

A Fuzzy Model for Identifying Significant Subtle Effects within a System of Objects' Responses

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Abstract

The paper deals with the problem of revealing subtle consistent patterns of functional connections between sets of data describing a system of objects' responses. The main strengths and weaknesses of utilizing fuzzy systems are explored afterwards. It outlines the main features of the fuzzy model for identifying significant subtle (implicit) effects within a system of objects' responses based on a cross between fuzzy set and data mining approaches. The tool for obtaining assessments in natural language resulting from exploiting an automated learning algorithm is later suggested. After that the model's applicability to examining the system of images' responses is discovered. The article lastly brings evidence for its adaptability to simulating mutual impacts of a range of economic objects.

1 Introduction

Finding behavior patterns of systems is a topical problem of recent studies, since it allows us to make an assumption concerning main features and proprieties of these systems, and to generate probable patterns of their responses to incoming signals based on such findings. The information, obtained in this fashion and related to features of interaction of the system with the ambient environment, has a significant importance to resolving practical tasks, associated with control establishment over the system activity, particularly, to removing main constraints and obtaining defined results.

However, in some cases, definition of the system expected behavior based solely on the key categories is straitened due to its multifunction nature, substantial quantity of elements, proprieties, couplings, relations, presence of subtle, implicit effects, sophisticated structure etc. In addition, if it is necessary to obtain sufficiently accurate results, we inevitably need to process big data [1], detailing the states of internal and external environments of the system, thus predetermining the requirement for Business Intelligence [2] type solutions and resulting in additional difficulties. One of the ways to solve this problem is the test object simulation based on the available data to determine the relationship between the applied input effect and the response (output reaction) of the system using the data mining tools.

Generally, we consider the signal as a carrier of the measurement information [3]. In addition, we treat subtle, implicit effects as non-evident impacts within the system caused by implicit factors and capable of producing

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a synergistic effect. Furthermore, we classify implicit factors as non-evident factors, having a significant effect on the system performance based on subtle information, previously not taken into account, which is practically useful and available for generating knowledge and making decisions [4].

An important prerequisite to determine the relationship between the system output response and the input effect on it is identification of the communication channel providing transmission of the signal from the message source to the receiver through a set of elements. The incorrect recognition of the communication channel may result in a false understanding of the relationship, and finally lead to inapplicability of this model in practice. We propose to apply fuzzy logic means to find such elements.

2 Pros and Cons of Utilizing Fuzzy Systems

Despite the availability of a substantial number of methodological approaches to evaluate the system response dependence on the input effect (see studies by I.I. Salnikov [5], A.V. Sedov [6], Yu.A. Koval, E.V. Ivanova, A.A. Rostyrya, A. Al-Tverzhi [7], A. Sandryhaila, J. Moura [8] etc.), the authors believe the developed tools do not completely recognize the uncertainty factor. This feature may be critical in conditions of the implicitness. Therefore, we should focus on application of fuzzy set means, which allow us to overcome the abovementioned disadvantages in the analysis.

Application of fuzzy systems has the following advantages:

- possibility of studying the parameters beyond the conventional formalization methods;
- development of solutions in conditions nonsufficient to perform another data analysis (e.g. statistical analysis);
- application of crude data in calculations;
- versatility [9, 10] etc. [11, 12].

Moreover, the fuzzy representation of the model indices improves the study results, allowing the user to evaluate not only the index values, but the verisimilitude of each value and its degree of confidence as well.

The fuzzy logic enables the reliability evaluation of fuzzy ranking of variations by defined, most credible values of indices, characterizing these variations, increasing the output validity as a result. In addition, application of fuzzy set means allows us to study stability of the model endogenous indices with respect to variation of its exogenous indices. It leads to the possibility of the quantitative evaluation of consequences of higher or lower volatility of various input variables for the input parameter stability.

The main disadvantages of this method are the absence of principles of selection of a proper membership function specified by the fuzzy set theory, biased formulation of an initial set of rules for fuzzy input etc. [11, 12]. However, the aforesaid approach disadvantages are collateral during processing of fuzzy data and do not reduce the relevance of this approach, which is confirmed by a high scientific appeal to study such fuzzy systems.

3 Fuzzy Set Approach to Identifying and Assessing Implicit Influences

The basic idea behind application of fuzzy set approach consists in the fact that a certain index assumed to be interval and to be determined (fuzzified) by a span instead of a number. This point aims to reflect an actual situation of more or less exactly specified threshold values, within which a parameter can vary.

As a consequence, the researcher needs to formalize the understanding of the concerned index probable values along with indication of the set of probable values and degrees of uncertainty of their adoption. Then, after calculation of the probability distribution of the overall index, we should pass on defuzzification and interpretation stages based on the system of rules and using developed output tools.

The model-building task is to find and to quantify effects of subtle, implicit factors, which are communication channel elements, on directly related parameters and through them on certain key indices. The important aspect is finding the very implicit factors.

These subtle factors typically involve difficulties at formalization and modelling stages of the system simulation process as they normally couple with linguistic uncertainty of the notions they relate to. That fact results in the need to treat subtle factors as linguistic variables and analyse their conceptual background before introducing these into the model itself. In this respect, linguistic variables regarded as having their values expressed in the formal shape of notions rather than numbers can be associated with natural language.

The linguistic approach lays the foundation for fuzzy logic and soft computing applied to realistic modelling of complex systems. The use of linguistic variables in specific process formalization allows for description in terms decision-makers and experts got used to. The above explanation leads to the feasibility of any notion formal characterization by means of the fuzzy set theory which contains linguistic variables in its conceptual basis.

Therefore, the fuzzy set approach provides the user with a range of methods for performing proper structural examination of a linguistic variable and investigating its interconnections with the set structure. Moreover, it's capable of taking into account the specific relation towards a group of objects being studied and producing above all a relevant procedure for assessing the system elements behavior.

The basic model-building tools are fuzzy binary relations, their composition, and data mining algorithms. The model-building logic is illustrated below.

4 Logic of Constructing a Fuzzy Model for Identifying Significant Subtle Effects within a System of Objects' Responses

Suppose we are given a set of $A = \{a_1, a_2, \dots, a_n\}$. With the specified degree of probability we can find two elements among the elements of the set, which have a relatively small mutual effect, but there is an element distinctive from the above two elements, with introduction of which the effect becomes significant.

Some definitions are introduced below. Suppose U is any set, U^2 is Cartesian square of this set ($U^2 = U \times U = \{(a; b) : a, b \in U\}$). The fuzzy binary relation on the U set is the U^2 fuzzy subset. Conventional notation format of the fuzzy binary relation for the discrete and continuous sets are represented in (1) and (2), respectively.

$$\Gamma = \sum_{U^2} \mu_{\Gamma}(u_i, u_j)/(u_i, u_j), \quad (1)$$

$$\Gamma = \int_{U^2} \mu_{\Gamma}(x, y)/(x, y). \quad (2)$$

Matrices elements of which are values of the membership function of $\mu_{\Gamma}(x, y)$ fuzzy binary relation are denoted as J_{Γ} .

The composition of Γ_1 and Γ_2 fuzzy binary relations is specified by such a fuzzy binary relation of $\Gamma = \Gamma_1 \circ \Gamma_2$, that (3) is valid:

$$\mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \bigcup_{z \in U} ((\mu_{\Gamma_1}(x, z)/(x, z)) \cap (\mu_{\Gamma_2}(z, y)/(z, y))). \quad (3)$$

Given that the intersection of $\mu_{\Gamma_1}(x, z)/(x, z)$ and $\mu_{\Gamma_2}(z, y)/(z, y)$ single-point fuzzy sets is generally performed by the logical T norm, and its union is performed by the logical T conorm: $a \cap b = \min(a, b)$, $a \cup b = \max(a, b)$, (3) takes the form of (4).

$$\mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \max_{z \in U} (\min(\mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y)))/(x, y). \quad (4)$$

Equations (5) and (6) are the relation composition graph for discrete and continuous sets, respectively.

$$\Gamma_1 \circ \Gamma_2 = \sum_{U^2} \mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \sum_{U^2} (\max_{z \in U} (\min \mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y)))/(x, y), \quad (5)$$

if U is a finite set;

$$\Gamma_1 \circ \Gamma_2 = \int_{U^2} \mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \int_{U^2} \max_{z \in U} (\min \mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y)))/(x, y), \quad (6)$$

if U is part of the number axis or the entire number axis. Therefore, from (4) for a U finite set we can obtain:

$$J_{\Gamma_1 \circ \Gamma_2} = J_{\Gamma_1} \circ J_{\Gamma_2} = (\max_k (\min(\mu_{\Gamma_1}(u_i, u_k), \mu_{\Gamma_2}(u_k, u_j)))_{n \times n} = (\mu_{\Gamma_1 \circ \Gamma_2}(u_i, u_j))_{n \times n},$$

where n is a number of U set elements. Build-up the J matrix for the A set.

$$J_{\Gamma} = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \dots & \dots & \dots & \dots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix},$$

where s_{ij} ($0 \leq s_{ij} \leq 1; i = 1, 2, \dots, n; j = 1, 2, \dots, m$) is an extent of the a_i index effect on the a_j index.

Then, to find mediate effects we can calculate values of J_{Γ^2} matrix using (4):

$$J_{\Gamma^2} = J_{\Gamma} \cdot J_{\Gamma} = \begin{pmatrix} f_{11} & f_{12} & \dots & f_{1m} \\ f_{21} & f_{22} & \dots & f_{2m} \\ \dots & \dots & \dots & \dots \\ f_{n1} & f_{n2} & \dots & f_{nm} \end{pmatrix}.$$

The reflexive selection of mediated factors [13] performed as part of data mining assumes existence of such a pair of s_{ij} and f_{ij} , that $s_{ij} \ll f_{ij}$, and indicates the presence of an implicit effect, which becomes apparent due to an intermediate factor within the system [4].

5 Model Adjustments to Handle Fuzzy Binary Correspondence

When necessary, the model may be adapted to work with fuzzy binary correspondences. The conceptual similarities and differences of this adaptation are described below.

Binary correspondence in the case of $A \times B$ set means a Γ subset of the Cartesian product of sets A and B : $\Gamma \subseteq A \times B$. The Cartesian product $A \times B$ describes the product of all the sets that feature A set element in the first position, and B set element in the second position. In the case when $A = B$, binary correspondences amount to the mode of binary relations $\Gamma \subseteq A^2$.

Compositions of binary correspondences and binary relations are defined similarly. Compositions of fuzzy binary correspondences $\Gamma_1 \subseteq A \times B$ and $\Gamma_2 \subseteq B \times C$ are defined by (7), provided that (8) is correct for the membership function.

$$\Gamma = \Gamma_1 \circ \Gamma_2 \subseteq A \times C, \quad (7)$$

$$\mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \bigcup_{z \in B} (\mu_{\Gamma_1}(x, z)/(x, z) \cap \mu_{\Gamma_2}(z, y)/(z, y)) \quad (8)$$

$(x \in A, y \in C).$

The intersection and the union of single-point fuzzy sets in the case of binary correspondences are made according to the abovementioned rules. In this case, (8) goes over to (9):

$$\mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \left(\max_{z \in B} (\min(\mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y))) \right) / (x, y) \quad (9)$$

$(x \in A, y \in C).$

The graph of finite set correspondences' composition follows (10); (11) defines the graph for the sets that represent an interval of the number axis or the entire number axis.

$$\Gamma_1 \circ \Gamma_2 = \sum_{A \times C} \mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \sum_{A \times C} \left(\max_{z \in B} (\min(\mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y))) \right) / (x, y), \quad (10)$$

where A , B , and C are finite sets;

$$\Gamma_1 \circ \Gamma_2 = \int_{A \times C} \mu_{\Gamma_1 \circ \Gamma_2}(x, y)/(x, y) = \int_{A \times C} \max_{z \in B} (\min(\mu_{\Gamma_1}(x, z), \mu_{\Gamma_2}(z, y))) / (x, y), \quad (11)$$

if A , B , and C are an interval of the number axis or the entire number axis.

From (10) it follows that the matrix of the composition of $J_{\Gamma_1 \circ \Gamma_2}$ relations, when A , B , and C are finite sets, is nothing but a maximin matrix product of J_{Γ_1} and J_{Γ_2} :

$$J_{\Gamma_1 \circ \Gamma_2} = J_{\Gamma_1} \cdot J_{\Gamma_2} = \left(\max_{k=1,2,\dots,p} (\min(\mu_{\Gamma_1}(x_i, z_k), \mu_{\Gamma_2}(z_k, y_j))) \right)_{m \times n} = (\mu_{\Gamma_1 \circ \Gamma_2}(x_i, y_j))_{m \times n},$$

where p is the amount of B set elements; m is the amount of A set elements; n is the amount of C set elements.

Building a model for evaluation of significant subtle effects in the object response system applying fuzzy binary correspondences falls into two steps:

- building sub-models A including the set of implicit factors, B incorporating the set of indirect indices, and C involving the set of key parameters;
- integration of sub-models into a general model, its analysis and solution of the problem set.

The operation sequence of the first step may include the following procedures:

1. Primary specification of a number index set for each sub-model.
2. Making lists of number index sets.

The operation sequence of the second step includes the following:

1. Evaluation of interdependence between the indices in pairs: (A, B) , (A, C) , (B, C) .
2. Detecting of indirect effects that indices of the sub-model A have on indices of the sub-model C .
3. Explanation of the obtained results.

Dependences are defined using J_{AB} , J_{BC} and J_{AC} matrices for the set of A , B , and C indices:

$$\begin{aligned}
 A &= \{a_1, a_2, \dots, a_n\}, \\
 B &= \{b_1, b_2, \dots, b_m\}, \\
 C &= \{c_1, c_2, \dots, c_k\}, \\
 J_{AB} &= \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \dots & \dots & \dots & \dots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix}, \\
 J_{AC} &= \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1k} \\ z_{21} & z_{22} & \dots & z_{2k} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nk} \end{pmatrix}, \\
 J_{BC} &= \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1k} \\ u_{21} & u_{22} & \dots & u_{2k} \\ \dots & \dots & \dots & \dots \\ u_{m1} & u_{m2} & \dots & u_{mk} \end{pmatrix},
 \end{aligned}$$

where s_{ij} ($0 \leq s_{ij} \leq 1; i = 1, 2, \dots, n; j = 1, 2, \dots, m$) is an extent of the a_i index effect on the b_j index, z_{ij} ($0 \leq z_{ij} \leq 1; i = 1, 2, \dots, n; j = 1, 2, \dots, k$) is an extent of the a_i index effect on the c_j index, u_{ij} ($0 \leq u_{ij} \leq 1; i = 1, 2, \dots, m; j = 1, 2, \dots, k$) is an extent of the b_i index effect on the c_j index.

The extent of the direct effect a_i makes on c_1 is determined by the z_{i1} element of the J_{AC} matrix. Similarly, the extent of the direct effect a_i makes on c_2, \dots, c_k is determined by z_{i2}, \dots, z_{ik} numbers. In addition to the direct effect, the a_i index affects c_1, c_2, \dots, c_k through the intermediate factor b_j , which is an index of the sub-model B . The extent of the indirect effect a_i has on c_1, c_2, \dots, c_k through b_j are taken as values $z_{i1}^*, z_{i2}^*, \dots, z_{ik}^*$, which are minimums of s_{ij} and corresponding $u_{j1}, u_{j2}, \dots, u_{jk}$:

$$z_{i1}^* = \min(s_{ij}, u_{j1}), z_{i2}^* = \min(s_{ij}, u_{j2}), \dots, z_{ik}^* = \min(s_{ij}, u_{jk}).$$

Equation (12) specifies combined indirect effect that a_i element produces on c_j :

$$z_{ij}^* = \max(\min(s_{i1}, u_{1j}), \min(s_{i2}, u_{2j}), \dots, \min(s_{im}, u_{mj}). \quad (12)$$

The matrix J_{AB} and J_{BC} product as per (13) specifies the indirect effect A set elements cause on C set elements through B :

$$J_{AC}^* = J_{AB} \cdot J_{BC} = \begin{pmatrix} z_{11}^* & z_{12}^* & \dots & z_{1k}^* \\ z_{21}^* & z_{22}^* & \dots & z_{2k}^* \\ \dots & \dots & \dots & \dots \\ z_{n1}^* & z_{n2}^* & \dots & z_{nk}^* \end{pmatrix}, \quad (13)$$

where z_{ij}^* is calculated using (12).

If the extent of direct effect A makes on C , determined by following the steps of hierarchy analysis, exceeds the indirect effect, then it is not worth being taken into account. If the inequality $z_{ij}^* - z_{ij} > 0$ is valid, then we showed the indirect and previously ignored effect that i -th implicit factor makes on j -th resulting index. Moreover, evaluation of the extent of such effect may be considered as $z_{ij}^* - z_{ij}$ difference.

Consequently, using compositions of binary correspondences implicit, indirect relations and cross-effects between the elements of A and C sets may be found out, when correspondences for $A \times B$ and $B \times C$ sets are given; intermediate factors may be specified as well.

6 Model Application to Examining the System of Images' Responses

To explain the described model we give an example for the $\Gamma = \{a, b, c\}$ set. Suppose a , b , and c are three characteristics of a bitmap image: a is a color depth, b is an image volume, and c is an image size. The J matrix specifies the mutual effects of the above characteristics:

$$J_{\Gamma} = \begin{pmatrix} 0.9 & 0.9 & 0.4 \\ 0.7 & 0.8 & 0.7 \\ 0.5 & 0.9 & 0.9 \end{pmatrix}.$$

To determine a mediate mutual effect of these characteristics we can calculate the composition matrix:

$$J_{\Gamma^2} = \begin{pmatrix} 0.9 & 0.9 & 0.4 \\ 0.7 & 0.8 & 0.7 \\ 0.5 & 0.9 & 0.9 \end{pmatrix} \cdot \begin{pmatrix} 0.9 & 0.9 & 0.4 \\ 0.7 & 0.8 & 0.7 \\ 0.5 & 0.9 & 0.9 \end{pmatrix} = \begin{pmatrix} 0.9 & 0.9 & 0.7 \\ 0.7 & 0.8 & 0.7 \\ 0.7 & 0.9 & 0.9 \end{pmatrix}.$$

The J_{Γ^2} matrix indicates the existence of a significant mediate effect of the c factor on the a factor: $\mu_{\Gamma}(c, a) = 0.5$, $\mu_{\Gamma^2}(c, a) = 0.7$, and of the a factor on the c factor: $\mu_{\Gamma}(a, c) = 0.4$, $\mu_{\Gamma^2}(a, c) = 0.7$. Comparison of the cross impact between the color depth and the image size, the intermediate factor taken into account and neglected, is graphically demonstrated in Fig. 1.

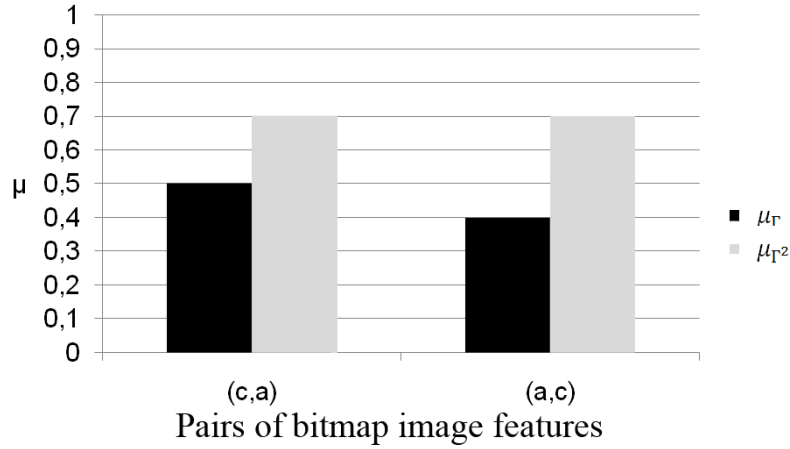


Figure 1: Comparison of the cross impact between the color depth and the image size, the intermediate factor taken into account and neglected

To demonstrate the process of detection of the intermediate factor we can perform the analysis of evaluation of the membership function $\mu_{\Gamma^2}(c, a) = 0.7$:

$$\mu_{\Gamma^2}(c, a) = \max(\min(0.5; 0.9), \min(0.9; 0.7), \min(0.9; 0.5)) = 0.7$$

We make the following statement concerning the indirect effect of the c factor on the a factor: "The image size is determined by its volume ($\mu_{\Gamma}(c, b) = 0.9$), the image volume makes effect on the color depth ($\mu_{\Gamma}(b, a) = 0.7$)." As a result, this leads us to the conclusion: "The image size via its volume determines the color depth ($\mu_{\Gamma^2}(c, a) = 0.7$)."

Thus, the image volume acts as an unobvious element of the communication channel, which is an intermediary between the color depth and the image size that are relatively weakly interconnected without considering the effect thereof. Identification of this significant factor based on data mining, that features the analysis of the image response system, makes it possible to perform an adequate object behavior evaluation, which otherwise would be considerably distorted.

7 Model Implementation

The above algorithm is automated and implemented in the form of a web-service developed on Joomla! 1.5 platform, written in PHP 3.1 language, and published on the website <http://bi.usue.ru/nauka/imin/>.

The latest available version 1.0.3 performs the following functions:

- makes it possible for the user to enter the basic data using the "fuzzy controller";
- carries out the analysis of correctness of the input values;
- displays a result with specification of the implicit factor and the effect strength.

The Fuzzy controller panel basically simulates expert evaluation of a linguistic variable by means of natural language and helps the researcher to describe the influence exerted with the use of words weak, moderate, strong, etc.

Crucial options of the model when operated on a certain object response system ensure its applicability for the analysis of a wide range of data. The model using Goguen's fuzzy implication [14] and defuzzification according to the method of the centre of gravity [15, 16] was officially approved in the research of the response system of the corporate culture [17], of the military-industrial complex of the Russian Federation [18], and demonstrated a high reliability (measure of inaccuracy of significant subtle effect finding was below 3% regardless of the subject of research).

Moreover, as one of the most promising methods of finding and evaluation of implicit effects, the aforesaid algorithm in the course of implementation of the system of machine learning may be applied for the purpose of handling the problems of adaptable control with application of artificial neural networks [12, 18].

8 Conclusion

Thus, the problem of finding hidden dependences and effects that exist among their elements, under conditions of increasing requirements to characterization of the current and projected status of the systems, remains topical. The object simulation based on the analysis of the system of its responses to the signal represents one of the promising research methods to solve control problems in order to obtain the required performance.

Taking into account the need to establish the communication channel appropriate for the signal, principal concern in the course of implementation of model design should be with the fuzzy logic means enjoying a number of crucial advantages over the existing methodological approaches that ensure an extensive application of fuzzy systems by virtue of versatility without compromising the reliability of the obtained results.

The fuzzy model presented in this work is designed to work with responses of different research subjects and makes it possible, based on the composition of fuzzy binary relations, fuzzy binary correspondences, and data mining, to identify significant subtle (implicit) effects in the object response system.

The above algorithm makes it possible to find out the unobvious parameters of the communication channel, as well as to evaluate their effects. The obtained results increase the appropriateness of assessments of patterns of the object behavior.

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