Filtration and Restoration of Satellite Images Using Doubly Stochastic Random Fields

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Abstract. The paper is devoted to filtering algorithms of satellite images. Inadvisability of applying the simplest mathematical models of random fields with non-uniform filtering material is shown. We consider the comparative analysis of effectiveness of the filtering and calculate the gain of the proposed algorithm. In addition, we have sufficient by adequate enough satellite image restore when applying doubly stochastic models. Restoration algorithm that can easily be implemented from different positions of the image is described. Dispersion values for recovery errors were found under using different models. We also have received the gain in image restoration by providing adequate description of satellite images unlike in application of autoregression (AR) models.

Keywords: Image processing, image filtration, Kalman filter, parameters estimation, image restoration, doubly stochastic models, random fields

1 Introduction

In many cases, transfer of multidimensional data with errors shadowing images or badly damaged them by noise arises the problem of recovering the missing fragments of images [1–3, 8] or filtering [4–6].

The white noise filtering is possible in the case of using the well-proven Kalman filter, which allows one of the reasonably accurate estimation without requiring significant computing expenditures.

One of the methods of restoration, essentially is in an image replacement by some model in the damaged area. However, in real-world images the damaged area can contain any objects, description of which is possible using inhomogeneous models. Therefore, to use this method, we must find adequate model. Most of the existing models [2, 7] are unable to provide adequate replacement of damaged areas due to some reasons. However, we can use the combination-mixed models of the images.

Quite a common option of such models is doubly stochastic ones [3, 5, 9] or models, which vary its parameters from pixel to pixel.

Another important feature of the filtering and restoration results is the need for their use in solving problems of signal detection in images [10, 11].

Thus, the purpose of this work is to improve effectiveness of the image filtering and restoration by applying models of images with varying parameters. Note that a comparison will be made on the criterion of minimum error dispersion.

2 Images filtration

Although signal detection is very important, effectiveness of the work of all algorithms significantly depends on the source material, and usually images are distorted versions of the raw data. So, received images may have different shifts, shading, as well as, be quite noisy nuisance. Moreover, strong interference leads to almost total loss of information at the site of exposure. Therefore, the most important stage of preprocessing is filtration.

We consider the following doubly stochastic model of random fields:

$$x_{i,j} = \rho_{xi,j} x_{i-1,j} + \rho_{yi,j} x_{i,j-1} - \rho_{xi,j}\rho_{yi,j} x_{i-1,j-1} + \xi_{i,j}, \tag{1}$$

where $\rho_{xi,j} = \tilde{\rho}_{xi,j} + m_{\rho x}$ are the row correlation parameters field; $\rho_{yi,j} = \tilde{\rho}_{yi,j} + m_{\rho y}$ is the column correlation parameters field; $m_{\rho x}$ and $m_{\rho y}$ are the average values of correlation parameters random field for row and column respectively; $\xi_{i,j}$ is the random field of independent Gaussian random values having average $M\{\xi_{i,j}\} = m_{\xi_{i,j}} = 0$ and dispersion $M\{\xi_{i,j}^2\} = \sigma_{\xi_{i,j}}^2 = \sigma_x^2 \left(1 - \rho_{xi,j}^2\right)\left(1 - \rho_{yi,j}^2\right)$; σ_x^2 is the base random field dispersion.

The random fields that describe changes in the correlation coefficients are described as follows:

$$\begin{aligned} \tilde{\rho}_{xi,j} &= r_{1x}\tilde{\rho}_{xi-1,j} + r_{2x}\tilde{\rho}_{xi,j-1} - r_{1x}r_{2x}\tilde{\rho}_{xi-1,j-1} + \varsigma_{xi,j}, \\ \tilde{\rho}_{yi,j} &= r_{1y}\tilde{\rho}_{yi-1,j} + r_{2y}\tilde{\rho}_{yi,j-1} - r_{1y}r_{2y}\tilde{\rho}_{yi-1,j-1} + \varsigma_{yi,j}, \end{aligned} \tag{2}$$

where $r_{1x}, r_{2x}, r_{1y}, r_{2y}$ are the constant correlation parameters of the internal random fields; $\varsigma_{xi,j}$ and $\varsigma_{yi,j}$ are the independent Gaussian random values with zero average and dispersions $M\{\varsigma_{xi,j}^2\} = \sigma_{\varsigma x}^2 = \sigma_{\rho_x}^2 \left(1 - r_{1x}^2\right)\left(1 - r_{2x}^2\right)$, $M\{\varsigma_{yi,j}^2\} = \sigma_{\varsigma y}^2 = \sigma_{\rho_y}^2 \left(1 - r_{1y}^2\right)\left(1 - r_{2y}^2\right)$; $\sigma_{\rho_x}^2$ and $\sigma_{\rho_y}^2$ define the dispersions of the basic random fields of correlation parameters for the row and column, respectively.

Thus, in order to solve the problem of parameter estimation, it is necessary to estimate random fields $\rho_{xi,j}$ and $\rho_{yi,j}$ in model (1). It should be noted that for the case of doubly stochastic models the important property is the ability to apply recurring evaluation procedure [2] that would only slightly increase the computational expenditures.

Suppose that the input signal of a monitoring system at the entrance is the sum of the useful signal (1) and additive white Gaussian noise $\{n_{i,j}\}$ with average $m_n = 0$ and dispersion σ_n^2

$$z_{i,j} = x_{i,j} + n_{i,j}. \tag{3}$$

We shall use the vector nonlinear Kalman filter to make the filtering process of the flat image. Therefore, we must obtain a vector of the image line items that can be written as follows:

$$\boldsymbol{x_i} = (x_{i_1},\ x_{i_2}, \dots ,\ x_{i_N})\,. \tag{4}$$

In this case, we write a generalized expression model for the flat image in accordance in the following form:

$$\begin{aligned} &\boldsymbol{x_i} = diag\,(\boldsymbol{\rho_{x_i}})\,\boldsymbol{x_{i-1}} + \vartheta\,(\boldsymbol{\rho_{x_i}}, \boldsymbol{\rho_{y_i}})\,\boldsymbol{\xi_i},\\ &\boldsymbol{\rho_{x_i}} = r_{1x}\boldsymbol{\rho_{x_{i-1}}} + \vartheta_{\rho_x}\boldsymbol{\xi_{xi}},\\ &\boldsymbol{\rho_{y_i}} = r_{1y}\boldsymbol{\rho_{y_{i-1}}} + \vartheta_{\rho_y}\boldsymbol{\xi_{yi}}, \end{aligned} \tag{5}$$

where $diag\,(\boldsymbol{\rho_{xi}})$ is the diagonal matrix with elements $\left(\rho_{x_{i_1}},\ \rho_{x_{i_2}}, \dots ,\ \rho_{x_{i_N}}\right)$.

Finally, expression for the process of line-by-line estimation is written as follows:

$$\hat{\boldsymbol{x}}_{pi} = \hat{\boldsymbol{x}}_{epi} + P_i \frac{\partial \Phi^T}{\partial \boldsymbol{x}_{pi}} (\boldsymbol{z}_i - \hat{\boldsymbol{x}}_{epi})\,. \tag{6}$$

It should be noted that application of the nonlinear vector Kalman filter is possible if the signal model is known. Thus, to make signal model known, it is necessary to have information about coefficients r_{1x}, r_{2x}, r_{1y}, and r_{2y} and, in addition, the statistical characteristics of the model such as $m_{\rho x}, m_{\rho y}$, and $\sigma^2_{\rho x}, \sigma^2_{\rho y}, \sigma^2_x$. If the receiving part has no *a priori* information tagged with parameters, we must perform a preliminary assessment, for which it is proposed to use the algorithm for pseudogradient search [3, 7].

In addition to filter (6), we will explore a number of filtering algorithms both for AR models and doubly stochastic ones.

1) Vector Kalman filter for the AR model with $\rho_{xi,j}$ =const and $\rho_{yi,j}$ =const.

2) Wiener filter for AR model with the covariance function

$$B\,(l, k) = \sigma^2_x m^{|l|}_{\rho x} m^{|k|}_{\rho y}.$$

3) Vector Kalman filter for the AR model with reverse swing (interpolation).

4) Vector Kalman filter for the doubly stochastic model with reverse swing (interpolation), for which

$$\begin{pmatrix} \hat{x}_{i,j} \\ \hat{\rho}_{xi,j} \\ \hat{\rho}_{yi,j} \end{pmatrix} = \Phi\,(x_e, \rho_{xe}, \rho_{ye}, V_n)\,.$$

Figure 1 presents filtering error dispersion dependencies on noise dispersion. We can see that if the image is close to identical and similar only in certain segments, we have effective filtration by only the fourth algorithm.

As for the gain (Figure 2), note that the maximum gain is achieved at low values of the noise dispersion, and then we get the stabilization of the gains. We note that for a single noise dispersion, algorithm of the Kalman filter with vector interpolation works almost 2 times more precisely than the Kalman and Wiener filters with interpolation configured for the AR model. Furthermore, the gain compared to Kalman without interpolation is much larger.

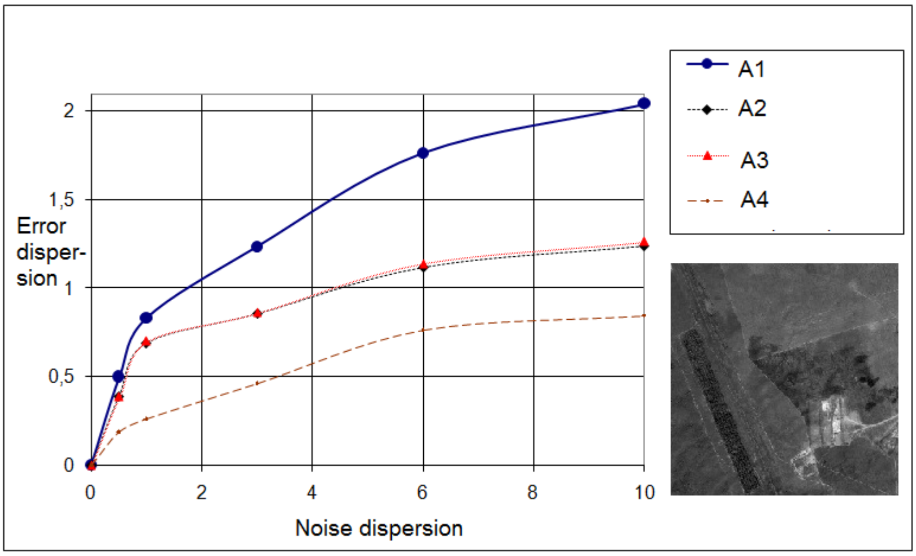

Fig. 1. Filtration efficiency of the heterogeneous image.

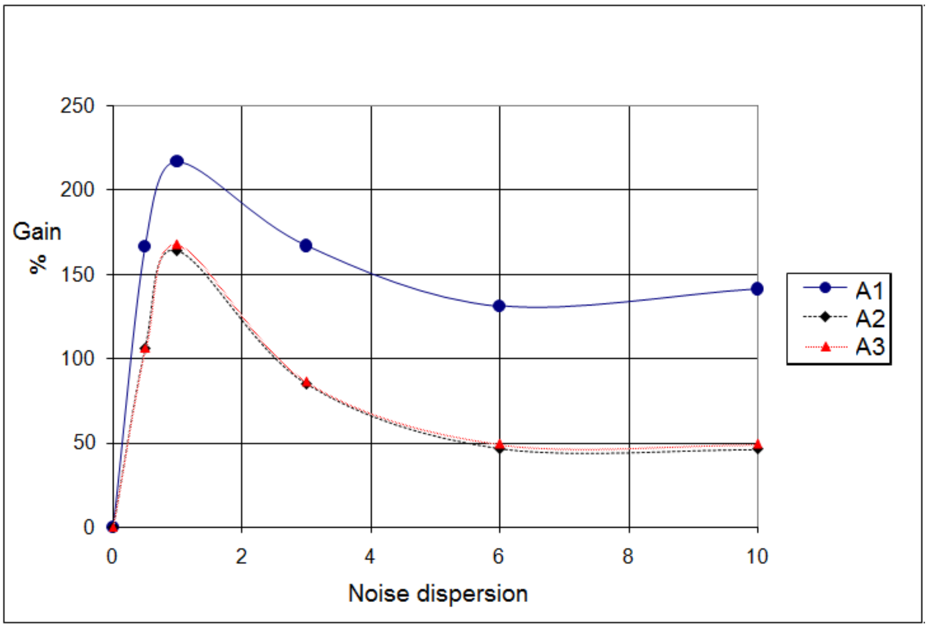

Fig. 2. Gain when heterogeneous images are filtered.

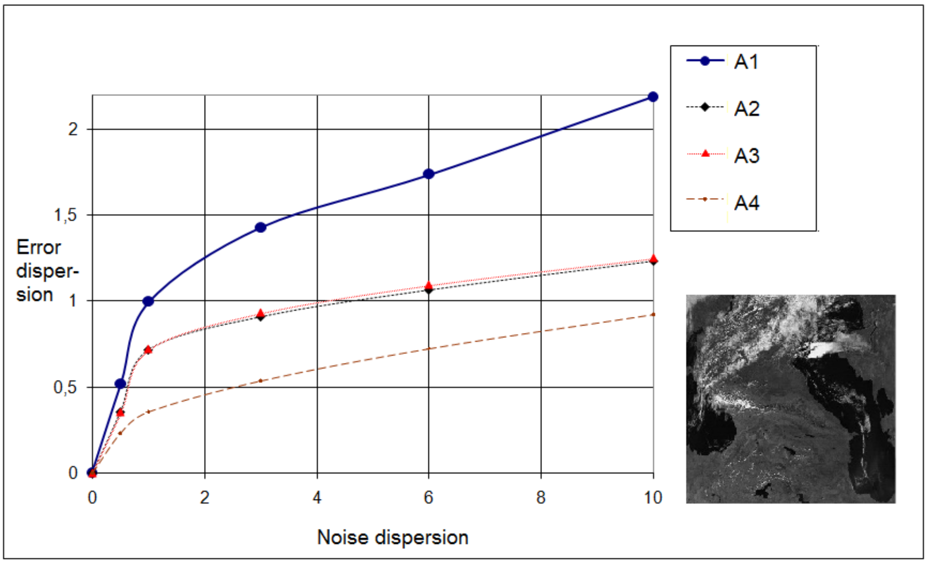

Fig. 3. Filtration efficiency of image with changes in brightness.

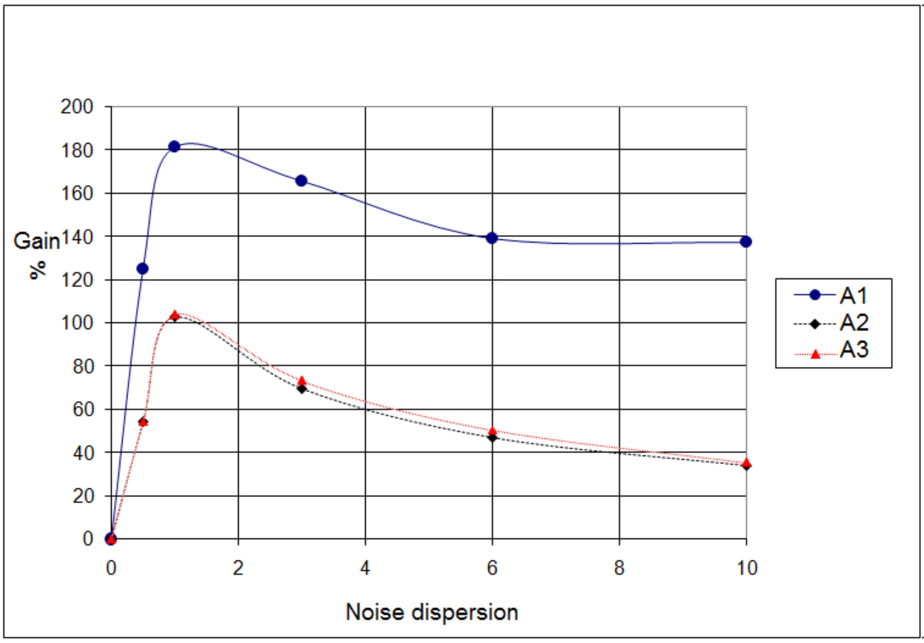

Fig. 4. Gain when images with brightness variations are filtered.

Obviously, the heterogeneous image filtering with variations in brightness (Figure 3) can not be used without analysis of correlation parameters. So, here, there is the fourth algorithm that is also applied more accurately. However, in terms of gains (Figure 4), such algorithm provides smaller indicators since the main image has the heterogeneity of the sharp variations in brightness.

Thus, filtering algorithm based on the doubly stochastic model provides the best results for real images.

3 Images restoration

Consider restoring the square area on the image using a model with variable parameters. Let the brightness values of the image representing the random field be $\{Z_{i,j}; i = 1, 2,\ldots, M_1; j = 1, 2,\ldots, M_2\}$. There is the damaged area starting at the point (i_0, j_0) of the $c \times c$-dimension. Denote this area as D. We introduce the following restoration model [12]

$$X_{i,j} = \left\{ \begin{array}{ll} Z_{i,j}, & \text{if (i,j)}\notin\text{D} \\ \rho_{1i,j}\left(X_{i-1,j} - \boldsymbol{X_{i,j}}\right) + \rho_{2i,j}\left(X_{i,j-1} - \boldsymbol{X_{i,j}}\right) - & \\ -\rho_{1i,j}\rho_{2i,j}\left(X_{i-1,j-1} - \boldsymbol{X_{i,j}}\right) + \boldsymbol{X_{i,j}} + \xi_{i,j}, & \text{if (i,j)}\in\text{D} \end{array} \right\}, \tag{7}$$

where $\rho_{1i,j}, \rho_{2i,j}$ are the estimations of the correlation coefficients for row and column at the point (i, j); $\boldsymbol{X_{i,j}}$ is the estimation of the average value at the point (i, j); $\xi_{i,j}$ is the Gaussian random field with average $M\{\xi_{i,j}\} = 0$ and dispersion $\sigma_\xi^2 = \sigma_{i,j}\sqrt{(1 - \rho_{1i,j}^2)(1 - \rho_{2i,j}^2)}$, where $\sigma_{i,j}^2$ is the estimation of the dispersion at the point (i, j).

It is advisable to assess parameters in the sliding window excluding the points that lay in the damaged area. Model (7) is a Habibie one with variable parameters in the area D. For the window with $N \times N$-size, estimates are determined by the formulas

$$\begin{array}{l} \boldsymbol{X_{i,j}} = \frac{1}{N^2}\sum_{u=-\frac{N}{2}}^{\frac{N}{2}}\sum_{q=-\frac{N}{2}}^{\frac{N}{2}} Z_{i+u,j+q} \\ \sigma_{i,j}^2 = \frac{1}{N^2}\sum_{u=-\frac{N}{2}}^{\frac{N}{2}}\sum_{q=-\frac{N}{2}}^{\frac{N}{2}} \left(Z_{i+u,j+q} - \boldsymbol{X_{i,j}}\right)^2 \\ \rho_{1i,j} = \frac{1}{\sigma_{i,j}^2 N^2}\sum_{u=-\frac{N}{2}}^{\frac{N}{2}}\sum_{q=-\frac{N}{2}}^{\frac{N}{2}} \left(Z_{i+u,j+q} - \boldsymbol{X_{i,j}}\right)\left(Z_{i+u-1,j+q} - \boldsymbol{X_{i,j}}\right) \\ \rho_{2i,j} = \frac{1}{\sigma_{i,j}^2 N^2}\sum_{u=-\frac{N}{2}}^{\frac{N}{2}}\sum_{q=-\frac{N}{2}}^{\frac{N}{2}} \left(Z_{i+u,j+q} - \boldsymbol{X_{i,j}}\right)\left(Z_{i+u,j+q-1} - \boldsymbol{X_{i,j}}\right) \end{array} \tag{8}$$

To restore the damaged area, we shall make an assessment in neighbourhood of the area D. In addition, to ensure greater heterogeneity, we divide area D into sub-areas, onto each of which we shall expand the model with the estimated parameters (8).

Evaluation system (8) gives estimates based on the motion of the window from left to right and from top to bottom, *i.e.*, the model unfolds from the top left corner of the area D. You can get similar expressions for motion of the window from other corners. It is clear that the estimates for different starting points of evaluation will vary. This is due to the fact that the basic values of the model implementation will depend on intact neighborhood, and it, in the turn, is determined by its position on the image. Thus, the nearest surroundings will change when the starting point for deployment model changes.

To assess the performance of the proposed algorithm, we shall implement restoration method on the different images. When we do this we compare restoration (7) with the restoration by the model of Habibie that can be written:

$$X_{i,j} = \left\{ \begin{array}{ll} Z_{i,j}, & \text{if (i,j)}\notin\text{D} \\ \hat{\rho_1}\left(X_{i-1,j} - \boldsymbol{X}\right) + \hat{\rho_2}\left(X_{i,j-1} - \boldsymbol{X}\right) - & \\ -\hat{\rho_1}\hat{\rho_2}\left(X_{i-1,j-1} - \boldsymbol{X}\right) + \boldsymbol{X} + \xi_{i,j}, & \text{if (i,j)}\in\text{D} \end{array} \right\}, \qquad (9)$$

where $\hat{\rho_1}, \hat{\rho_2}$ are the estimation of correlation parameters for the row and column; $\boldsymbol{X}$ is the average value estimation; $\xi_{i,j}$ is the Gaussian random field with average $M\{\xi_{i,j}\} = 0$ and dispersion $\sigma_\xi^2 = \sigma\sqrt{(1-\hat{\rho_1}^2)(1-\hat{\rho_2}^2)}$, where σ^2 is the estimation of the dispersion.

So, the restoration algorithm with the Habibie model (9) requires only one assessment of the image parameters.

Figures 5-7 show the different damaged images and the result of their recovery: a) damaged image, b) restore from the upper left corner, c) restore from a right corner, d) restore from the left bottom corner, e) restore from a right corner, f) restore based on the Habibie model from the upper-left corner.

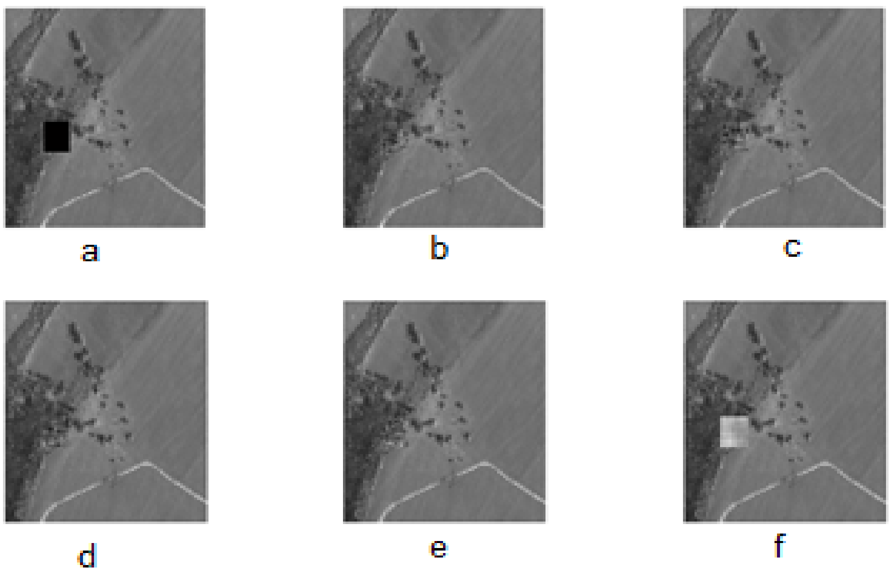

Fig. 5. Restoration of the area of the image on the border of two dissimilar surfaces.

Dispersions of the restoration error (Figure 5) are the following:
b: 0.046, **c:** 0.060, **d:** 0.045, **e:** 0.043, **f:** 0.335.

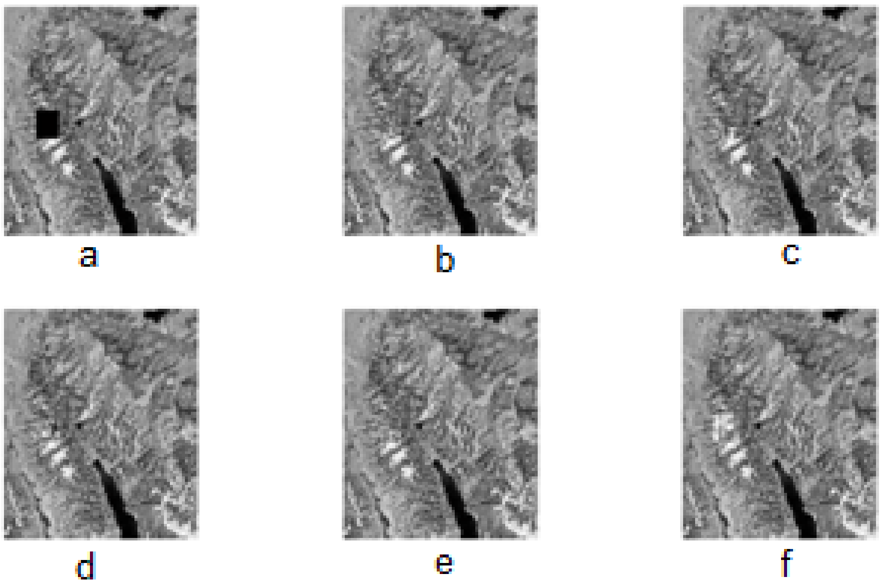

Fig. 6. Restoration of the image area close to uniform.

Dispersions of the restoration error (Figure 6) are the following:
b: 0.031, **c:** 0.040, **d:** 0.035, **e:** 0.034, **f:** 0.043.

Analysis of the errors for restoration in Figure 6 shows that in the case of a homogeneous area of the image, results in the implementation of algorithms are close enough regardless the initial point. However, slight variation of the dispersion values of the error may be due to the fact that the implementation of a model uses a random field. Consequently, the value of the restored pixel brightness is accidental. It also should be noted that the image selected in Figure 6 consists the inhomogeneities. Such structure also affected the calculation of variance of the restoration error. However, the restoration results in Figure 6b — e are significantly better than restoration results in Figure 6f. Firstly, it is due to the fact that the model with variable parameters is better suited for the description of the original image. Secondly, implementation of the Habibie model leads to using the constant correlation coefficients, although the connection between the real image pixel does not correspond to this description. Finally, we consider the restoration of the image area when the neighborhood is different from different sides, *i.e.*, there are either diverse objects at the corners of the damaged section or there is a difference in the brightness values.

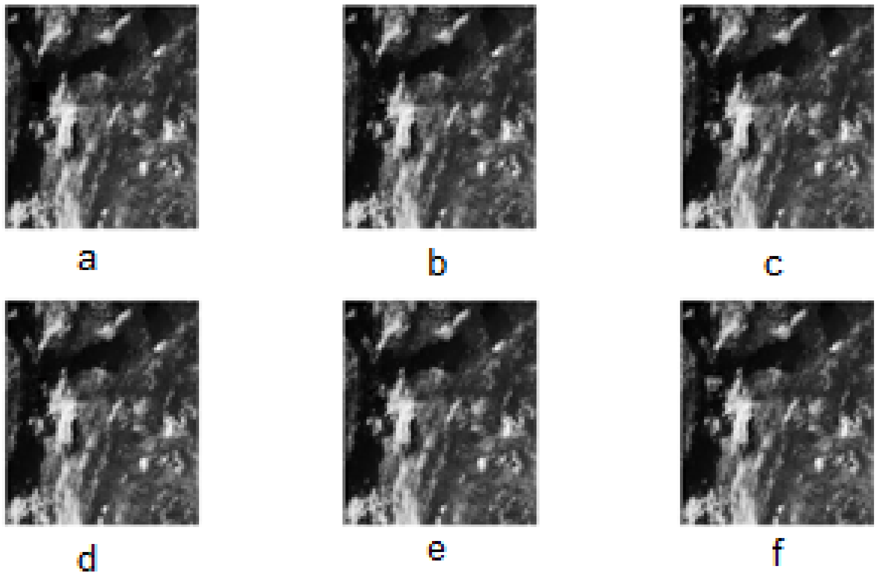

Fig. 7. Restoration of the image area limited by different structures.

Dispersions of the restoration error (Figure 7) are the following:
b: 0.005, **c:** 0.003, **d:** 0.006, **e:** 0.008, **f:** 0.015.

An error dispersion investigation for the Figure 7 revealed that when the damaged area is bounded on different sides of the pixels with different brightness, it is a very important factor what side is basic for restoration. Indeed, restoration from the right upper corner of the neighborhood is based on the dark area closest to the brightness of the damaged portion. Further, the order of similarity of the neighborhood brightness coincides exactly with the order of increasing error variance. Therefore, when the brightness of the vicinity parts is closer to the brightness of the damaged area, we have the better restoration results. Thus, for Figure 7, restoration algorithm using the Habibie model is considerably inferior to the algorithm based on the use of complex (doubly stochastic) models. This is primarily due to the heterogeneity of the original image.

The size of the damage is $c = 40$ for all images. The images sizes are the following: Figure 5 290×290, Figure 6 330×330, Figure 7 440×440. Dispersions of the error were calculated from relations for dispersion of the images.

The analysis shows that restoring (7) is better suited to heterogeneous images. Restoration using the Habibie model looks much worse even in visual perception. In addition, the value of the error restoring dispersion in the cases examined also depends on the ratio of size of the damage to the image size. Obviously, if it is smaller, then the accuracy is higher.

It should be noted that application of the doubly stochastic model allows one to have information on the undamaged neighborhoods in the form of parameter fields that provides better restoration. Analysis of the results shows that the efficiency of restoring depends on the starting position of the model implementation that makes it possible to increase the efficiency of the restoration of the damaged area due to splitting into the smaller subareas. For these subareas, we choose the best neighborhood, in which estimation is made.

Thus, considered restoring algorithm based on the use of models with varying parameters unlike the Habibie model restoration is able to restore the heterogeneity areas and is generally superior to the latter.

4 Conclusion

Comparative analysis of four algorithms of filtering was described in details. Researches were performed for the different images.

It was found that the gain of the vector Kalman filter stabilizes with noise dispersion increasing.

The vector Kalman filter for the doubly stochastic models with reverse swing provides the significant gain (40 — 50%) for the actual images that cannot be adequately described by the AR models of random fields.

The algorithm of restoring damaged areas on images based on mathematical modeling was suggested. Analysis of the results obtained shows that to restore satellite images, it is appropriate to use models with varying parameters.

Improvement of efficiency of the algorithm in future can be obtained by aggregation of the restoration results obtained for different directions.

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