

Amplitude and polarization transformations of the Bessel beam as it passes through an anisotropic crystal perpendicular to the axis of the crystal

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Abstract

A comparative numerical calculation of the propagation of a zero-order Bessel laser beam in a uniaxial crystal perpendicular to its axis is performed using the Rayleigh-Sommerfeld integral operator, generalized for an anisotropic environment. Numerical simulation is performed with a different type of beam polarization and different characteristics of the Bessel beam. Patterns of the beam intensity during the passing of different distances in the crystal are obtained, showing the degree of astigmatic transformation, which makes it possible to determine the conditions under which the greatest astigmatic distortion of the beams occurs. The above analysis can be useful in practice for determining the anisotropy characteristics of a crystal.

Keywords: diffraction axicon; birefringent crystal; polarization transformations; amplitude transformations, Bessel beams; astigmatism

1. Introduction

Optical devices are becoming more and more interesting and practical. They allow to transform certain properties of electromagnetic radiation into others. Most often, modal transformations (from the fundamental mode to higher order distributions) and polarization (from homogeneous linear polarization to more complex ones) are required. One of the tools of such transformations are anisotropic crystals. The propagation of laser modes with a high numerical aperture in an environment with strong anisotropy leads to complex polarization-mode transformations [1-6].

In particular, when propagating along the crystal axis, the spin angular momentum is transformed, which has a circularly polarized beam at the orbital angular momentum [7-13]. It was shown in [6, 7, 11-13] that when propagating along the crystal axis, nonparaxial Bessel beams undergo a periodic change in intensity, corresponding to a transformation into a higher-order beam. In publications [14-20], polarization transformations of beams focused along the crystal axis were considered.

The propagation of various types of laser beams perpendicular to the axis of the crystal was investigated in [21-26]. The most interesting transformations were observed for Bessel beams [16, 21, 24, 27], since in this case there is a visually pronounced astigmatic distortion of the ring structure of the beam. A similar distortion can be observed with oblique incidence of a plane wave on a diffraction axicon [28-30], and also with a cylindrical lens [31]. This analogy was noted in [24], and the analytical basis for such an effect was given in [27].

In this paper, the effect of the astigmatic transformation of Bessel beams propagating perpendicular to the crystal axis is studied in detail on the basis of numerical simulation. The calculation was carried out using the Rayleigh-Sommerfeld integral operator, generalized for an anisotropic environment [32, 33]. Numerical simulation is performed for different types of beam polarization and different characteristics of the Bessel beam. The formation of Bessel beams [34-37] was carried out with the diffraction axicon with different period of the radial lattice. The effect of the relative position of the polarization plane of the radiation and the c-axis of the crystal on the intensity distributions formed in different vector components of ordinary and extraordinary beams is investigated. Patterns of the beam intensity are obtained during the passing of different distances in the crystal, showing the degree of astigmatic transformation, which makes it possible to determine the conditions under which the greatest astigmatic distortion of the beams occurs. The above analysis can be useful in practice for determining the anisotropy characteristics of a crystal.

2. Theoretical analysis

Consider an anisotropic crystal whose axis is oriented perpendicular to the propagation axis and coincides with the Oy axis. In this case, the field propagation in a crystal with dielectric permittivities, (ordinary and extraordinary) can be described by an expression similar to the Rayleigh-Sommerfeld integral [32, 33]:

$$\mathbf{E}(u, v, z) = \frac{2\pi z}{\lambda^2} \sum_{j=1}^2 \iint \mathbf{e}_j(\alpha_{jc}, \beta_{jc}) \left[\mathbf{w}_j(\alpha_{jc}, \beta_{jc})^T \mathbf{E}_\perp(x, y, 0) \right] \frac{\sqrt{d_j s_j t_j}}{R_j^2} \exp \left\{ ik \sqrt{\frac{d_j}{s_j t_j}} R_j \right\} dx dy, \quad (1)$$

where the indices correspond to the ordinary ($j = 1$) and extraordinary ($j = 2$) waves, $d_1 = d_2 = \varepsilon_o$, $s_1 = t_1 = 1$, $s_2 = t_2 = \varepsilon_o / \varepsilon_e$.

For transverse (x- and y- components):

$$\begin{aligned}
 e_{1x}(\alpha, \beta) &= 1, \\
 e_{1y}(\alpha, \beta) &= 0, \\
 e_{2x}(\alpha, \beta) &= \alpha\beta, \\
 e_{2y}(\alpha, \beta) &= \beta^2 - \varepsilon_o.
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 \mathbf{w}_1(\alpha, \beta)^T &= \left(1, \frac{\alpha\beta}{(\varepsilon_o - \beta^2)} \right), \\
 \mathbf{w}_2(\alpha, \beta)^T &= \left(0, -\frac{1}{(\varepsilon_o - \beta^2)} \right).
 \end{aligned} \tag{3}$$

$$\begin{cases} \alpha_{1c} = \sqrt{\varepsilon_o} \frac{(u-x)}{R_1}, \\ \beta_{1c} = \sqrt{\varepsilon_o} \frac{(v-y)}{R_1}, \end{cases} \quad \begin{cases} \alpha_{1c} = \sqrt{\varepsilon_o} \frac{(u-x)}{R_1}, \\ \beta_{1c} = \sqrt{\varepsilon_o} \frac{(v-y)}{R_1}, \end{cases} \tag{4}$$

where

$$\begin{aligned}
 R_1 &= \sqrt{(u-x)^2 + (v-y)^2 + z^2}, \\
 R_2 &= \sqrt{\frac{\varepsilon_e}{\varepsilon_o} \sqrt{(u-x)^2 + \frac{\varepsilon_o}{\varepsilon_e} (v-y)^2 + z^2}}.
 \end{aligned} \tag{5}$$

Similar results can be obtained if the crystal axis is directed along the Ox axis.

3. Results of numerical simulation

During the experiment, the axicon was used. The scheme of the axicon's work is shown in Fig. 1

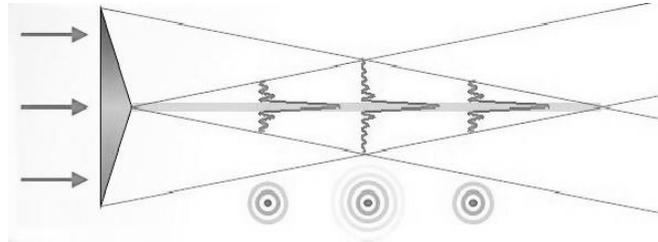


Fig. 1. The scheme of the axicon's work.

In order to carry out the simulation as an anisotropic medium, a lithium niobate crystal of the X-cut was chosen in this study, the dielectric constant of which is $\varepsilon_0 = 5.2273505956$, $\varepsilon_e = 4.8517551289$. The refractive indices of this crystal are: $n_0 = 2.28634$, $n_e = 2.20267$. For the formation of zero-order Bessel beams, diffraction axicons with periods $d_1 = 1.2 \mu\text{m}$, $d_2 = 2 \mu\text{m}$, $d_3 = 4 \mu\text{m}$ were used and illuminated with light polarized linearly along the OY axis with a wavelength of $\lambda = 632.8 \text{ nm}$. We also compared the results of the transformation for different crystal thicknesses, which were chosen $h_1 = 1047 \mu\text{m}$ and $h_2 = 843 \mu\text{m}$. To analyze the transformation of Bessel beams with axicons, the results of the simulation were presented in the form of patterns of light distribution of propagating beams separately for polarized light along OX, separately for OY and their superposition. The results of the simulation are presented in Table 1.

It can be noted that the picture of the Y component almost does not differ from the superposition picture of the X and Y components, which means that the X component has a negligible intensity, and the linearly polarized light at the exit from the lithium niobate crystal has not changed its polarization.

As can be seen from the modeled intensity distribution maps of lithium niobate transformed into an anisotropic lithium crystal by Bessel beams, the beams formed by axicons with the minimal period are subjected to the strongest astigmatic distortions. With an increase in crystal thickness, the degree of astigmatism increases in proportion to the propagation length.

When analyzing patterns of light intensity distribution at the output of an anisotropic crystal for linearly polarized light along the Y axis, with circular polarization, polarization rotated through an angle of 45° about the X axis, the above-described character of the Bessel beam transformation is preserved.

Table 1. Patterns of propagation of Bessel beams formed with axicons under illumination by light polarized along the OY axis through an anisotropic X-cut crystal.

	$d_1=1,2 \mu\text{m}$		$d_2=2 \mu\text{m}$		$d_3=4 \mu\text{m}$	
Component	$h_1=1,047 \text{ mm}$	$h_2=0,843 \text{ mm}$	$h_1=1,047 \text{ mm}$	$h_2=0,843 \text{ mm}$	$h_1=1,047 \text{ mm}$	$h_2=0,843 \text{ mm}$
General						
x						
y						

4. Conclusion

In the work, to analyze the dependence of the propagation of the zero-order Bessel beam on the polarization angle, on the period and the radius of the axicon, we used the calculation with the Rayleigh-Sommerfeld integral operator generalized for an anisotropic medium. The Bessel beams formed by an axicon with the smallest period and passing through an anisotropic Crystal at the greatest distance. The described regularities can be used in practice to determine the degree of anisotropy or the exact thickness of the crystal cuts.

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