

Visual Metaphor of Mathematical Abstractions and Their Visualization through Newly Uprised PDF-document Facilities

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Abstract. PDF format documents with new interactive and dynamic facilities essentially improve the effectiveness of teaching/ learning process, especially in mathematics education. The main problem in preparation of PDF documents is developing an adequate visual metaphor and a suitable and inexpensive program instrument for metaphor implementation. The \LaTeX (\pdf\LaTeX) is the most popular text processor in academic society, that produces an excellent mathematical papers. Graphs and pictures can be produced by many graphical programs (MetaPost, TikZ, Asymptote, etc.) and systems like the Maple with various output formats. This especially refers to 3D images with a vast collection of formats. Here a brief survey is done for the role of mathematical visualizations in teaching, and the implementation examples are presented for transforming initial 2D and 3D mathematical images into a PDF file by the embedding technology.

Keywords: Visualization, embedding 3D images, mathematical image

1 Introduction

Man perceives information mainly via his eyes and ears. Human brain plays a great role in visual perception process together with eyes and the whole nervous system. So, the graphical language takes full advantage of capabilities of human visual system.

The word *visualization* has been used in many areas of knowledge: architecture, medicine, science [1], software [2], information visualization [3], “big data” visualization [4] and so on. In this paper, the visualization of mathematical abstract entities and concepts is used to facilitate teaching/learning process and communication between teacher and students [5].

A word *visualization* has nowadays many senses: architecture, medical, scientific [1], software [2], Information Visualization [3], “big data” visualization [4] and so on.

Our particular focus is the visualization of mathematical abstract entities and concepts for pedagogical purpose and communication [5].

An inseparable element of the educational technology is imaging. That is used as actual teaching tool and as the genuine knowledge component of the studied discipline. Spectrum of the visual facilities is determined by the technical equipment of the educational process and essentially depends on the teacher's ability and skills.

The open education concept caused radical restructuring of the classical scheme of direct interaction Teacher-Student and not only on the organizational and management level. Learning materials need to be able to provide feedback in the mediated chain of Teacher-Teaching Media-Student. The visual metaphor gives students more concise and acceptable understanding.

The main purpose of this paper is using dynamical 2D and 3D images in the form of PDF slides and pages for visualization mathematical mental objects and their properties while teaching students.

2 Role of mathematical visual mathaphor in education process

Various types of artistic drawing (art of drawing), especially, painting have at their disposal a variety of techniques for creating in the viewer mind the illusion of natural perception. An alternative system of image objects is classical engineering drawing reincarnated now in the AutoCAD software and the similar ones. This is actually a rigorous specific language with his semiotics (a set of symbols), syntax (the rules of combining individual graphic elements into a holistic image), semantics, and pragmatics. It should be noted that there are known attempts to systematize scientific (and others) visualization [6].

Preparation of engineering drawings, as well as reading them is impossible for user without training and time-consuming preparation. But good trained user (such as a designer, engineer, worker receives a comprehensive description of individual parts shape and materials and how to unite them into whole devices of all arbitrarily complex products.

For scientific or educational purposes, neither one of the above mentioned drawing systems is suitable in the whole, because now we have to deal with abstract structures, concepts, and notions. This raises the problem of adequate visual metaphors [5], reflecting on the one hand, the properties of the abstract and, on the other hand, taking into account peculiarities of the human visual perception and thinking.

One can note existence of a large number of well-established conventional visual metaphors for mathematical objects, concepts, and properties with different graphical complexity. Also, students are familiar with many of such objects by longstanding scholarship practice; for example, the function graphic, i. e., the curve, which shape displays mapping from domain into region, or the slope of the tangent line as a function derivative.

New a original author's metaphor instruments should satisfy the requirement: one can comprehend the depicted concept or idea.

Modern graphical tools allow one display dynamical images. One can control the level of inputs and outputs for logical transformations, formulae exploration, and cover and recover the text fragments.

The instructor want to explain to students how to solve some difficult problem. He talks with students and at the same time showing a dynamic slide (Fig. 1). Every steps of his explanation connect with certain picture associated with a particular image via appearing arrows. This is an example of the most attractive feature of AcroJS (Acrobat Java Script): the opportunity it provides in the way of Form validation and interactivity. Nice and effective tool is animation or “multiplication” — a consecutive display of the pictorial sequence. According to one student: “I lose track if I try to trace staff in my mind, but animation kept me from losing the track.”

One more example is shown on Fig.2. Convolution of a piece-wise functions calculation is a very hard task for many students. We offer them to draw “multiplication” frames — aka in Russian for animation, because it is native language for my students — for “moving” graph of one function relative to the “stationary” other. So we use a familiar mental pattern, and students easily organize the calculation — to correctly place the integration limits.

Dedactical animation should provide the control for required speed and zooming. The example above mentioned is about “zero-speed” framerate — the student should understand what caused the reduction of the integration limits. In other words frames are the depicted base for intuitive conjecture.

The more faster animation can illustrate the Law of Large Numbers graphically. One demonstrates convolution of a probability density functions of uniform distribution. Number of terms grows: two, three, four, to ten. As a result one can see: a triangle, and other curves, shown on Fig. One more example is an experiment modeling two falling dice 4. An observer gets only general impression.

A flat image has a fairly limited ability to adequately transfer the form of spatial objects to spectator, since there is only one figure projection onto the plane. So the human visual perception system has also as “source code” a projection of the object onto the retina.

However, this projection, first, reflects the perspective distortion of the image, secondly, it dynamically changes it due to reflex motions of the eyeballs and lens deformation. In addition, the nerve and brain structures participate together in this process. As a result of the combined action of all elements of the human visual system, the person receives information about a material object position in the space relatively to other objects.

In the communication process, the very essential thing is physical and material embodiment of a visual metaphor. In this aspect, a crucial role is played by presence of a modern infrastructure for the creation, transmission, and display of graphical information. Perhaps the development of computer technology has initiated the emergence of a wide range of types of visualizations including more than one hundred elements [8].

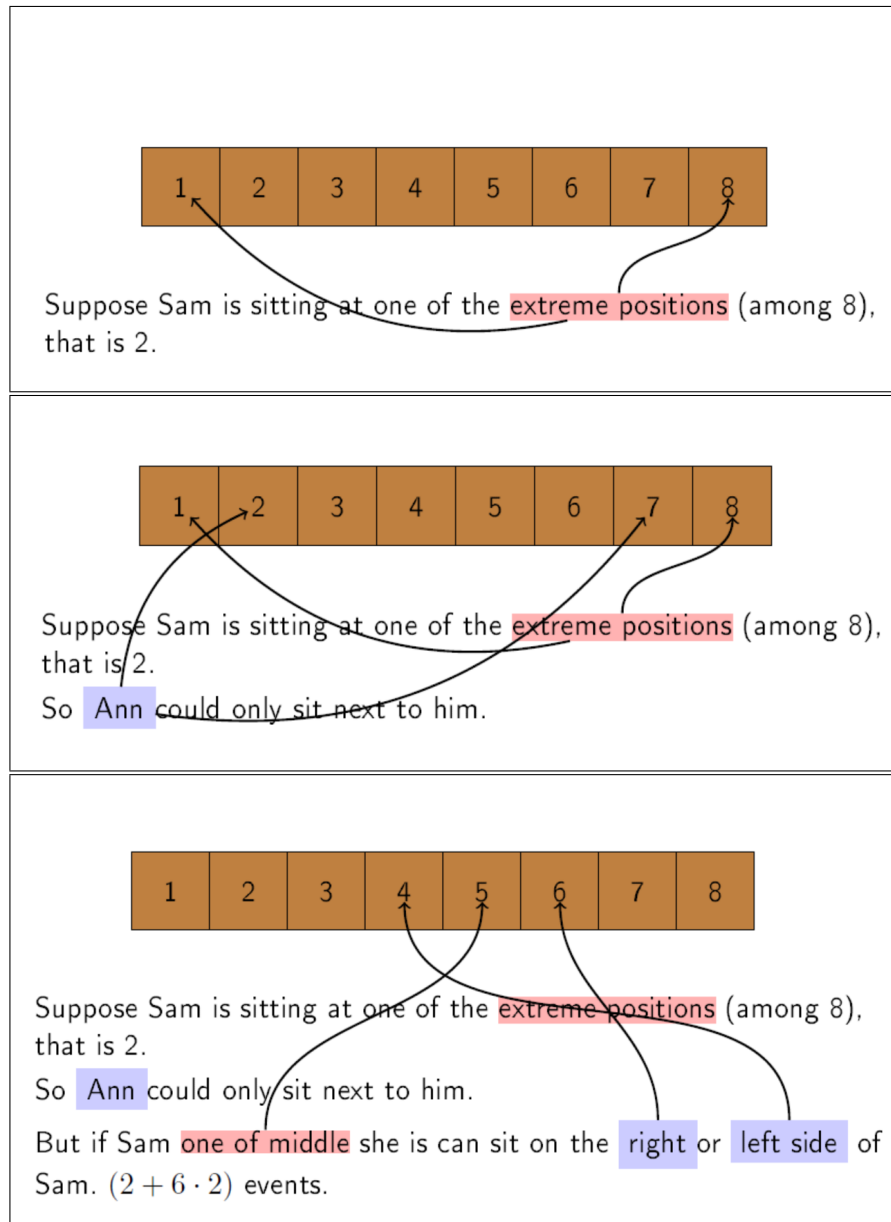


Fig. 1. The fragments of the lecture slides with the visual explanation — “the moving arrows”.

3 Learning PDF materials with 2D and 3D images

Portable Document Format (PDF) (developed, improved and maintained by Adobe) is one of the wide-spreading document standards for platform-independent exchange.

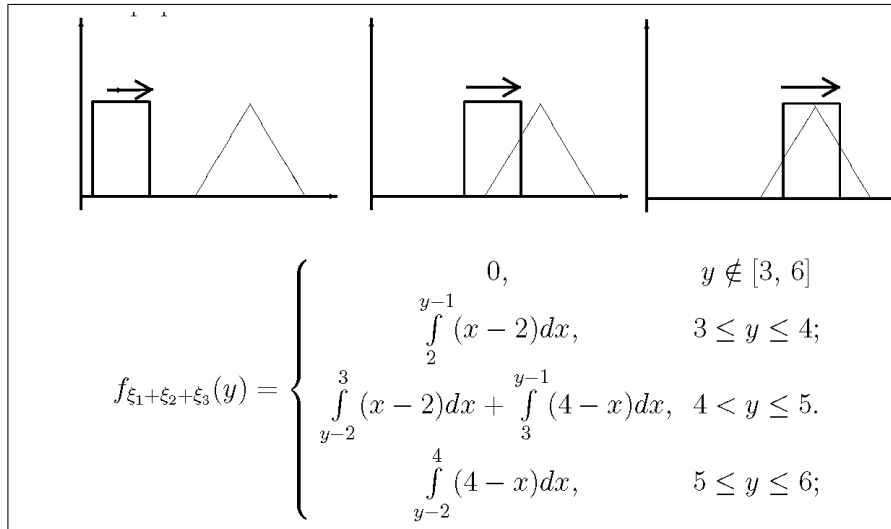


Fig. 2. Fragment of one page [7] with the visual explanation of two functions convolution

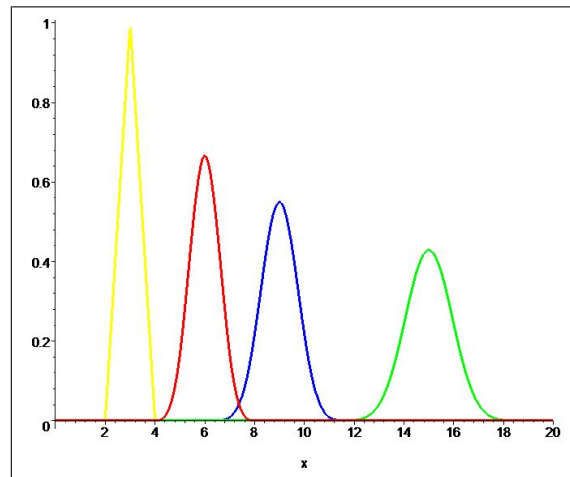


Fig. 3. Distribution functions for the sums of two, four, six, and ten uniformly distributed stochastic variables. Calculated via the Maple [7].

The main features of the file formats in the use of learning are:

- presence of the built-in JavaScript programming language;
- controlled multi-layered structure of the slide, which supports animation and quasi-animation;
- possibility of introducing (embedding) multimedia objects;

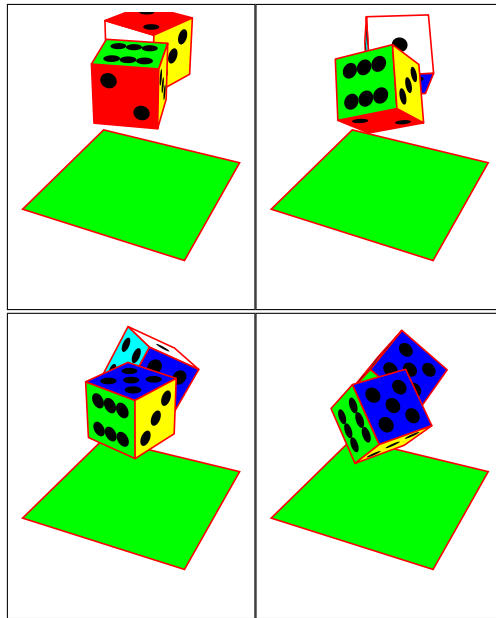


Fig. 4. Frame of a lecture animated slide in the experiment with two falling dice.

- a built-in mechanism for interactive viewing of 3-dimensional objects from the embedded U3D and PRC formats source.

Internal JavaScript programming language allows using many practical features. For example, an author may vary layer transparency of the slide frame and places snippets of text, formulas, and drawings on these layers. Further, we show fragments of diagrams, charts, and chain formulas in an arbitrary order that provides the dynamics of the slide. Switching transparency occurs “on event” by pressing the mouse button or control button.

The source LaTeX code of these features are given by special tags, which in translation are handled by the respective expansion packages. Detailed description is not necessarily here, because the popular assembly like MikTeX or TeXlive contain packages themselves, and the author’s guidance documentation are pre-installed. In the absence of a needed package the package manager makes a request to install it. User can elaborate the operating system to manage this package.

The PDF document can be embedded with animation reproducible both by external program or by free Adobe Reader. Suitable programm tool (which provides source and converts it) named as *animate* package is also included into the assembly and is well documented.

Making teaching animations is very easy. Each frame of the animation is made in a graphical program tool (MetaPost or similar one) like regular pictures.

Unlike standard movie/video formats, package “animate” allows for animating vector graphics, and can be used to assemble animations from sets of existing graphics files or from multipage PDF.

Other option —the environment “*animateinline*”. One can prepare scripts with PSTricks or pgf/TikZ, and input the typeset material into the environment.

Animation with different frame rates and reverse scrolling allows the student to understand better the whole way of the process or the derivation of the formulae.

It teaching mathematics, it is impossible for lecturer in the to speak without demonstration the images of geometric solids, surfaces, and other multidimensional structures.

Let us distinguish an ordinary static drawing of 3D object named as *quasi 3D* and *real* or *dynamical 3D* images. Somebody can rotate letter, vary size, and change angle and direction of vision. The last picture is much more informative and expressive that is important for educational purposes.

Perhaps, the dynamic 3-dimensional images are the most interesting and spectacular. The learning process has been actively used as a quasi 3-dimensional drawings and diagrams, as well as, the standard projection and section. Psychological perception of patterns allow the viewer to get a fairly complete picture of the form and structure of the object (see. Fig. 5).

Dynamic 3-D images can be rotated, zoomed, and panned. You can change the character rendering style receiving wire frame or textured surface (see. Fig. 6).

This opportunity is given by expensive commercial packages Maple, Mathematica, and MatLab. By this packages we can import an image and incorporate it into the training PDF document; all manipulations are provided by the free Adobe Reader.

4 The process of creating dynamic PDF training documents

As metioned above PDF document format has very useful features. The main challenge is how to realize these opportunities. There is free tool for demonstrate PDF documents in its entirety — Adobe Reader. But a convenient and suitable tool for making teaching materials is absent.

Authors from the academic and university environment use \LaTeX and similar tools. Graphical parts of documents are prepared by means of numerous free tools, occasionally, commercial programs. The choice of tool is determined by the objectives of each specific work. Popular assembly software based on \LaTeX contains almost all free graphic-aimed tools (MetaPost, TikZ, and Ps-tricks). So, the \LaTeX source scripts “collecting” text, graphics, and multimedia components and transform all in whole standard PDF document.

The choice of specific programs for creating graphical objects is determined by the specific features of visualization of mathematical mental images. Typically, in systems like Maple or MathCad, the graphics object (say the graph of

the function) is based on the calculated coordinates on the basis of a predetermined formula (see Fig.5).

Other methods are in systems with GUI. Here their is a menu of graphics primitives and drawing comes down to choose the right one and click directly by the mouse the characteristic points of the desired graphics primitive from the available list.

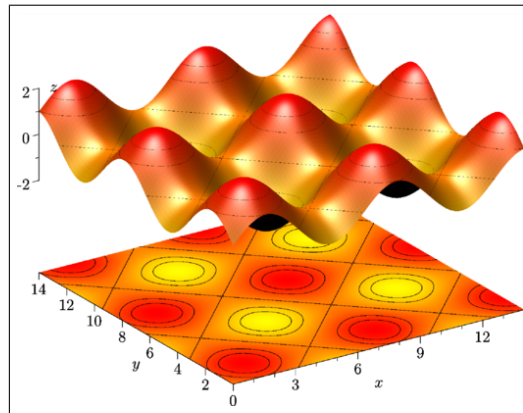


Fig. 5. Figure projection; its appearance is determined by the its generation and can not be change by the viewer. Surface form is better transmitted by the quasi 3-dimensional picture than the level curves of the system.

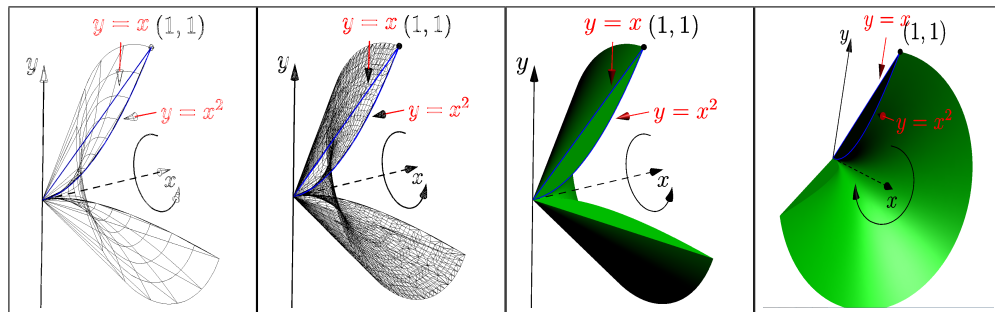


Fig. 6. Wire frame surface as one of the alternatives managed audience. Looking together it make entire surface shape for spectator.

In our opinion, implementation of visual mental image metaphors implies, on the one hand, “free” drawing not limiting and not tied to a given mathematical formulae.

But, on the other hand, there are certain requirements to get some important properties of the image: presence of the right intersection points of lines, desired character of the convexity/concavity, perpendicular or other angular characteristics determined the relative position of parts of the picture, put the arrow, place labels, and so on.

In other words, all we want to get should be clearly described by an appropriate way for a graphical program-executor.

Just such opportunities are provided by the Meta language of MetaPost program, language of a bunch PGP/TikZ, and PostScript with PSTricks add-on. The output can be either a standard graphic file or a natural structural element of the PDF document. However, initially the system is intended to describe the flat drawings and diagrams. Additional packages of various authors make it possible to obtain 3-dimensional images (quasi 3D) as a projection onto the plane of the drawing.

The system Asymptote also has its own C-like programming language. It allows you to get a “real” 3D objects. This refers to the geometric model in its own 3-dimensional coordinate system.

It can be rendered and viewed in a special window through the OpenGL and changed dynamically in an interactive mode: rotate, scale, move, etc. The most interesting and useful property is in the following. This model can also be imported into PRC format and inserted into a PDF document with possibility of the interactive manipulation remain. In Figure 7 screen shots of Adobe Reader are given, which show the drop-down menu with control commands.

One can insert 3D images from other programs into a PDF document if only they were transformed into U3D or PRC formats. Unfortunately, this is not so simple if not to resort to commercial products. The author uses a few free programs. There are maplex3s2prc, U3DIntelWin, and MeshLab.

File formats U3D and PRC are embedded into the PDF documents (Beamer-slides or text) directly while the conversion process from the L^AT_EX-source via utility media9. The necessary scripts are in the documentation for the Asymptote and the media9.

The latest versions of Maple and Matlab also allow one to import dynamic 3D graphics. Here, there are some other output formats, but the most convenient is X3D.

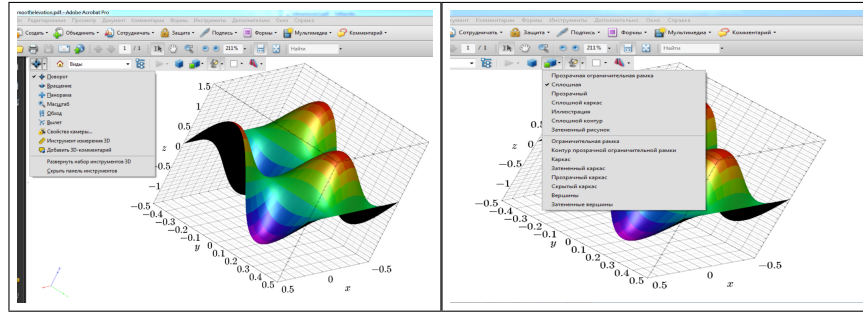


Fig. 7. Adobe Reader screen shots 3D model with drop-down control menu.

5 Conclusion

The visual metaphor approach facilitates deep learning, especially in mathematics. The \LaTeX -based assemblies are a convenient tool for the preparation of effective training materials with 2D dynamical and interactive images. We can note the growing popularity of the PDF in terms of dynamic 3D documents. Now many online magazines on astronomy and Biomedicine present the research results visualized and converted into PRC/U3D components of articles in PDF [10].

The only problem is the process of flow transforming the initial results into appropriate 3D file format, the components of which are not consistent, and the choice of components depends on the nature of the source data.

However, the active interest of researchers and the work of enthusiastic programmers is encouraged.

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