Implementation of Weibull's Model for Determination of Aircraft's Parts Reliability and Spare Parts Forecast

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Abstract. Planning of aircraft's maintenance activities, failure occurrences and necessary spare parts are essential for minimizing downtime, costs and preventing accidents. The aim of this paper is to propose an approach that supports decision making process in planning of aircraft's maintenance activities and required spare parts. Presented mathematical model is based on Weibull's model and calculates aircraft's reliability characteristics by using data on previous failure times of an aircraft part. Further, by capitalizing the random nature of failure time, the number of spare parts and the costs of negative inventory level are determined.

Keywords: aircraft's spare parts, reliability, forecast, Weibull's model.

1 Introduction

Optimized maintenance can be used as a key factor in organization's efficiency and effectiveness. Maintenance in aviation industry requires replacing of parts to assure aircraft availability. Aviation companies are often facing aircraft's downtime due to spare parts shortage because they simply follow manufacturers' or suppliers' recommendation regarding the required number of spare parts to be kept on inventory [1]. Furthermore, that leads to unexpected costs of urgent orders or the passenger accommodation costs in case of flight cancellation, etc. Adequate spare parts management in the aircraft maintenance system improves the aircraft availability and reduces downtime. Spare parts forecasting and provisioning is a complex process and there are numerous paper dealing with this issue [2–6]. In aviation industry some methods described in papers [7–11] found their application but due to stochastic nature of demand they often failed to provide accurate results. In recent times, spare parts forecasting with respect to techno-economical issues (reliability, maintainability, life cycle costs) have been studied [12–14] but not that extensively in aviation industry. In [15] a methodology to forecast the needs for expendable or non-repairable aircraft parts has been presented. That methodology was based on observing total unit time (Tut) provided by manufacturer as stochastic process. In the case when parameter (Tut) is not available, we herewith present a new approach for determination of spare parts requirements. Described approach relies on historical data of previous failure times of an aircraft part and their stochastic nature. In order to determine the reliability characteristic of each aircraft part, the Weibull's model has been used. The Weibull's probability density function (PDF) is given by:

$$f(w) = \frac{\beta}{\eta}\Big(\frac{w}{\eta}\Big)^{\beta-1} \exp^{-(\frac{w}{\eta})^{\beta}}, f(w) \geq 0, \quad w \geq 0, \quad \beta > 0, \quad \eta > 0, \tag{1}$$

where w denotes flight hours, β denotes shape parameter or slope, η denotes scale parameter or characteristic life. Based on previous, the cumulative distributive function (CDF) can be determined as given in eq. (2):

$$F(w) = 1 - \exp^{-(\frac{w}{\eta})^{\beta}}. \tag{2}$$

Further, reliability function of Weibull's model can be calculated as follows:

$$R(w) = \exp^{-(\frac{w}{\eta})^{\beta}}. \tag{3}$$

Also, there is a possibility to calculate the conditional reliability i.e. the reliability for the additional period of w duration for the parts having already accumulated W flight hours. It can be calculated as given in eq. (4):

$$R(w|W) = \frac{R(W+w)}{R(W)} = \frac{\exp^{-(\frac{W+w}{\eta})^{\beta}}}{\exp^{-(\frac{W}{\eta})^{\beta}}} = \exp^{-\left((\frac{W+w}{\eta})^{\beta} - (\frac{W}{\eta})^{\beta}\right)}. \tag{4}$$

The mean time to failure (MTTF) of Weibull's PDF can be determined as in eq. (5):

$$\mathrm{MTTF} = \eta \cdot \Gamma\Big(\frac{1}{\beta} + 1\Big), \tag{5}$$

where Γ is Gamma function. Failure rate function is given in eq. (6):

$$\lambda(w) = \frac{f(w)}{R(w)} = \frac{\beta}{\eta}\Big(\frac{w}{\eta}\Big)^{\beta-1}. \tag{6}$$

In order to calculate reliability characteristic of an aircraft part it is necessary to estimate the parameters of Weibull's model. There are several ways to achieve that, but in the case when we have limited historical data on previous failures, it is best to perform rank regression on Y [16]. Rank regression on Y is a method based on the least squares principle, which minimizes the vertical distance between the data points and the straight line fitted to the data as presented in Fig. 1. The idea is to bring our function to linear line. In order to achieve that we are taking natural algorithm of the both sides of the eq. (2).

$$\ln[1 - F(w)] = \ln[exp(-w/\eta)^{\beta}]$$

$$\ln\left[-\ln[1 - F(w)]\right] = \beta \ln(w/\eta)$$

$$\ln\left[-\ln[1 - F(w)]\right] = \beta \ln w - \beta \ln \eta.$$

Then by setting:

$$y = \ln[-ln(1 - F(w))], \quad x = \ln w, \quad a = \beta \quad and \quad b = -\beta \ln \eta.$$

the previous equation can be rewritten as $y = ax + b$. Now, assume that we have sample of failure data set as (x_1, y_1), (x_2, y_2), ... , (x_n, y_n) plotted and x values are predictor variables. According to least square principle, the straight line that best fit to these data is $y = \hat{a} + \hat{b}x$, such that:

$$\sum_{i=1}^{N} (\hat{a} + \hat{b}x_i - y_i)^2 = min \sum_{i=1}^{N} (\hat{a} + \hat{b}x_i - y_i)^2$$

where $\hat{a}$ and $\hat{b}$ are the least squares estimates of a and b and N is the number of failure data. The equations can be minimized by estimates $\hat{a}$ and $\hat{b}$ as in Eqs. (7) and (8)

$$\hat{b} = \frac{\sum\limits_{i=1}^{N} x_i y_i - \frac{\sum\limits_{i=1}^{N} x_i \sum\limits_{i=1}^{N} y_i}{N}}{\sum\limits_{i=1}^{N} x_i^2 - \frac{\left(\sum\limits_{i=1}^{N} x_i\right)^2}{N}}. \tag{7}$$

and

$$\hat{a} = \frac{\sum\limits_{i=1}^{N} y_i}{N} - \hat{b}\frac{\sum\limits_{i=1}^{N} x_i}{N} = \bar{y} - \hat{b}\bar{x}, \tag{8}$$

The variable $\bar{y}$ is the mean of all the observed values and $\bar{x}$ is the mean of all values of the predictor variable at which the observations were taken. Now, according to the previous, we can easily obtain y_i and x_i

$$y_i = \ln[-ln(1 - F(w_i))], \quad x_i = \ln(w_i). \tag{9}$$

The $F(w_i)$ are values determined from the median ranks, and after we calculate $\hat{a}$ and $\hat{b}$, we can easily estimate parameters η and β.

2 Numerical analysis

According to the previous formulas we can further perform numerical analysis on sample of 14 failure-time data for aircraft part number 302634-2 (Igniter plugs for aircraft Cessna Citation 560XL - provided by Prince Aviation Company, Serbia). Data are sorted by ascending order and presented in Table 1.

First, it was concluded by using Weibull's probability plotting that data are following Weibull's distribution, as can be seen in Fig. 1. Since the table provide the sample size less than 15 failed times, rank regression on Y method, presented in previous section, has been used for parameter estimation. We applied this method since it has been considered as more accurate [16]. It has been calculated

Table 1. Failure time (flight hours for part no. 302634-2 Igniter Plug)

No. of part	Failure time (flight hours)
1	3258
2	4321
3	5183
4	5223
5	5786
6	5920
7	6004
8	6321
9	6550
10	6893
11	6906
12	7221
13	7305
14	7400

that shape parameter (β) is 4.86 and characteristic life (η) is 6,572.98. According to the previous conclusions and eq. (3), we further determined reliability function of the part Igniter plug. Reliability of the part Igniter plug is given in Fig. 2 and the failure rate is presented in Fig. 3.

According to these figures we can conclude after how many flight hours this part would most likely stop working.

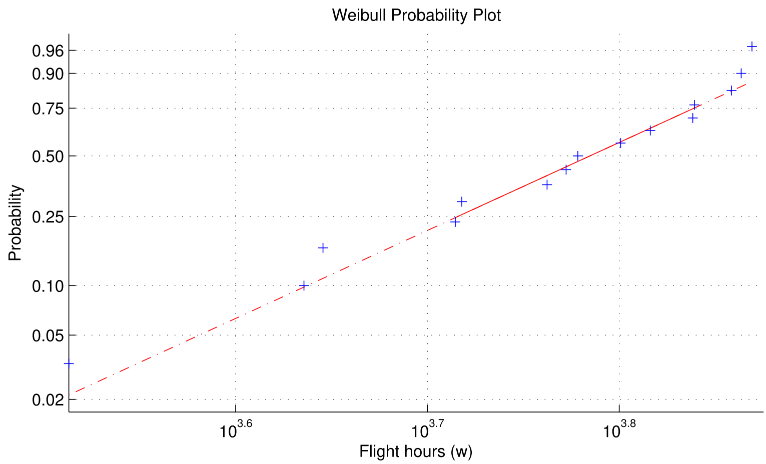

Fig. 1. Weibull Probability Plot.

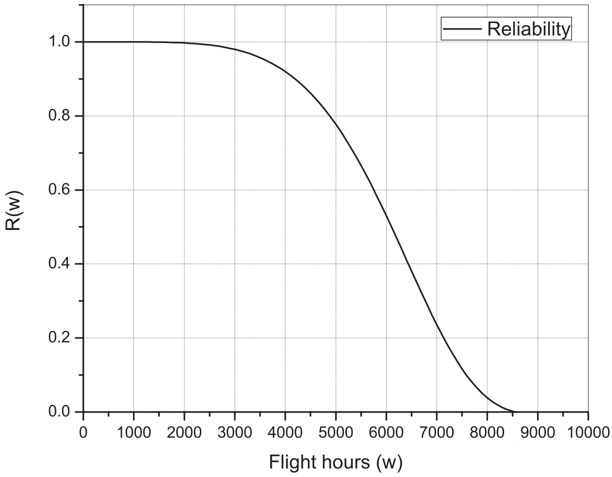

Fig. 2. Reliability function of the part Igniter plug.

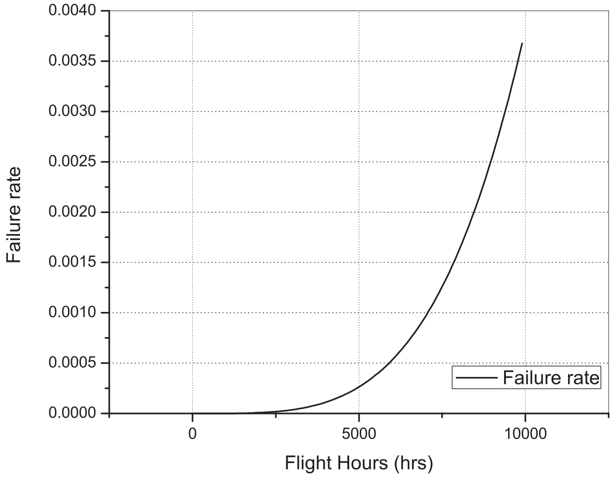

Fig. 3. Failure rate function of the part Igniter plug.

3 Method evaluation

The major contribution of this paper is to determine the number of spare parts that should be kept on stock in interval $[0, w]$. In order to achieve that we are using an approach presented in paper [15] where the number of part exposed to failure in certain time frame was calculated. These calculation are based on Rayleigh's model in the case when only total unit time (usually provided by parts manufacturer) is available. Similar approach is applied in this paper but in the case when data of previous failures are available so the reliability characteristics of the aircraft parts are determined by using the Weibull's model. PDF of Weibull's distributed failure time is given by eq. (1), while the PDF of Rayleigh's distribute failure time is:

$$f(\mu) = \frac{\mu}{\sigma^2} \exp\Big(- \frac{\mu^2}{2\sigma^2}\Big). \tag{10}$$

In the eq. (10), μ presents Rayleigh's random variable, while the PDF of Weibull's model has been given by eq. (1). According to the above stated equations it can be concluded that $\sigma = \eta/\sqrt{2}$ and $\mu = w^{\frac{\beta}{2}}$.

In order to create relation between these models we are using the following transformation:

$$p_{w\dot{w}}(w, \dot{w}) = p_{\mu\dot{\mu}}\Big(w^{\frac{\beta}{2}}, \dot{w}\frac{\beta}{2}w^{\frac{\beta}{2}-1}\Big)|J|, \tag{11}$$

where $|J|$ presents Jacobian transformation of random variables given by the following equation:

$$|J| = \begin{vmatrix} \frac{d\mu}{dw} & \frac{d\mu}{d\dot{w}} \\ \frac{d\dot{\mu}}{dw} & \frac{d\dot{\mu}}{d\dot{w}} \end{vmatrix} = \frac{\beta^2}{4} w^{\beta-2}.$$

So, the eq. (11) further transforms into:

$$p_{w\dot{w}}(w, \dot{w}) = \frac{\beta^2}{4} w^{\beta-2} p_{\mu\dot{\mu}}(\mu, \dot{\mu}).$$

Based on the random nature of failure time of an aircraft part, we are observing the expected number of variations of Rayleigh's random variable μ within an interval $(\mu, \mu + d\mu)$, for a given slope $\dot{\mu}$ within a specified open neighborhood $d\mu$. Actually, $\dot{\mu}$ is a gradient of Rayleigh's random variable, while $\dot{w}$ is gradient of Weibull's random variable. The number of parts that will be exposed to failure can be determined as:

$$n = \int_0^{+\infty} \dot{\mu} p_{\mu\dot{\mu}}(\mu, \dot{\mu}) d\dot{\mu} = \int_0^{+\infty} \dot{\mu} \frac{\mu}{\sigma^2} exp\Big(- \frac{\mu^2}{2\sigma^2}\Big) \frac{1}{\sqrt{2\pi\sigma^2}} exp\Big(- \frac{\dot{\mu}^2}{2\sigma^2}\Big) d\mu$$

$$n = \int_0^{+\infty} \dot{w} p_{w\dot{w}}(w, \dot{w}) d\dot{w}.$$

According to the previous equations, the number of spare parts exposed to failure in time w can be finally determined as:

$$n = \frac{4\sqrt{2} w^{\frac{\beta}{2}}}{\eta} exp\Big(- \frac{w^\beta}{\eta^2}\Big).$$

After we calculated average number of parts that are exposed to failure in interval $[0, w]$, we can determine the number of parts that should be on inventory. We are using the approach presented in paper [15] where we observed the expected amount of time when random variable w is below total unit time as quotient of Rayleighs CDF and n. Since the characteristic life parameter of Weibull's distribution η is the time at which 63.2% of the units will fail and it is approximately equal to MTTF [17], in this case, we are assessing the amount of time when w is below η by dividing CDF function of Weibull's distributed variable and the average number of parts to fail in time interval $[0, w]$ as:

$$q = \frac{F(w)}{n}. \tag{12}$$

As presented in Fig. 4 for the part Igniter plug it can be concluded at what time the spare part should be available. In the case that this part is not available when needed, the underage costs appear. The underage costs are difficult to determine due to their nature. Also, in this paper we are using the well known Newsvendor method [18] in order to calculate these costs. This method gives good results when it is necessary to estimate a stochastic variable. The result of this estimation is a compromise between losses when we decide to order more spare parts than needed and losses when we order less than required. In both cases we have costs, either unnecessary inventory costs or costs of urgent orders. Newsvendor method should provide optimal quantity of spare parts. Since we determined that number in eq. (12), we are using the following formula to

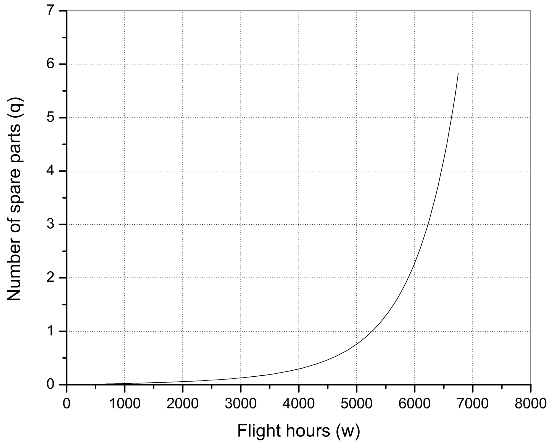

Fig. 4. Number of spare parts for part Igniter plug.

calculate the underage costs:

$$q = \Phi^{-1}\Big(\frac{c_u}{c_u + c_o}\Big),$$

where Φ^{-1} presents inverse distribution function (complementary error function), c_u are underage costs and c_o are overage costs, which in our case is the spare part price. Fig. 5 presented the underage costs for aircraft part Ignition plug. The overage costs for this part are are $1.925,00 and it can be noticed that the underage costs are growing exponentially in relation to time.

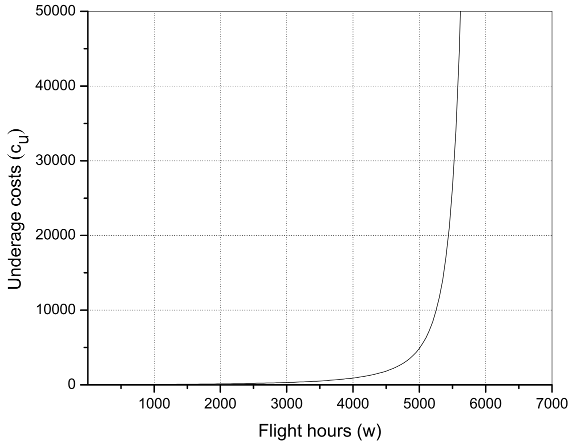

Fig. 5. Underage cost of the part Igniter plug.

4 Conclusion

This paper presents an approach to determine reliability parameters of each aircraft part. This has been achieved by using the observed failure times for certain aircraft part and Weibull's model. Also, a new methodology for calculation of parts that are exposed to failure in observed period of time is presented. This approach was based on random nature of failure or total unit time of each aircraft's part. According to the obtained number, we further calculated the quantity of the aircraft spare parts that should be kept on stock in order to avoid necessary costs. Also, the Newsvendor model was used in order to assess the potential underage costs in certain time period. All these calculations aim to support the decision making process in planning of aircraft maintenance activities and spare parts needs. As presented in the paper, we evaluated the method for one specific aircraft part and presented results. Same could be done for any other aircraft part. Also, these analysis could be applied to other industries with no massive production of spare parts such as weapons industry.

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