Dynamic programming method for optimization problem of multi-modal transportation

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Abstract

An optimization problem of multi-modal transportation is investigated. The multi-step decision-making process is constructed. Dynamic programming method for solving the problem is proposed. It should be noted that the dynamic programming is not provided computational advantages, but facilitates changes and modifications of tasks.

Keywords: Optimization Problem, Dynamic Programming, Multi-Step Decision Making Process, Multi-Modal Transportation.

1 Introduction

There are many approaches to the planning of multi-modal transportations. In [Flo11] authors deal with a real-world problem and in particular focus on intermodal freight transport. A system that solves multi-modal transportation problems in the context of a project for a big company is described. It is combined Linear Programming with automated planning techniques in order to obtain good quality solutions. A new hybrid algorithm, combining linear programming and planning to tackle the multi-modal transportation problem, exploiting the benefits of both kinds of techniques is proposed there. The system also integrates an execution component that monitors the execution, keeping track of failures if it is necessary, maintaining most of the plan in execution.

A parameterized model for the optimization of a multi-modal transportation network has been developed and implemented in [Per13] aiming at evaluating the possibilities to increase the absorption rate by a multi-modal transport service. Different software packages have been integrated in order to implement the transport model and an optimization algorithm. A novel formulation of the optimization process has been developed, by combining the differential evolution algorithm and a constructive heuristic.

In [Sun15] a review on the freight routing planning problem in the multi-modal transportation network from the viewpoints of the model formulation and solving approaches is presented. In this study, the formulation characteristics are distinguished and classified into six aspects, and the optimization models in the recent studies are identified based on their respective formulation characteristics. Furthermore, the solution approaches developed to solve the optimization models are discussed, especially the heuristic algorithms.

In this paper, we deal with an optimization problem of multi-modal transportation. Transportation of goods by several types of transport such as road, rail, sea or river are considered. Decision maker can select the
carrier of the goods for a given initial point of departure and final destination of its receipt. Optimization of the
decision–making process is carried out by the dynamic programming method.

2 Problem Statement

Integrated route planning may include:

- Estimation of the cost of services for each carrier;
- The study of the geographic features and limitations in terms of binding to the warehouses, railway stations,
sea and river ports;
- Analysis of the cost of transhipment of goods;
- The timetable for delivery.

In the model scheme shown in Fig.1, is planned four segments of the route with three overloads: the supplier
– the initial segment AB or AC is serviced by road transport to the station B or C, next segment – by railway
from the stations B or C to D or E ports, then by sea transport from port D or E to the ports of F or G, the
final section – motor transport from ports F or G to the consumer H.

![Figure 1: Example of multi–modal transportation graph](image)

Selection of decision maker is to implement a route through nodes B or C, D or E or F and G in such a way
that the total transportation cost was minimal. Possible taking into account additional criteria, such as delivery
time or the various risks.

<table>
<thead>
<tr>
<th>Type of transport</th>
<th>Segment</th>
<th>Cost (RUB per container)</th>
</tr>
</thead>
<tbody>
<tr>
<td>road transport</td>
<td>AB</td>
<td>30000</td>
</tr>
<tr>
<td></td>
<td>AC</td>
<td>40000</td>
</tr>
<tr>
<td>railway</td>
<td>BD</td>
<td>40000</td>
</tr>
<tr>
<td></td>
<td>BE</td>
<td>50000</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>20000</td>
</tr>
<tr>
<td></td>
<td>CE</td>
<td>30000</td>
</tr>
<tr>
<td>sea transport</td>
<td>DF</td>
<td>60000</td>
</tr>
<tr>
<td></td>
<td>DG</td>
<td>90000</td>
</tr>
<tr>
<td></td>
<td>EF</td>
<td>70000</td>
</tr>
<tr>
<td></td>
<td>EG</td>
<td>80000</td>
</tr>
<tr>
<td>road transport</td>
<td>FH</td>
<td>60000</td>
</tr>
<tr>
<td></td>
<td>GH</td>
<td>50000</td>
</tr>
</tbody>
</table>

Data [Cos16] to construct the optimal route for multi–modal transportation model example listed in Table 1.

3 Formation of Multi–Step Decision–Making Process

To construct a mathematical model of a multi–modal transportation, allowing the use of dynamic programming
for finding a solution to the optimal route search task required to describe a multi–step decision–making process,
the scheme of which is shown in Fig.2.
Figure 2: Graph of multi-step decision-making process

Suppose that the system is in the state $X_0$. As a result of the decision $U_0$, the system goes into a state of $X_1$ and the price of such decision is $C(X_0, U_0)$. From the standpoint of a model example (see Fig.1), the state $X_0$ corresponds to locating the goods from the supplier A, solution $U_0$ is to use the first motor carrier to station B or C. Next, the system will be in the state $X_1$, i.e. at station B or C. As a result of the decision $U_1$, the system goes into a state of $X_2$ which corresponds to the carriage by rail to stations of D or E, and the price of the decision will be $C(X_1, U_1)$. This process will continue until the final state of $X_4$ when the goods will be delivered to the consumer.

Optimization of the above constructed multi-stage decision-making process is carried out by the method of dynamic programming [Wag75].

4 Optimization

Using the method of dynamic programming and Bellman’s principle of optimality [Bel53] will provide greater flexibility in terms of possible inclusion in the model of various modifications, for example in case emergency situations. The minimum of decision-making price in $i$ steps from the $j$ state will be described by the Bellman’s recurrence relation

$$f_i(X_j) = \min_{U_j} \{C(U_j, X_j) + f_{i-1}(X_{j-1})\}. \tag{1}$$

Here $U_j$ is control on step $j$, $C(U_j, X_j)$ is the price of making a decision on step $j$. As $U_j$ implemented to set the minimum price of $j$-th freight. Algorithm transitions from one state to another (see Fig.2) is described below.

1–st step. Suppose that the system was in the penultimate state $X_3$. Then the price for the decision of the final step according to (1) will be

$$f_1(X_3) = \min_{U_3} \{C(U_3, X_3)\}. \tag{2}$$

2–nd step. Suppose that the system was in the state $X_2$. Then the price decision of the last two steps will be

$$f_2(X_2) = \min_{U_2} \{C(U_2, X_2) + f_1(X_3)\}. \tag{3}$$

3–rd step. If the system was in a state $X_1$ then the price of the decision in the last three steps can make one of the alternatives

$$f_3(X_1) = \min_{U_1} \{C(U_1, X_1) + f_2(X_2)\}. \tag{4}$$

4–th step. Finally, let the system is in the initial state $X_0$. Then the price decision for all four steps will be

$$f_4(X_0) = \min_{U_0} \{C(U_0, X_0) + f_3(X_1)\}. \tag{5}$$

This price corresponds to the minimum value of the entire carriage.

Now, using the data shown in Table 1, it is possible to construct the optimal route for multi-modal transportation for model example (see. Fig.1) in accordance with the above algorithm.

1–st step. Price decision $U_3$ at a final step in the transition of the system from the penultimate state $X_3$ will be

$$f_1(X_3 \supset F) = \min\{60\} = 60,$$
\[ f_1(X_3 \supset G) = \min\{50\} = 50. \]

2–nd step. Being in a state \( X_2 \) and making decision \( U_2 \) the price of the last two steps will be

\[
\begin{align*}
f_2(X_2 \supset D) &= \min \left\{ \begin{array}{c} 60 + f_1(X_3) \\noalign{\smallskip} 90 + f_1(X_3) \end{array} \right\} = \min \left\{ \begin{array}{c} 60 + 60 \\noalign{\smallskip} 90 + 50 \end{array} \right\} = \min \left\{ \begin{array}{c} 120 \\noalign{\smallskip} 140 \end{array} \right\} = 120, \\
f_2(X_2 \supset E) &= \min \left\{ \begin{array}{c} 70 + f_1(X_3) \\noalign{\smallskip} 80 + f_1(X_3) \end{array} \right\} = \min \left\{ \begin{array}{c} 70 + 60 \\noalign{\smallskip} 80 + 50 \end{array} \right\} = \min \left\{ \begin{array}{c} 130 \\noalign{\smallskip} 130 \end{array} \right\} = 130.
\end{align*}
\]

3–rd step. Suppose that the system was in a state \( X_1 \). Then the price decision of the last three steps can make one of the alternatives

\[
\begin{align*}
f_3(X_1 \supset B) &= \min \left\{ \begin{array}{c} 40 + f_2(X_2) \\noalign{\smallskip} 50 + f_2(X_2) \end{array} \right\} = \min \left\{ \begin{array}{c} 40 + 120 \\noalign{\smallskip} 50 + 130 \end{array} \right\} = \min \left\{ \begin{array}{c} 160 \\noalign{\smallskip} 180 \end{array} \right\} = 160, \\
f_3(X_1 \supset C) &= \min \left\{ \begin{array}{c} 20 + f_2(X_2) \\noalign{\smallskip} 30 + f_2(X_2) \end{array} \right\} = \min \left\{ \begin{array}{c} 20 + 120 \\noalign{\smallskip} 30 + 130 \end{array} \right\} = \min \left\{ \begin{array}{c} 140 \\noalign{\smallskip} 160 \end{array} \right\} = 140.
\end{align*}
\]

4–th step. Finally, let the system is in the initial state \( X_0 \). Then the price decision for all four steps will be

\[
\begin{align*}
f_4(X_0 \supset A) &= \min \left\{ \begin{array}{c} 30 + f_3(X_1) \\noalign{\smallskip} 40 + f_3(X_1) \end{array} \right\} = \min \left\{ \begin{array}{c} 30 + 160 \\noalign{\smallskip} 40 + 140 \end{array} \right\} = \min \left\{ \begin{array}{c} 190 \\noalign{\smallskip} 180 \end{array} \right\} = 180.
\end{align*}
\]

This price corresponds to the minimum value of the entire carriage.

To find a proper sequence of the optimal route with a minimum value of 180 enough to write a minimum prices decision-making at each step.

![Diagram of the optimal scheme of multi-modal transportation](image)

The optimal scheme (see. Fig.3) for the considered model example of multi-modal transportation following

\[ A \rightarrow C \rightarrow D \rightarrow F \rightarrow H. \]

5 Conclusion

The model example of multi-modal transportation is studied. The Multi-Step Decision-Making Process is constructed. Optimization of the decision–making process is carried out by the dynamic programming method. Using the method of dynamic programming and Bellman’s principle of optimality will provide greater flexibility in terms of possible inclusion in the model of various modifications, for example in case emergency situations [Zav15].

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References


