Forecast of Thermokarst Lakes Dynamics in Permafrost Based on Geo-Simulation Modeling and Remote Sensing Data

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Abstract. The information technology of forecast of the dynamics of lake's fields in permafrost was developed using geo-simulation approach to modeling. The model properties were determined on base of analysis of data on climatic changes and satellite images. The program complex for predicting geocryological changes under global warming using computer experiments with the model is presented. A new forecast assessments of changes of thermokarst processes on the territory of West-Siberian permafrost were obtained on base of computer experiments. model. It is shown that the gradual increase in temperature to 2 - 3 °C in future decades will cause a reduction in the area of thermokarst lakes, what is an indicator of the continuing degradation of permafrost by the end of the century. The developed information technology can be used for solving problems of predicting the dynamics of greenhouse gas emissions from thermokarst ponds in Western Siberia under the impact of global warming. *abstract* environment.

Keywords: modeling, geo-simulation, forecast, permafrost, thermokarst lakes, information technology, satellite images, climate changes

1 Introduction

The global warming leads to an increase of accidents on pipelines and other oil and gas facilities due to lower strength of permafrost. Moreover permafrost, as a repository of carbon conserved in the vast frozen peat bogs of northern Eurasia and America may cause even more warming if greenhouse gas release. The development of measures to reduce the damage of oil and gas companies require the use of forward-looking assessments of the dynamics of thermokarst processes. Due to considerable bogging and inaccessibility of the territory of Western Siberia, where is located the main oil and gas complex of Russia the research of these processes is impossible without the use of remote sensing data. The information technology of modeling and forecasting of the dynamics of thermokarst lakes fields was developed using satellite images for the period 1973-2010 years. An important issue is the creation of a mathematical model. The complexity of modeling the field of thermokarst lakes has led to the need to use geo-simulation approach to modeling of natural objects with a spatial structure.

We know that global warming leads to the northern territories to the growth of accidents on pipelines and other oil and gas facilities. Reducing the strength of permafrost caused by the acceleration of thermokarst processes under the influence of warming, is accompanied by the growth of economic and environmental damages on the domestic oil and gas companies, located in the permafrost zone. The development of measures to reduce the damage of oil and gas companies is impossible without predictive estimates of the dynamics of the morphological structure of thermokarst lake fields, the preparation of which requires the use of mathematical modeling of the dynamics of thermokarst processes on the territory of permafrost under a global warming.

Due to the high degree of waterlogging and remote areas of permafrost, these studies both in our country and abroad are carried out with the use of remote sensing data. At the same time as the most suitable geomorphological indicator of changes in permafrost is used thermokarst lakes which is well seen on satellite images [1]. So important is the question of forecasting the dynamics of fields of thermokarst lakes.

Thermokarst processes can be modeled mathematically with analytical models based on theory. Matt has shown [2] that such models are efficient for studying processes in a single thermokarst lake, but unsuitable for modeling spatio-temporal changes of thermokarst lake fields. The methods of mathematical morphology developed by Victorov [3]are of great importance here as they are designed to use analytical models for territory dynamics modeling. These methods enable long-term dynamics of the state of a territory to be predicted; but they are not designed for the study of the spatio-temporal changeability of fields of thermokarst lakes. A new approach to modeling the dynamics of spatio-temporal systems proposed in [4,5] allowed to develop geo-simulation model of thermokarst lake's dynamics. This model allows to take into account important regularity of dynamics of thermokarst fields - reducing areas of thermokarst lakes in the permafrost in last decades, confirmed in a large number of remote studies, for example, [6,7]. On the basis of this model authors [8] carried out a forecast of changes in the permafrost zone of Western Siberia thermokarst lakes area up to 2030 using data on temperature and presipitation obtained by linear extrapolation of the reanalysis data.

However, modern forecasts of temperature changes for the north of Western Siberia [9] shown that the trend of average annual temperatures in the long term is differ from the linear type. At present the forecast estimates of dynamics of thermokarst lakes under climate changes in coming decades after 2030 year are not available. Therefore, it is interesting to carry out predicting dynamics of thermokarst processes in West-Siberian permafrost on base of the forecast estimates of climate changes obtained Klimenko et al [9], what is the aim of the present work.

2 Geo-simulation model of lake field dynamics based on experimental data from satellite images

Simulation modelling is one of the most important mathematical modelling types. According to Moiseev and Svirezhev [10], simulation modelling is a research method which can build an approximate model of a studied object; the simulation model describes a real object with accuracy sufficient for current research. Kosolapova and Kovrov [11], and Low and Kelton [12] claim that simulation modelling is used to construct models in cases where, firstly, there is no analytical solution or this solution is very complex and requires huge computer capacity and, secondly, the amount of experimental data about a modelled object is insufficient for statistical method. In such a case a mathematical model is developed in simulation modelling. For modelling spatial objects Polishchuk and Tokareva [13] and later Zhao and Murayama [14] have introduced a special term "geo-simulation modeling". Problems of creating a geo-simulation model of thermokarst lake fields will be considered further.

Creation of a geo-simulation model of thermokarst lakes fields requires knowledge of the basic properties of these fields, which can be obtained experimentally. Because of the inaccessibility of the northern territories of Siberia, thermokarst experimental studies were carried out by remote sensing. For remote study twenty-nine test sites were chosen in different zones of the West-Siberian permafrost (sporadic, discontinuous and continuous). Remote study of the shape of thermokarst lakes boundaries was carried out via satellite images in our research [15]. Research conducted in test sites in sporadic, discontinuous and continuous permafrost showed that the error in estimating lakes areas while replacing their real lakes boundaries by a circle is comparatively small (about 5%). It may serve as a reason to choose a circle as a model for a lake in geo-simulation modelling thermokarst lake fields. In addition, the formation of geo-simulation model of thermokarst lakes fields in the form of a population of random circles requires experimental knowledge about the distribution of coordinates of lakes centres and the distribution of lakes sizes (areas).

To state the regularities for distribution of random coordinates of lakes and size-distribution of them, satellite images Landsat obtained in period 1984-2014 years were used. All space images are selected from the public archive - Global Land Cover Facility and these images are georeferenced in the UTM projection. Processing of space images was carried out by using the software ENVI 4.7 and ArcGIS 9.3. Lakes classification on the Landsat images was carried out by the method of a binary coding (encoding binary classification algorithm in the software ENVI 4.7). Lakes areas are defined by using ArcGIS 9.3. At each test site were identified from hundreds to thousands of lakes. Received data about lake areas were used to determine the average area of lakes and to build both distribution histograms of coordinates centers and of lakes areas for each test site.

Analysis of histograms of distribution of latitude and longitude values of location of lakes centers given in [4,16] showed that experimental regularities of distribution of coordinates of lakes centers correspond to the law of uniform density according to criterion with a probability of 95% [17]. Histograms of size-distribution of lakes were built for all the test sites, located in different permafrost zones. Examples of the histograms are represented in [4]. Comparison of the histograms shows that they have, in general, an exponential character by means of the experimental law of distribution, which makes it possible for thermokarst lake fields to be modelled easily. We may choose a one-parameter exponential law to describe thermokarst lake area distribution in the following form:

$$y = \lambda \times \exp^{-\lambda S} \tag{1}$$

where λ - a parameter of distribution law.

The value of parameter λ can be determined with the help of experimental data according to the formula:

$$\lambda = 1 \div \bar{S} \tag{2}$$

where

$$\bar{S} = \frac{1}{n} \sum_{i=1}^{n} S_i, i = \overline{1, n}$$

S_i is area of i-th lake in test site; n - number of lakes in this test site.

Testing correspondence of exponential law of lake area distribution given by Eq. (1) to experimental histograms shows that in all researched test sites this law corresponds to experimental data in accordance with criterion χ^2 with average probability 90%. Consequently, the stated law of distribution of lake area in form Eq. (1) does not contradict the experimental data. The analysis of the experimental distribution of lakes according to their areas shows that λ in all test sites varies in the range of 0.034 – 0.086 with average values 0.06.

Accordingly, the following fundamental principles determining substantial properties of a model of spatial-temporal structure of thermokarst lake field can be formulated:

1. Lake coastline shapes can be represented by a circle equation with centres coordinates x_i, y_i, and area S_i (i- lake serial number).
2. Spatial changes in the position of centres of circles and their areas are statistically independent.
3. Random distribution of circle centres coordinates $x_i, y_i (i = \overline{1, n})$ is governed by a uniform law.
4. Random distribution of number of circles over their areas conforms to the exponential law of distribution as in (1) with λ as a parameter.

5. Time changes in statistical properties of population of random circles and their dependence on climatic changes are determined by dependency of parameter λ on time and climatic characteristics in the following equation:

$$\lambda = f(T, P, t) \tag{3}$$

where T - temperature, P - level of precipitation and t - time.

Model of field of thermokarst lakes is a collection of random circles, the statistical properties of which correspond to the above principles (1-5). Consequently, major elements in the model description are characteristics of lake shapes, parameters of their random location on surface and random distribution of lakes over their size (areas).

It is necessary to discuss questions of study of interrelation of climate and geo-cryological changes in permafrost and its accounting in the model. To analyse a correlation of area change of thermokarst lakes and climatic indices (average annual temperature and precipitation level) an alternative approach was taken to obtain data on air temperature and precipitation. The approach is based on re-analysis of meteorological data [18] which makes it possible to estimate the value of climatic characteristics in test sites. On base of the re-analysis approach tables of temporal series of annual average value of air temperature and annual sum of precipitation for each test site were obtained.

To study the interrelation between the changes of thermokarst lake areas and changes of air temperature and precipitation level we shall compare coefficients of a linear trend of time changes of average values of the lakes' areas and climate characteristics. It is the analysis of the data obtained for developing a model of thermokarst lake fields suitable for prediction that is of most interest. It is necessary to study temperature dependence of parameter , which determines the kind of law for thermokarst lakes' distribution in accordance with their areas, discussed in [8]. The data exist only for the years when cloudless images were taken, which made it possible to calculate the value of parameter λ.

Previously the equation of dependence of parameter λ on time and climate features was introduced in implicit form (3). To develop a model of actual thermokarst lake dynamics it is necessary to define this dependence in explicit form. This was the reason for doing multidimensional regression analysis [19] of time series of the values of parameter λ and climate features in the West Siberian territory under study represented in [16].

The results of multidimensional regression analysis of the data on parameter λ and climate features can be presented as an equation of multiple regression in the form:

$$\lambda = c_0 + c_1 \times x_1 + c_2 \times x_2 + c_3 \times x_3 \tag{4}$$

where x_1 - average annual air temperature, x_2 - precipitation level, x_3 - time, c_i - coefficients of regression equation $i = 0, \ldots, 3$.

In the result of the regression analysis of time series of the values of parameter and climate features, the following values of regression equation coefficients were obtained (4):

$c_0 = -0.585ha^{-1}$; $c_1 = 0.00062ha^{-1}/°C$; $c_2 = 0.000014ha^{-1}/mm$; $c_3 = 0.00032ha^{-1}/year$.

The stated regression dependence of parameter λ on time and climate changes is a basis for developing algorithms for modelling random thermokarst lake fields, discussed in the next section.

3 Methodic questions of predicting of lake's field based on geo-simulation modeling

In a general case, mutual density of probabilities of random coordinates of centres and areas of circles imitating lakes in a mathematical model of random thermokarst lake fields can be presented in the form:

$$f(x, y, s) \tag{5}$$

where x and y - coordinates of circle center in a model; s - area of a circle imitating a lake.

Consequently, the totality of circles in the model of lake fields will be presented as a totality of groups of three random values (x, y, s). To develop an algorithm for modelling thermokarst lake fields, it is necessary to take into consideration statistical connections between changes in lakes coordinates and their areas.

Further to equation (5), the random-number sequence determining characteristics of location of circles centers (x and y) is generated using the antenna of pseudo-random numbers distributed in accordance with the law of even distribution. And to form circles of random size whose areas are distributed according to the law conforming to equation (1) it is necessary to generate random-number sequences distributed in accordance with the exponential law. Consequently, together with using software generators for even distribution of pseudo-random numbers, the software realization of an imitation model of thermokarst lake fields includes creating a generator for pseudo-random number sequences, distributed in accordance with the exponential law.

We should consider a geo-simulation model of spatial structure of thermokarst lake field $M_{Sp}(t)$ that is a totality of circles and reflects the state of a thermokarst lake field at the moment of time t. To model the dynamics of thermokarst lake fields, we should consider a general model of spatio-temporal structure of a thermokarst lake field in the form:

$$M_{SpTm} = \{M_{Sp}(t_1), \ldots, M_{Sp}(t_j), \ldots, M_{Sp}(t_n)\}, j = 1, \ldots, n \tag{6}$$

which is a time sequence of geo-simulation models of a thermokarst lake field $M_{Sp}(t_j)$, $j = 1, \ldots, n$ where each model relates to a particular moment of time.

Fig. 1 gives a visual presentation of the general model for spatio-temporal structure of thermokarst lake fields in the form of geo-information system (GIS) layers that relate to given time moments $t_1, t_2, \ldots, t_n \in \overline{(t_1, t_n)}$.

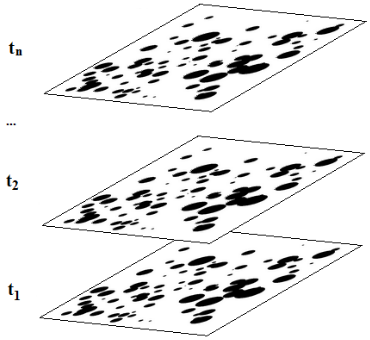

Fig. 1. Visual presentation of a general model of dynamics of thermokarst lake fields. Legend: t_i - time (year), $i = \overline{1, n}$

When modelling spatio-temporal structure of thermokarst lake fields it is important to take into consideration both time dependence and climate features (temperature, precipitation level). Accordingly, the dependence of parameter λ on time and climate features is determined by the equation of multiple regression in the form (4). This is the reason why equation (4) was used to develop an algorithm for numerical modelling dynamics of thermokarst lake fields.

The developed algorithm for modelling dynamics of thermokarst lake fields can be presented as follows:

step 1 — the year of modelling is specified $t_j,\ j = 1, \dots, m$;

step 2 — the areas (S_{MA}) of the model area (MA) under study are specified;

step 3 — lake density (σ_{MA}) in MA is specified;

step 4 — the number of circles within MA is determined in accordance with formula: $N_{MA} = S_{MA} \times \sigma_{MA}$;

step 5 — the centre of MA location in the map is specified;

step 6 — parameter λ is determined in accordance with formula (4) for given values of temperature and time (year of modelling);

step 7 — pseudo-random number is generated, distributed in accordance with uniform law;

step 8 — using the number obtained at the previous step, a pseudo-random number is calculated to characterize the value of circle area according with formula:

$$s_i = -\frac{1}{\lambda} \ln z_j \qquad (7)$$

where z_j - pseudo-random number distributed in accordance with uniform law in interval $(0, 1)$, $j = 1, \dots, m$;

step 9 — two pseudo-random numbers are generated, distributed in accordance with uniform law, determining the coordinates for circle centre location on the screen;

step 10 — using the values of a number triple (x, y, s) obtained at previous steps 8 and 9, in accordance with equations

$$x_{ki} = R_i \times \cos\theta_i + x_i \tag{8}$$

and

$$y_{ki} = R_i \times \sin\theta_i + y_i \tag{9}$$

a circle is formed on the screen;

step 11 — if the number of circles obtained is less than N_{MA} , determined at step 4, the algorithm repeats beginning with step 7, otherwise it is completed.

The given algorithm allows formation of a model of spatial structure for a given time moment $M_{Sp}(t_j)$, where $j = 1, \ldots, m$. To make a general model of dynamics of a thermokarst lake field by means of forming a time sequence of models $M_{Sp}(t_j)$ for a given set of moments $t_j (i = 1, \ldots, m)$ the algorithm repeats for the number of times (m) needed.

Accuracy of modelling dynamics of thermokarst lake fields was studied in the form of computer experiment on the model. Values of parameter λ in this case are calculated according to the multiple regression formula (4) using the data about average annual temperature and precipitation level determined for each S by re-analysis. Then a model field is formed in accordance with the algorithm described above. Estimation showed that the error of determination of average values of lakes areas on base of modelling with use of experimental data is 17%. This may well be regarded as a suitable result of modelling thermokarst lake fields for predicting thermokarst lake fields dynamics.

4 Software of predicting of lake's field dynamics

Geo-simulation models are considered as more promising and make it possible to study the dynamics of the thermokarst lakes fields in today's global warming. Recently, within the framework of the ideology of the simulation formed one of the new areas of computer modeling, which is called geo-simulation modeling. It is a simulation of complex objects with a spatial structure and realized with the use of methods and means of geoinformatics. The geo-simulation model of thermokarst lakes fields in the form of the random circles set is described in [16]. This model takes into account the properties of the main characteristics of the real thermokarst lakes fields, which are identified by the experimental data of remote sensing. However, issues of information technology of the geo-simulation thermokarst lakes fields, in particular, software implementation, is now not enough considered and this fact defined the purpose of the present work.

The implementation of a mathematical model of thermokarst lakes fields considered in [20] involves the creation of the pseudo-random number sequences - triples: the first two pseudo-random numbers are distributed over a uniform density of the law, and the third - exponentially. The software package is designed, and its structural diagram is shown in fig. 2.

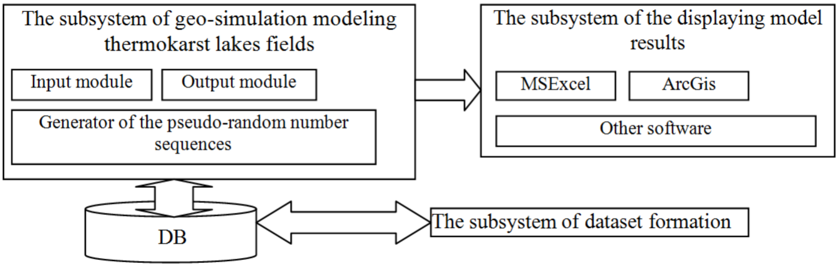

Fig. 2. Generalized diagram of the software package of geo-simulation modeling thermokarst lakes fields

The structure of the developed software package includes the following main components:

1. *subsystem of geo-simulation modeling thermokarst lakes fields;*
2. *the subsystem of the displaying model results;*
3. *database (DB);*
4. *the subsystem of dataset formation.*

The following describes components of the software package. The subsystem of geo-simulation modeling thermokarst lakes fields, developed by the author, it is a set of software modules that provide the input parameters of the model, the formation of pseudo-random number sequences and output of simulation results. The structure of the subsystem of geo-simulation modeling thermokarst lakes fields includes the following main blocks:

- *input module* is designed to provide the pseudo-random number sequences values of model parameters;
- *generator of the pseudo-random number sequences* is a major component of the subsystem of geo-simulation modeling thermokarst lakes field and it is designed to generate random number sequences in the algorithms implementation for numerical simulation of thermokarst lakes fields. The numerical simulation algorithm is described in details in [16];
- *output module* is designed to convert the simulation results in one of the following formats: Microsoft Excel (* .xls), a vector format (* .shp), bitmap format (* .jpeg)).

The subsystem of the displaying model results allows showing the output given either on a digital map by means of geographic information system (ArcGIS), or in the form of electron tables and graphics in MSExcel. The database is a part of the software package. It is a store spatial and attributes information on the study sites obtained during the field experiment. Description of the database structure and capacity is given in [16]. *The subsystem of dataset formation* allows extracting from the database the information about the object of research and forming data sets for model experiments.

5 Sample of predicting of lake's field dynamics in West-Siberian permafrost

To generate the forecast of dynamics of the thermokarst lakes fields in permafrost of Western Siberia for future decades, it is necessary to have projections on climate changes in the study area. Temperature forecast for the north of Western Siberia to 2300 are presented in [9,21]. Data on precipitation forecasts in the coming decades are not existed in the literature. A comparison of the coefficients of the regression equation (4) shows, that in predicting the dynamics of thermokarst processes in the permafrost zone of Western Siberia can neglect the effect of precipitation and take into account only the temperature changes. Therefore, predicting the dynamics of thermokarst lakes fields may be carried out using the temperature forecast data [21] for the north of Western Siberia by the computer experiments with the model in accordance with the below considered scenario.

Scenario of computer simulation experiment: Predicting the dynamics of thermokarst lake fields on the basis of the geo-simulation model using predictive estimates of temperature changes in the north of Western Siberia [21] for the period up to 2050 year.

As shown in [21], an increase in temperature, which began after 1970, will continue in the coming decades. According to [21], the maximum warming could reach by the end of the forecast period almost 1°C compared to the present time. The result of forecasting the dynamics of thermokarst-lake fields in Western Siberia, is shown in Fig. 3 as plot of time dependence of the mean value of thermokarst lake area.

Because of the current lack of prognosis of precipitation changes in the prediction period, at obtaining of forecast assessments of lakes area changes are used data only about temperature changes. To substantiate the prediction possibility without precipitation we conducted a comparison of the regression equation coefficients (4), the values of which are given on p. 6. Comparison of these coefficients shows that $c_2 \ll c_1$ and $c_2 \ll c_3$. This allows us to ignore the contribution of term $c_2 \times x_2$ in the equation (4) in the value of the parameter λ. Comparison of the results of the lakes dynamics prediction by using the developed algorithm for the period up to 2030 [16] showed the unessential difference between the forecast estimates for both cases with and without precipitation, that proves our statement.

The most important result of the analysis of forecast estimates presented in Fig. 3, is the conclusion about continuation of the reduction of the average area of thermokarst lakes in the West Siberian permafrost. The graph in Fig. 3 shows that at the end of the forecast period, the average area of the lakes can be reduced to 14.5 hectares, i.e. approximately 20% compared to 2010 year. Thus, the continued increase in the coming decades the average annual temperature of surface atmosphere will be accompanied by a reduction in the average area of thermokarst lakes in the permafrost zone of Western Siberia, what is the result of permafrost degradation and reduce its strength.

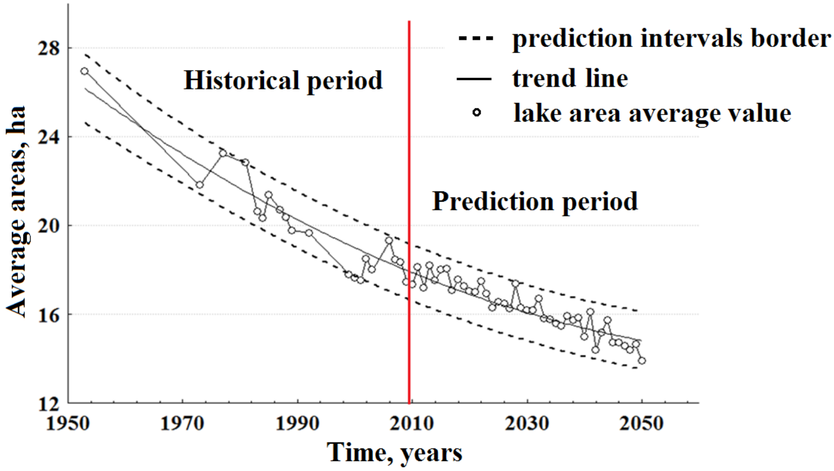

Fig. 3. Predicted time dependence of the lake area average values

6 Conclusion

Designed geo-simulation model of the dynamics of thermokarst lake fields, taking into account the relationship between regional geocryological and climate changes, allowed to carry out forecasting changes of lake sizes in conditions of continuing global warming. Long-term prognosis of the dynamics of lake thermokarst-fields using this model showed that with the growth of the air temperature in West-Siberian permafrost lake areas will be reduced on average by approximately 20% by the end of the forecast period. A new forecast assessments of changes of thermokarst processes on the territory of West-Siberian permafrost were obtained using computer experiments with the model. It is shown that the gradual increase in temperature to 1°C by 2050 year will cause a reduction in the area of thermokarst lakes, what is an indicator of the continuing degradation of permafrost in coming decades.

The information technology of modeling and forecasting the dynamics of thermokarst lakes fields can be used for solving the problems of reducing the accident rate on the infrastructure facilities in the permafrost territories and predicting the dynamics of greenhouse gas emissions from thermokarst ponds in Western Siberia under the impact of global warming.

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