Set-valued linear dynamical system state estimation with anomalous measurement errors

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Abstract

The article deals with set-valued dynamical system state estimation problem when there is no statistical information on initial state, disturbances and noises but sets of their possible values are available. In practice some measurements can be performed with anomalous errors (failures, fallings out). That is at some time instants the measurement errors were realized outside of the given set of possible values. The article considers the level of measurement error falling out that can be reliably recognized using set-valued state estimation.

Keywords: linear dynamical systems; set-valued estimation; anomalous measurement errors.

1 Introduction

Nowadays set-valued dynamical system state vector estimation under condition of statistical uncertainty is being developed in control and identification theory. A lot of publications are devoted to this area [2, 4, 8, 10, 11, 12, 13, 14, 18]. The main subject of set-valued estimation is a feasible set, that is a set of all possible dynamical system states at a time instant.

The processes in the control system are described with equations:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + \Gamma w_k, \\ y_{k+1} &= Gx_{k+1} + Hv_{k+1}, \quad k = 0, 1, \dots N. \end{aligned} \tag{1}$$

where $x_k \in R^{n_x}$, $u_k \in R^{n_u}$, $w_k \in R^{n_w}$, $y_k \in R^{n_y}$,$v_k \in R^{n_v}$ denote state, control, disturbance, measurement, noise vectors at time instant k correspondingly; A, B, Γ, G, H are known matrices. The system (1) is supposed to be controllable and observable.

Initial state x_0, disturbances w_k and noises v_k are unknown but they can take some value from given convex sets:

$$x_0 \in X_0, \quad w_k \in W, \quad v_k \in V. \tag{2}$$

Set-valued estimation involves a construction of feasible sets $\bar{X}_{k+1}$, which are guaranteed to contain all state vectors $x_{k+1} \in \bar{X}_{k+1}$ at the time k:

$$X_{k+1/k} = A\bar{X}_k + \Gamma W. \tag{3}$$

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$$X[y_{k+1}] = \{x \in R^{n_x} | Gx + Hv = y_{k+1}, \forall v \in V\}, \tag{4}$$

$$\bar{X}_{k+1} = X_{k+1/k} \cap X[y_{k+1}]. \tag{5}$$

Operations in (3)-(5) are set operations, that is Minkowski sum, set intersection, linear set transformation. According to equations (3)-(5) the shape and size of feasible sets depend not only on the sets $\bar{X}_0$, W and V but also on the values of disturbance w_k and measurement error v_k. The feasible set is calculated as intersection of reachable set $X_{k+1/k}$ and measurement consistent set $X[y_{k+1}]$, the location of these sets depends on the values of realized disturbance w_k and measurement error v_k. The smaller feasible set is constructed the more accurate set-valued estimate we have. Therefore if a feasible set is a point it means that the exact value of the dynamical system state is calculated.

Feasible set shape and structure can be complex that is it can contain a lot of vertices and facets. But when the system state vector dimension increases troubles in performing set operations in real-time occur. Then feasible set outer approximations with some canonical forms, like ellipsoids [3, 8, 9, 10], parallelotopes [7] and zonotopes [18] can be applied.

In practice some measurements can be performed with anomalous errors (failures, fallings out) [1, 5, 11, 15]. Anomalous measurements can appear because of abrupt violation of data measurement equipment performance condition. Anomalous measurements can lower efficiency of classical data processing algorithms [5, 15]. That is why it is important to timely recognize anomalous measurements and exclude them from the following processing. There are heuristic approaches to anomalous measurement errors filtration like application of appropriate threshold criteria for selection and elimination of failures, least absolute deviation method and others. However these methods are effective only when the anomalous measurements significantly differ from other measurements. Besides these methods do not allow to get borders of parameter estimation errors.

2 Anomalous measurements

Let us consider that at a time k the measurement error v_k was outside the given set of possible values $v_k \notin V$. Two cases from feasible set construction algorithm (3)-(5) depending on the realized value of the measurement error v_k are possible:

- the feasible set is empty;
- the feasible set is not empty.

If the feasible set $\bar{X}_k$ is empty at a time k (fig.1), it means that at this time k or earlier the falling of v_k out of the set V has happened, i.e. $v_k \notin V$. In this case estimation algorithm can be restarted with new initial data, for example the borders of set V can be extended. Another way is to exclude this measurement from data processing and do not perform estimation at this time k. If the feasible set $\bar{X}_k$ is not empty (fig.2) the constructed set may not contain the real system state $x_k \notin \bar{X}_k$. But in this case it is impossible to recognize the failure. Therefore the guaranteed feature of anomalous measurements at set-valued estimation is the empty feasible set. For comparison when Kalman filter is used for system state estimation there are no guaranteed features of anomalous measurements because the system state estimation is probabilistic [6, 16, 17].

Let us consider what level of measurement error v_k falling out can be reliably recognized using set-valued state estimation that is at what measurement error v_k falling out the feasible set is empty. The feasible set $\bar{X}_k$ is empty if the corresponding reachable set $X_{k/k-1}$ and measurement consistent set $X[y_k]$ do not intersect (fig.3). These sets certainly do not intersect if their projections on coordinate axes do not intersect . Let us consider projections of the sets $X_{k/k-1}$ and $X[y_k]$ on coordinate axes, on which the measurements are performed.

Let us consider the outermost case when the real system state is on the border of reachable set $X_{k/k-1}$ projection (fig.3). If the measurement error was realized on the border v_{max_i} of the set V on the axis $x(i)$ the measurement y_k would get to the point y'_k. Let us suppose that the falling out of measurement error happened and the measurement is in the point y_k.

$$\begin{aligned} y'_k &= x_k + v_{max_i}, \\ y_k &= x_k + v_k. \end{aligned} \tag{6}$$

Then the value of measurement error falling out is equal to the distanse between points y'_k and y_k:

$$\delta v_{x(i)} = |v_k - v_{max_i}| = |y'_k - y_k|. \tag{7}$$

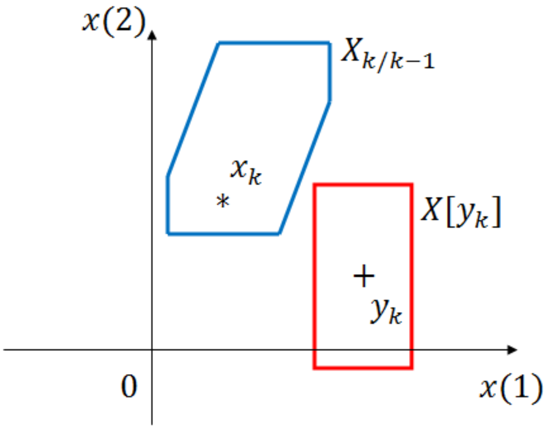


Figure 1: Example of anomalous measurement

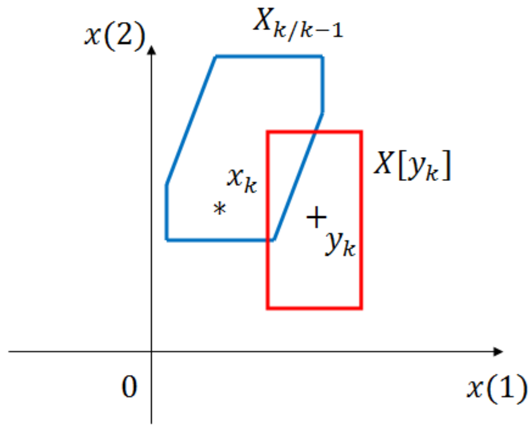
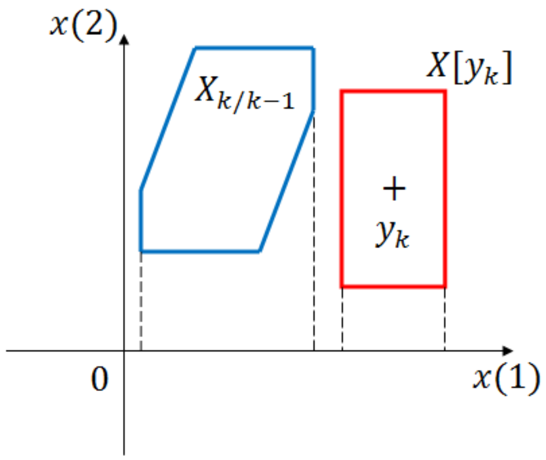
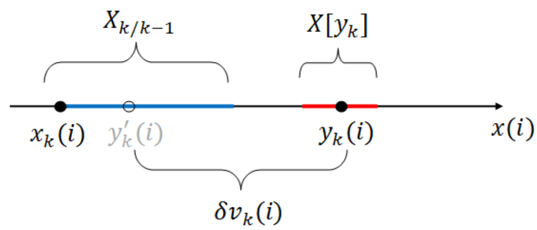


Figure 2: Example of anomalous measurement



a) Sets $X_{k/k-1}$, $X[y_k]$



b) Projections of sets $X_{k/k-1}$, $X[y_k]$ on coordinate axis $x(i)$

Figure 3: Anomalous measurement

Then the minimum falling out $\delta v_{x(i)}$ when the projections' intersection is empty is equal to

$$\delta v_{x(i)} = |v_k - v_{max_i}| = (d(X_{k/k-1})_{x(i)} - r(X[y'_k])_{x(i)}) + r(X[y_k])_{x(i)} = d(X_{k/k-1})_{x(i)}, \tag{8}$$

where $d(\bullet)_{x(i)}$ denotes the projection of set diameter to the axis $x(i)$, $r(\bullet)_{x(i)}$ denotes the projection of set radius to the axis $x(i)$.

Let us estimate the diameter of the set $X_{k/k-1}$. The set $X_{k/k-1}$ is constructed as Minkowski sum of sets $A\bar{X}_{k-1}$ and ΓW. The set-valued estimate $\bar{X}_{k-1}$ is not worse than the set V on measured coordinates. Then

$$d(X_{k/k-1})_{x(i)} = d(A\bar{X}_{k-1})_{x(i)} + d(\Gamma W)_{x(i)} \leq d(AV)_{x(i)} + d(\Gamma W)_{x(i)}, \tag{9}$$

Therefore if the measurement error is outside of the given set V and the following condition is fulfilled for any of the coordinates $x(i)$

$$\delta v_{x(i)} \geq d(AV)_{x(i)} + d(\Gamma W)_{x(i)}, \tag{10}$$

then the measurement error falling out will be reliably recognized.

3 Example

Let us consider the model (1) with the following matrices:

$$A = \begin{pmatrix} 0.9976 & 0.04639 \\ -0.09278 & 0.8584 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0.1189 \\ 4.639 \end{pmatrix} \cdot 10^{-3}, \quad G = I_{2\times 2}, \quad H = I_{2\times 2}. \tag{11}$$

The initial state set, disturbance set and measurement error set are the following polyhedras:

$$\begin{gathered} X_0 : \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} x_0 \leq \begin{pmatrix} 7.5 \cdot 10^{-4} \\ 3 \cdot 10^{-2} \\ 7.75 \cdot 10^{-4} \\ 3 \cdot 10^{-2} \end{pmatrix}, \quad W : \begin{pmatrix} 1 \\ -1 \end{pmatrix} w_k \leq \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix}, \\ V : \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} v_k \leq \begin{pmatrix} 1.45 \cdot 10^{-4} \\ 2.28 \cdot 10^{-2} \\ 1.45 \cdot 10^{-4} \\ 2.28 \cdot 10^{-2} \end{pmatrix}. \end{gathered} \tag{12}$$

Let us compute from (10) the measurement error falling out level that can be recognized for this model.

For the coordinate $x(1)$ the falling out level is:
$d(AV)_{x(1)} = 0.0024, \quad d(\Gamma W)_{x(1)} = 0.00036,$
$\delta v_{x(1)} \geq 0.0024 + 0.00036 = 0.00276.$

For the coordinate $x(2)$ the falling out level is:
$d(AV)_{x(2)} = 0.039170, \quad d(\Gamma W)_{x(2)} = 0.013917,$
$\delta v_{x(2)} \geq 0.039170 + 0.013917 = 0.053087.$

Notice that the measurement error set is much smaller that the disturbance set at the first coordinate. That is why the measurement error falling out can be recognized if the measurement error v_k value is 19 times greater than the largest possible value of the set V at the first coordinate. For the second coordinate if the falling out is 2.3 times greater than the largest possible value of the set V at the second coordinate the anomalous measurement error can be recognized. However this estimate is upper-bound. In practice there are realizations when the smaller falling outs can be recognized. Let us consider a disturbance realization (fig. 4). We considered some measurement errors realizations with different falling outs at the steps $k = 10$ and $k = 20$. At the first realization the value of $v_k(2)$ is equal to $1.5 v_{max_2} = 0.0342$ at steps $k = 10$ and $k = 20$. At the second realization the values of $v_k(1)$ is equal to $3 v_{max_1} = 4.35 \cdot 10^{-4}$ at steps $k = 10$ and $k = 20$. The figures 5-7 show the measurement error realization with falling outs which were recognized although the falling out value was smaller than the estimate $\delta v_{x(2)}$. For these realizations the feasible set is empty at the time $k = 10$. For the realizations from fig.8, 11 the measurement error falling outs were not recognized. The feasible sets at the times $k = 10$ and $k = 20$ are not empty but they do not contain the real system state x_k (fig.8, 11).

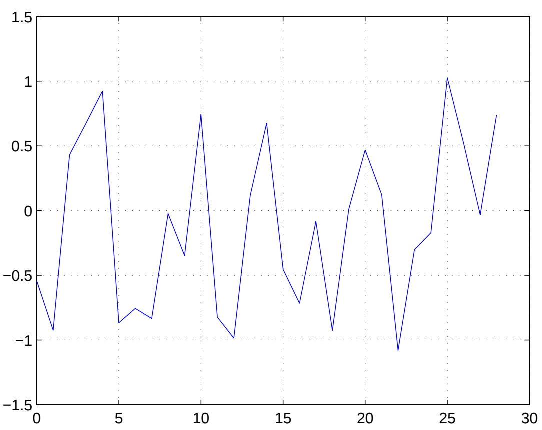


Figure 4: Disturbances.

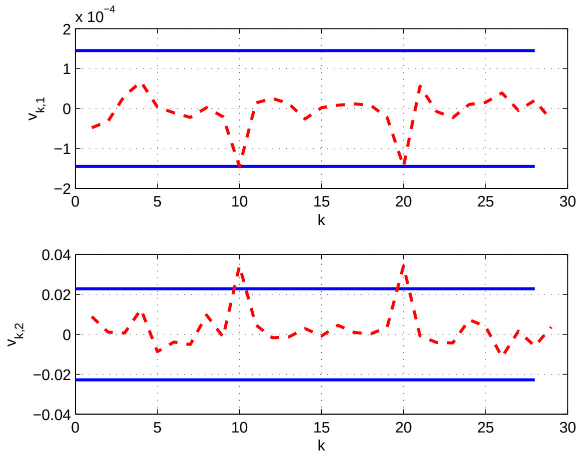


Figure 5: Measurement errors (realization 1). The solid line denotes the border of the given set V, dashed line denotes the measurement errors v_k.

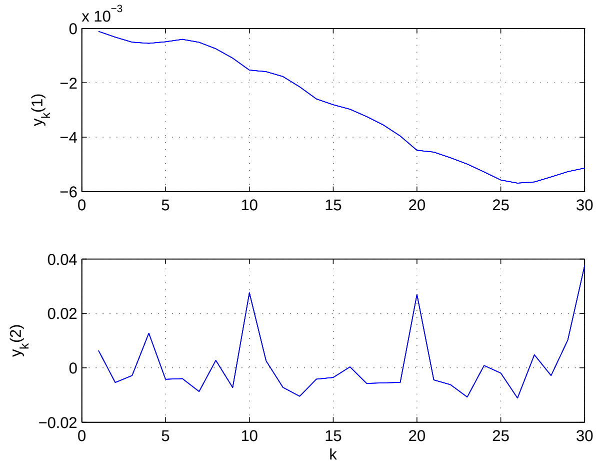


Figure 6: Measurements (realization 1)

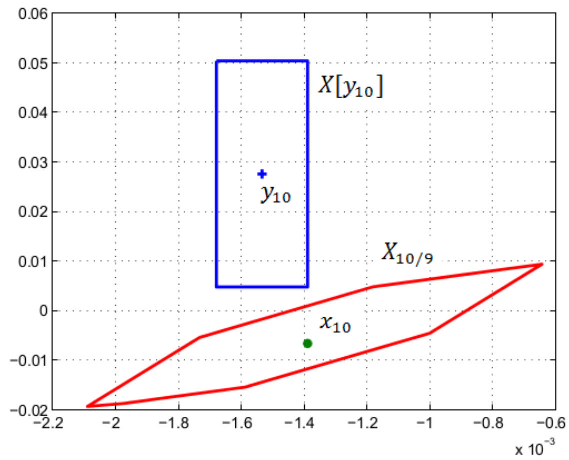


Figure 7: Reachable set $X_{10/9}$ and measurement consistent set $X[y_{10}]$ (realization 1)

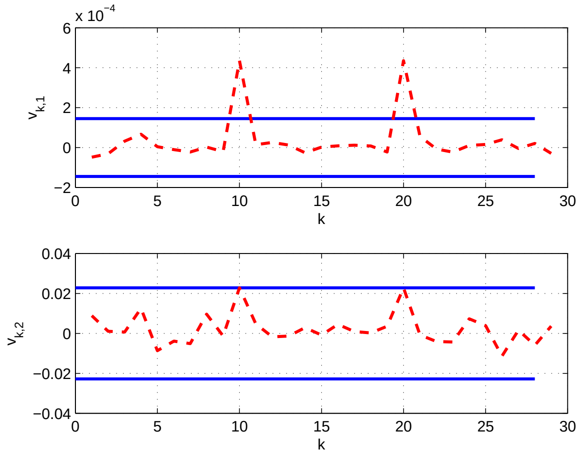


Figure 8: Measurement errors (realization 2). The solid line denotes the border of the given set V, dashed line denotes the measurement errors v_k.

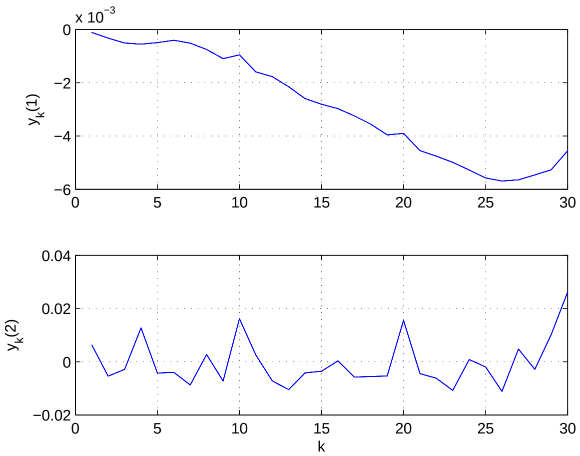


Figure 9: Measurements (realization 2)

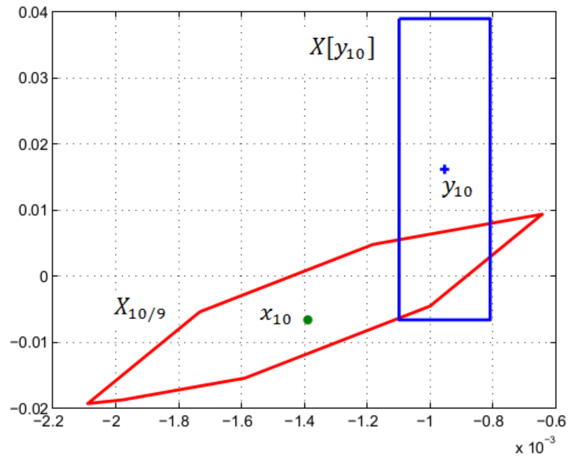


Figure 10: Reachable set $X_{10/9}$ and measurement consistent set $X[y_{10}]$ (realization 2)

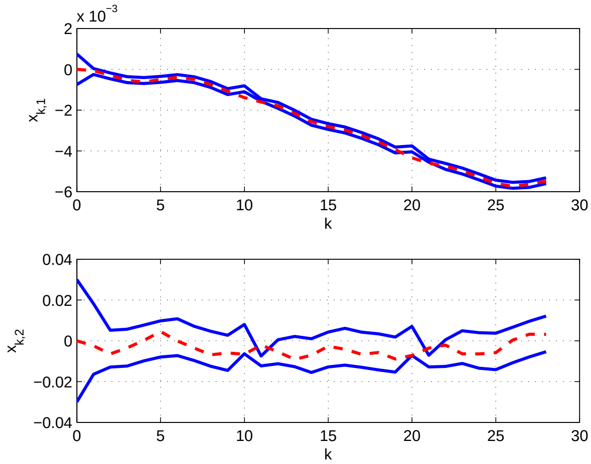


Figure 11: System state x_k estimation (realization 2).The solid line denotes the border of set-valued estimates, dashed line denotes the real value x_k.

4 Conclusion

The set-valued dynamical system state estimation with anomalous measurements was considered when the measurement error is realized outside the set of possible values. The reliable feature of anomalous measurement errors is empty feasible set. The anomalous measurement is reliably recognized if the falling out value $\delta\nu_k$ at a time k is greater than the sum of projections of the diameters of the sets AV and ΓW on any of coordinates. However in practice there are realizations when smaller falling outs can be recognized. When the anomalous measurement is recognized it can be excluded from the data processing or the estimation procedure can be restarted with new initial data.

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