathcal{A}(B)\} \leq \sup_{A \in Args} \min\{\mathcal{A}(A), A \twoheadrightarrow B)\}, \tag{2}$$

which equals to

$$\max\{1-z, \mathcal{A}(B)\} + \sup_{A \in Args} \min\{\mathcal{A}(A), A \twoheadrightarrow B\} \geq 1,$$

i.e. $\exists A \in Args$, such that

$$\max\{1-z, \mathcal{A}(B)\} + \min\{\mathcal{A}(A), A \twoheadrightarrow B)\} \geq 1.$$

Reversing the process, we have if there exists A in $Args$, such that

$$\max\{1-z, \mathcal{A}(B)\} + \min\{\mathcal{A}(A), A \twoheadrightarrow B\} \geq 1,$$

then $\mathcal{A}$ z-sufficiently attacks B.

Replacing z by 1, we have $\mathcal{A}$ 1-sufficiently attacks B iff $\exists A \in Args$ s.t.

$$\mathcal{A}(B) + \min\{\mathcal{A}(A), A \twoheadrightarrow B\} \geq 1.$$

□

In Equation (2), $\sup_{A \in Args} \min\{\mathcal{A}(A), A \twoheadrightarrow B\}$ is the degree $\mathcal{A} \twoheadrightarrow B$. Therefore, if we wish to determine whether $\mathcal{A}$ z-sufficiently attacks B, we only need to compare three values — $z, 1 - \mathcal{A}(B)$ and $\mathcal{A} \twoheadrightarrow B$. For 1-sufficient attacks, only two values must be compared, $1 - \mathcal{A}(B)$ and $\mathcal{A} \twoheadrightarrow B$.

This does however leave an open question, namely if $\mathcal{A}(B) = 1$, i.e., B is a crisp element of $\mathcal{A}$, then it is always the case that $\mathcal{A}$ (counter-intuitively) z-sufficiently attacks B. Understanding the reasons for this is left for future work.

3.2 y-defends

Theorem 4. *A fuzzy set $\mathcal{A}$ y-defends another fuzzy set $\mathcal{C}$, iff for any $B, C \in Args$, there is some $A \in Args$, such that*

$$\min\{y, \min\{(B \nrightarrow C), \mathcal{C}(C)\}\} \leq \min\{(A \nrightarrow B), \mathcal{A}(A)\}.$$

Particularly, $\mathcal{A}$ 1-defends $\mathcal{C}$, iff for any $B, C \in Args$, there is some $A \in Args$, such that

$$\min\{(B \nrightarrow C), \mathcal{C}(C)\} \leq \min\{(A \nrightarrow B), \mathcal{A}(A)\}.$$

Proof. $\mathcal{A}$ y-defends $\mathcal{C}$ iff $\forall B \in Args$, $(B \nrightarrow \mathcal{C}) \rightsquigarrow (\mathcal{A} \nrightarrow B) \geq y$, i.e.

$$\min\{y, (B \nrightarrow \mathcal{C})\} \leq (\mathcal{A} \nrightarrow B). \tag{3}$$

It is

$$\min\{y, \sup_{C \in Args} \min\{(B \nrightarrow C), \mathcal{A}(C)\}\} \leq \sup_{A \in Args} \min\{(A \nrightarrow B), \mathcal{A}(A)\}.$$

That is: $\forall B \in Args$, for any $C \in Args$, there is some $A \in Args$, such that

$$\min\{y, \min\{(B \nrightarrow C), \mathcal{C}(C)\}\} \leq \min\{(A \nrightarrow B), \mathcal{A}(A)\}.$$

Replace y by 1, we get: $\forall B, C \in Args$, $\exists A \in Args$, such that

$$\min\{(B \nrightarrow C), \mathcal{C}(C)\} \leq \min\{(A \nrightarrow B), \mathcal{A}(A)\}.$$

□

Equation (3) demonstrates that in order to determine whether $\mathcal{A}$ y-defends $\mathcal{C}$ or not, one must compare the values of $y, B \nrightarrow \mathcal{C}$ and $\mathcal{A} \nrightarrow B$, $\forall B \in Args$. Particularly, the following property holds, which shows the essence of JAF's defence.

Proposition 2. *$\mathcal{A}$ 1-defends $\mathcal{C}$ if and only if $B \nrightarrow \mathcal{C}$ is no stronger than $\mathcal{A} \nrightarrow B$, i.e., $\forall B \in Args$,*

$$(B \nrightarrow \mathcal{C}) \leq (\mathcal{A} \nrightarrow B).$$

A useful case is that both $\mathcal{A}$ and $\mathcal{C}$ are fuzzy points, i.e. $\mathcal{A} = (A, a)$ and $\mathcal{C} = (C, c)$.

Corollary 1. *(A, a) y-defends (C, c) if and only if, for any B in Args,*

$$\min\{y, \min\{B \nrightarrow C, c\}\} \leq \min\{A \nrightarrow B, a\}.$$

Particularly, (A, a) 1-defends (C, c) if and only if, for any B in Args,

$$\min\{B \nrightarrow C, c\} \leq \min\{A \nrightarrow B, a\}.$$

Example 2. *Given a FAF $(\{A, B, C\}, \{((A, B), 0.9), ((B, C), 0.4)\})$.*[1] *Then $(A, 0.8)$ 1-defends $(C, 0.7)$. But $(A, 0)$ doesn't 1-defend $(C, 0.4)$, instead $(A, 0)$ only 0-defends $(C, 0.4)$.*

This corollary also shows that the fuzzy defends can be calculated point by point.

Corollary 2. *A fuzzy set $\mathcal{A}$ of arguments y-defends $\mathcal{C}$ iff $(A, \mathcal{A}(A))$ y-defends $(C, \mathcal{C}(C))$.*
Particularly, $\mathcal{A}$ 1-defends $\mathcal{C}$ iff $(A, \mathcal{A}(A))$ 1-defends $(C, \mathcal{C}(C))$.

3.3 x-Conflict-free extensions

Theorem 5. *A fuzzy set $\mathcal{E}$ in Args is x-conflict-free, iff for any A, B in Args, one of $\mathcal{E}(A)$, $\mathcal{E}(B)$ or $A \nrightarrow B$ is no more than $1 - x$.*

Proof. $\mathcal{E}$ is x-conflict-free iff

$$\mathcal{E} \nrightarrow \mathcal{E} = \sup_{A, B \in Args} \min\{\mathcal{E}(A), \mathcal{E}(B), A \nrightarrow B\} \leq 1 - x.$$

Then for any $A, B \in Args$, $\min\{\mathcal{E}(A), \mathcal{E}(B), A \nrightarrow B\} \leq 1 - x$, i.e., one of $\mathcal{E}(A)$, $\mathcal{E}(B)$, $A \nrightarrow B$ is no more than $1 - x$. □

Corollary 3. *The fuzzy set $\mathcal{E} \subset Args$ is 1-conflict-free iff for any $A, B \in Args$, at least one of the following holds: $\mathcal{E}(A) = 0$, $\mathcal{E}(B) = 0$ or $A \nrightarrow B$=0, i.e., either A does not attack B, or A or B is not a member of the fuzzy set.*

[1] The degrees of all the attacks not mentioned are 0. And similar for the following.

3.4 y-admissible extensions

By Theorem 4, we immediately get:

Theorem 6. *A fuzzy set $\mathcal{E} \subset Args$ is y-admissible if and only if for any $B, C \in Args$, there is some $A \in Args$, such that*

$$\min\{y, \min\{B \nrightarrow C, \mathcal{E}(C)\}\} \leq \min\{A \nrightarrow B, \mathcal{E}(A)\}.$$

Corollary 4. *A fuzzy set $\mathcal{E}$ is 1-admissible iff $\forall B, C \in Args, \exists A \in Args$, s.t. $\min\{\mathcal{E}(C), B \nrightarrow C\} \leq \min\{\mathcal{E}(A), A \nrightarrow B\}$.*

If $\mathcal{A}$ is a 1-admissible extension and $\mathcal{A} \subset \mathcal{B}$, $\mathcal{B}$ may not be a 1-extension, as per the following example.

Example 3. *Given a JAF $(\{A, B, C\}, \{((A, B), 0.4), ((B, C), 0.9)\})$. Then $\mathcal{E} = \{(A, 1), (B, 0.7), (C, 0.7)\}$ is not 1-admissible, because $\min\{\mathcal{E}(C), B \nrightarrow C\} = 0.7 > \min\{\mathcal{E}(A), A \nrightarrow B\} = 0.4$.. But from Corollary 4, the empty set $\{(A, 0), (B, 0), (C, 0)\}$ is always 1-admissible.*

3.5 z-stable extensions

The following property can be directly obtained from Theorem 3.

Theorem 7. *A fuzzy set $\mathcal{E}$ is z-stable iff for any external argument $B \in Args$, there exists $A \in Args$, such that*

$$\min\{z, 1 - \mathcal{E}(B)\} \leq \min\{\mathcal{E}(A), A \nrightarrow B\}.$$

Corollary 5. *$\mathcal{E}$ is 1-stable iff for any argument $B \in Args$, there exists $A \in Args$, such that*

$$\mathcal{E}(B) + \min\{\mathcal{E}(A), A \nrightarrow B\} \geq 1.$$

Note, in JAF 1-stable may not be 1-admissible, thus not 1-preferred. The following is a counter example.

Example 4. *Given a JAF $(\{A, B, C\}, \{((A, B), 0.4), ((B, C), 0.9)\})$. Then $\mathcal{E} = \{(A, 1), (B, 0.7), (C, 0.7)\}$ is 1-stable, but not 1-admissible.*

Similarly, we can conclude that z-stable extensions may not be z-admissible.

4 Relation to relabelings in [Per11]

In [Per11], a fuzzy argumentation framework with fuzzy arguments and crisp attacks (as opposed to Janssen's crisp arguments with fuzzy attacks) was introduced. Within this system, given some initial fuzzy values for arguments, new values were computed according to a *rewinding procedure* by computing the fixed point for the following function.

$$\alpha_{t+1}(A) = \frac{1}{2}\alpha_t(A) + \frac{1}{2}\min\{\mathcal{A}(A), 1 - \max_{B:\,(B,A) \in Atts} \alpha_t(B)\},$$

In the above, $\mathcal{A}(A)$ is the original value of the argument A and $\alpha_0(A) = \mathcal{A}(A)$, for all $A \in Args$.

The sequence $\alpha_t(A)$, $\forall A \in Args$, was shown to converge, with the limit denoted by $\alpha(A)$. This $\alpha(A)$ was considered to be the *relabelled value* of A.

Given this approach, we ask whether, if α is considered as a fuzzy set of arguments, it yields the same semantics as JAF. However, even if JAFs are extended to permit fuzzy arguments with crisp attacks[2], then the following counterexample shows that the two semantics differ.

Example 5. *Given a fuzzy AF $(\{(A, 0.1), (B, 0.8), (C, 1)\}, \{((A, B), 1), ((B, C), 1)\})$. $\alpha = (\{(A, 0.1), (B, 0.8), (C, 0.2)\}$ is the relabeling result of [Per11]. And Corollary 1 shows that, $(A, 0.1)$ y-defends $(C, 0.2)$ only for $y \leq 0.1$.*

Now if the belief degree of A drops to 0, the framework becomes $(\{(A, 0), (B, 0.8), (C, 1)\}, \{((A, B), 1), ((B, C), 1)\})$ and $\alpha = (\{(A, 0), (B, 0.8), (C, 0.2)\}$ is the relabeling. In this case, $\{(A, 0), (B, 0.8), (C, 0.2)\}$ is not y-admissible, for any $y > 0$. However, it is meaningless to say a fuzzy set is 0-admissible*, because Theorem 4 shows every fuzzy set is 0-admissible.*

[2] such an extension is easy to instantiate by restricting the fuzzy subsets to the set of original arguments, and by treating the crisp attacks as a special case of fuzzy attacks.

In general, the relabelling approach of [Per11] will not result in y-admissible extensions, and will therefore not be y-preferred.

There is therefore a significant difference in the way the two approaches select arguments to be within an extension, and characterising these differences further forms a portion of our ongoing work.

5 Conclusion

In this work we analysed Janssen's fuzzy argumentation frameworks in detail. We began by reexamining the Janssen's definition, and described z-sufficient attack and y-defence based on how extensions are computed within JAFs. We then considered a special case of JAFs and described how the semantics of such a system can be efficiently computed. Finally, we demonstrated — by means of a simple example — the fact that JAFs cannot be used to encode the relabellings used by [Per11], and that further enhancements to JAFs are therefore required. We are at present investigating how the strengths of Janssen's approach to fuzzy argumentation can be combined with the benefits of the work described in [Per11].

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