Control of a one rigit-link manipulator in the case of non-smooth periodic trajectory

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Abstract

Mathematical model of a single-link manipulator is considered. It describes the motion of the manipulator in the case of non-smooth path. Interpolation of the trajectory of motion is used, which makes it possible to reduce the amount of calculations and allows you to take into account the restrictions on the movement of the manipulator. Integral manifold method is used for the system order reduction. As a result, the reduced system of the investigated object is obtained, and the control function for the manipulation robot model in the case of a non-smooth periodic trajectory is constructed.

Keywords: mathematical model; manipulation robot; integral manifold; singular perturbations; periodic trajectory

1. Introduction

In this paper, we consider a mathematical model of a robotic manipulator that describes its motion along a non-smooth periodic trajectory. After path definition of the manipulator movement control function is selected. It allows implementing the required movement accurately. To solve the problem, we use the method of integral manifolds [1-3]. As applied to control problems, this method was considered in [4-7].

2. Single-link manipulator model

The equations of motion of a single-point manipulator have the form [7-8]:

$$J_1\ddot{q}_1 + Mgl\sin q_1 + c(\dot{q}_1 - \dot{q}_m) + k(q_1 - q_m) = 0, \tag{1}$$

$$J_m\ddot{q}_m - c(\dot{q}_1 - \dot{q}_m) - k(q_1 + q_m) = u,$$

where J_m is the jet second moment; J_1 is the link second moment; M is the link mass; l is the link length; c is the attenuation factor; k is the hardness. Let q_1 is the link angular displacement; q_m is the output angle, and u is the control circuit. In Fig.1 the image of the single-link manipulator is presented.

Variables in the system are changed in the following manner:

$$x_1 = \frac{J_1q_1+J_mq_m}{J_1+J_m},\ x_2 = \dot{x}_1,\ y_1 = q_1 - q_m,\ y_2 = \varepsilon\dot{y}_1, \tag{2}$$

Then system (1) is transformed to:

$$\dot{x}_1 = x_2,\ \dot{x}_2 = \frac{Mgl}{J_1+J_m}\sin\left(x_1 + \frac{J_m}{J_1+J_m}y_1\right) + \frac{u}{J_1+J_m}, \tag{3}$$

$$\varepsilon\dot{y}_1 = y_2,\ \varepsilon\dot{y}_2 = -\left(\frac{1}{J_1}+\frac{1}{J_m}\right)y_1 - \varepsilon c\left(\frac{1}{J_1}+\frac{1}{J_m}\right)y_2 - \varepsilon^2\frac{Mgl}{J_1}\sin\left(x_1 + \frac{J_m}{J_1+J_m}y_1\right) - \varepsilon^2\frac{u}{J_m}. \tag{4}$$

This system is singularly perturbed with slow subsystem (3) and fast subsystem (4). Omitting all terms of $O(\varepsilon^2)$ order in the right hand side of the last equation the independent subsystem is obtained.

$$\varepsilon\dot{y}_1 = y_2,\ \varepsilon\dot{y}_2 = -\left(\frac{1}{J_1}+\frac{1}{J_m}\right)y_1 - \varepsilon c\left(\frac{1}{J_1}+\frac{1}{J_m}\right)y_2,$$

The solutions of system are characterized by quite high frequency $\frac{\sqrt{\left(\frac{1}{J_1}+\frac{1}{J_m}\right)}}{\varepsilon}$ and relatively low damping factor $c(\frac{1}{J_1}+\frac{1}{J_m})/2$, and differential system has a characteristic equation

$$\varepsilon^2\lambda^2 + c\left(\frac{1}{J_1}+\frac{1}{J_m}\right)\lambda + \left(\frac{1}{J_1}+\frac{1}{J_m}\right)$$

with complex roots

$$\lambda_{1,2} = -\frac{c}{2}\left(\frac{1}{J_1}+\frac{1}{J_m}\right) \pm \frac{i}{2}\sqrt{\left(\frac{1}{J_1}+\frac{1}{J_m}\right) - \varepsilon^2\frac{c^2}{4}\left(\frac{1}{J_1}+\frac{1}{J_m}\right)^2} \tag{5}$$

As far as a real part is negative, slow invariant manifold can be used for model analysis of the concerned manipulator.

3. Integral manifold construction

To calculate the slow integral manifold for the system (3)-(4) we use asymptotic expansions and obtain, within the accuracy of $O(\varepsilon^3)$, $y_1 = \varepsilon^2Y + O(\varepsilon^3)$ и $y_2 = O(\varepsilon^3)$ (5), where

$$Y = -\left[\frac{Mgl}{J_1}\sin(x_1) + \frac{u_0}{J_m}\right]\left(\frac{1}{J_1}+\frac{1}{J_m}\right)^{-1}$$

Here the representation $u = u_0 + \varepsilon^2u_1 + O(\varepsilon^3)$ is used.

Movement on the manifold is described by the following equations

$$\dot{x}_1 = x_2,\ \ \dot{x}_2 = -\frac{Mgl}{J_1+J_m}\sin\left(x_1 + \varepsilon^2\frac{J_m}{J_1+J_m}Y\right) + \frac{u_0+\varepsilon^2 u_1}{J_1+J_m} + O(\varepsilon^3) \quad (6)$$

Manipulator angular displacement q_1 is expressed using new variables

$$q_1 = x_1 + \frac{J_m}{J_m+J_1}y_1, \quad (7)$$

where $y_1 = \varepsilon^2 Y + O(\varepsilon^3)$. This allows to rewrite the system (5) on the slow integral manifold as

$$\ddot{q}_1 - \varepsilon^2\frac{J_m}{J_m+J_1}\ddot{Y} = -\frac{Mgl}{J_1+J_m}\sin(q_1) + \frac{u_0+\varepsilon^2 u_1}{J_1+J_m} + O(\varepsilon^3). \quad (8)$$

4. Control function

Let $q_d(t)$ be the required trajectory of the manipulator movement. Slow control function term is in the form $u_0 = (J_1 + J_m)u_d + Mgl\sin q_1$, где $u_d = \ddot{q}_d - a_1(x_1 + q_d) - a_2(\dot{x}_1 + \dot{q}_d)$. Using (8) and u_0 and u_d we obtain within the accuracy of the order $O(\varepsilon^2)$

$$\ddot{q}_1 - \ddot{q}_d + a_2(\ddot{q}_1 + \ddot{q}_d) + a_1(q_1 + q_d) = 0 \quad (9)$$

for $q_1 - q_d$, aand $q_1 = x_1 + O(\varepsilon^3)$ on the slow integral manifold.

Equation (9) gives the possibility to select control function u_d coefficients in such a way that the relevant control affords to achieve the required trajectory. Assume, for instance, M =1, k =100 , l =1, J_1 =1 , J_m =1, g =9.8, c =2 , at that a_1=3, a_2=4, and the required trajectory is of the form $q_d = \sin t$, then we obtain the following original variables control law

$$u = 2u_d + 9.8\sin(q_1) = 2[-\sin t - 4(\dot{q}_1 - \cos t) - 3(q_1 - \sin t)] + 9.8\sin(q_1)$$

The first stage of the control construction is to determine the desired trajectory of motion of the manipulator in the form of some analytically described function. In most cases, the manipulators do not move along smooth trajectories, so that its trajectory is a sectionally smooth line. For smoothing the interpolation of the chosen trajectory is used by polynomials of a certain class approximating the segments of the desired trajectory of the manipulation robot between the node points (for example, lines, arcs, parabolas, etc.). But there is a possibility that there will be a problem associated with the difficulty of calculating a polynomial of high degree. In this regard, to perform interpolation of the trajectory from the given nodal points, it is necessary to choose polynomials of low degrees or to break the trajectory of the manipulator's movement into separate sections.

In Fig. 1 there is a displacement-time diagram in case the required path q_d is written as

$$q_d = \begin{cases} x,\ \ 0 < x < \delta - 1 \\ a(x-1)^4 + b(x-1)^2 + 1, \delta - 1 < x < \delta + 1 \\ -x + 2,\ \ \delta + 1 < x < 2 \end{cases}$$

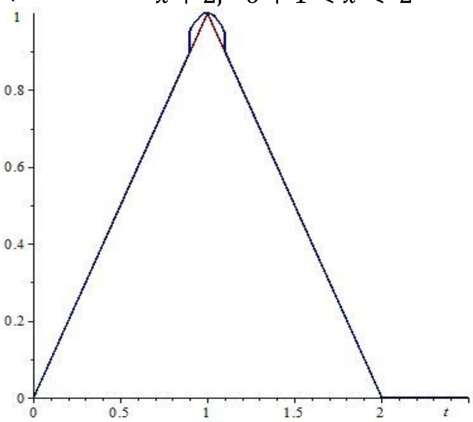

Fig. 1. Trajectory q_d.

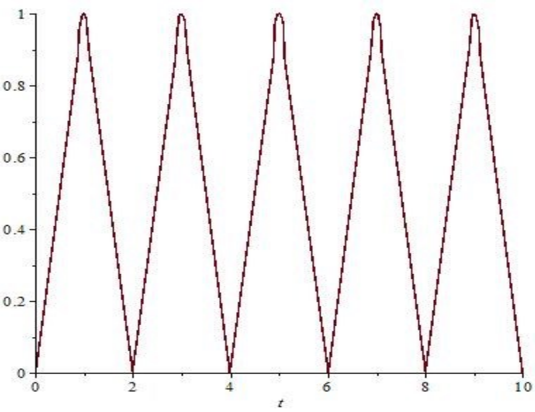

Fig.2. Periodic trajectory.

When the trajectory q_d is substituted in the system of equations of motion of the manipulation robot (1), the trajectory of motion will look as follows (fig. 3).

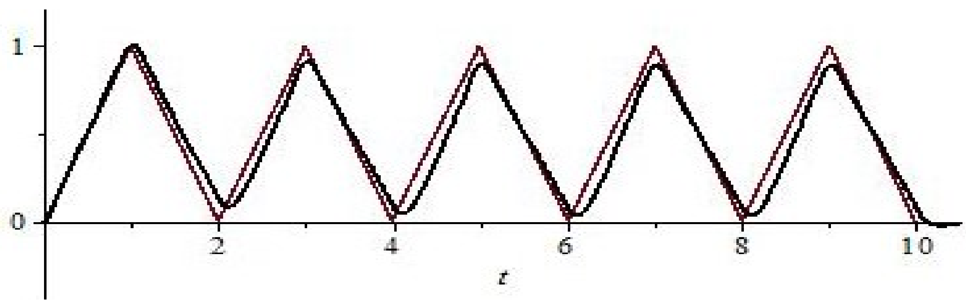

Fig. 3. Trajectory of motion of the manipulator in the case of a periodic trajectory.

Conclusion

The object of research is a manipulator model describing the manipulator motion in a non-smooth path. The interpolation of the trajectory of motion by polynomials is used that approximates the segments of the desired trajectory of the manipulation robot between the nodal points, which makes it possible to reduce the amount and time of calculations, and allows us to take into account the restrictions on the movement of the manipulator. Integral manifold method is used for the system order reduction.

As a result of the work done the reduced system of the object is obtained and the control function for a diagrammatic formulation of the manipulator model motion. It is established that manifold control provides the motion of the system along the trajectory near to the effective one.

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References

[1] Shchepakina E, Sobolev V, Mortell MP. Singular Perturbations: Introduction to system order reduction methods with applications, 2014; 121 p.
[2] Sobolev VA, Tropkina EA. Asymptotic expansions of slow invariant manifolds and reduction of chemical kinetics models. Comput. Mathematics and Math. Physics 2012; 52(1): 75–89.
[3] Strygin VV, Sobolev VA. Effect of geometric and kinetic parameters and energy dissipation on orientation stability of dual-spin satellites. Cosmic Research 1976; 14(3): 331–335.
[4] Smetannikova E, Sobolev V. Regularization of Cheap Periodic Control Problems. Automation and Remote Control 2005; 66(6): 903–916.
[5] Mikheev YuV, Sobolev VA, Fridman EM. Asymptotic analysis of digital control systems. Autom. Remote Control 1988; 49(9): 1175–1180.
[6] Sobolev VA. Singular perturbations in linearly quadratic optimal control problems. Autom. Remote Control 1991; 52(2): 180–189.
[7] Ghorbel F, Spong MW. Integral manifolds of singularly perturbed systems with application. to rigid-link flexible-joint multibody systems. Int. J. of Non-Linear Mechanics 2000; 35: 133–155.
[8] Spong MW. Modeling and control of elastic joint robots. J. of Dynamic Systems, Measurement and Control 1987; 109(4): 310–319.