Conditions for the loss of stability of eqiulibrium manifold in satellite model

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Abstract

The problem of stabilizing a spin satellite by means of passive dampers is considered. The application of the method of integral manifolds allows us to find conditions for the loss of stability in the analytical form.

Keywords: stability; stabilization; manifold of steady states; satellite

1. Introduction

A lot of work has been devoted to the study of dynamic models of stabilization of satellites with the help of gyroscopic forces. As the main apparatus, the Lyapunov function method and the stability criteria applied to first approximation systems are used. In addition to gyroscopic forces for stabilization, damping devices are used in a number of models to ensure the asymptotic stability of the required modes of satellite motion. In a number of works, passive dampers are considered as such devices. For the case of two co-axial bodies, on each of which one damper is installed, the stabilization problem was considered, for example, in [1-3]. In this paper, we confine ourselves to the study of a model of a satellite consisting of two bodies, on one of which a damper with a relatively small coefficient of viscous friction is installed. The damper is modeled by a particle of relatively small mass placed in a tube filled with a viscous liquid and attached by a spring. To analyze the system of differential equations, the method of integral manifolds [3, 4] is applied, which allows to significantly reduce the dimensionality of the model and simplify the analysis.

2. Equations

To study the conditions and the mechanism of loss of stability for a satellite stabilized by rotation, consider a dynamic model that is a system of ordinary differential equations for dimensionless variables and parameters of the form [3]:

$$q\dot{\omega} - \varepsilon\dot{x_1} = \varepsilon[2x_1 v_1 - \omega x_2 u_1],$$

$$[1 - 2Lu_1]\dot{x_1} - \varepsilon u_2 \dot{\omega} =$$

$$= -\Lambda x_2 + \varepsilon[-u_1 2L\omega x_2 + \varepsilon x_1 x_2 u_1 + 2L x_1 v_1],$$

$$[1 - 2Lu_1]\dot{x_2} - \varepsilon\dot{v_1} = \Lambda x_1 + \varepsilon[\omega^2 u_1 - 2Lu_1\omega x_1 + 2Lx_2 v_1 - \varepsilon x_1^2 u_1]$$

$$\dot{u_1} = v_1,$$

$$-\varepsilon\dot{x_2} + \varepsilon(1 - \varepsilon\rho_1)\dot{v_1} =$$

$$= -K_1 u_1 - \varepsilon\beta_1 v_1 + \varepsilon(x_1^2 + x_2^2)(u_1 - L) - \varepsilon\omega x_1.$$

Variables ω, x_1, x_2 play the role of projections of the absolute angular velocity of the main body on the axis of the coordinate system associated with it with the origin at the center of mass of this body. The variable u_1 characterizes the deviation of a particle moving inside the damper from its nominal position. In these equations, the nonlinear terms containing the factors $\varepsilon^2 u_1$ are omitted. The value of ε, which characterizes the moment of inertia of the mass moving in the damper, plays the role of a small parameter. Some details can be found in [5].

3. Manifold of steady states

The system of differential equations under consideration has a manifold of steady states:

$$\mathfrak{M} = \{\omega = \Omega = const, \qquad x_1 = x_2 = u_1 = v_1 = 0\}.$$

Following [6], we say that this manifold is stable with respect to variables

$$x_1, x_2, u_1, v_1,$$

If for any $\omega = \Omega$ and any neighborhood of zero W in the space of variables x_1, x_2, u_1, v_1 we can find a neighborhood of zero W_0 of this space such that for any point of this neighborhood the corresponding solution belongs to$W$ for $t \geq 0$.

We will say that $\mathfrak{M}$ is asymptotically stable with respect to variables

$$x_1, x_2, u_1, v_1,$$

if it is stable with respect to these variables and, in addition, the variables x_1, x_2, u_1, v_1 tend to zero with unlimited increase of t.

We will say that $\mathfrak{M}$ is stabilizable if it is asymptotically stable with respect to variables x_1, x_2, u_1, v_1 under $t \to \infty$ the solution tends to some point of the manifold $\mathfrak{M}$.

It follows from the results of [5, 6] that the manifold of steady states $\mathfrak{M}$ is stabilizable if all the roots of the characteristic equation, except for one zero root, have negative real parts. Any perturbed motion, sufficiently close to the unperturbed motion, tends to one of the possible steady motions belonging to the indicated manifold if $t \to \infty$.

4. Model reduction

The differential system under consideration is singularly perturbed one and has a three-dimensional manifold of slow motions:

$$u_1 = \varepsilon f(\omega, x_1, x_2), \qquad v_1 = \varepsilon g(\omega, x_1, x_2),$$

the motion along which is described by a system of three scalar differential equations of the form:

$$q\dot{\omega} = \varepsilon[2x_1\, g - (\Lambda + \omega)x_2 f]\,,$$

$$\dot{x_1} = -\Lambda x_2 + \varepsilon[x_2\big(x_2 - 2L\omega(\Lambda + \omega)\big) + f + 2Lx_1\, g],$$

$$\dot{x_2} = \Lambda x_1 + \varepsilon[(-K_1 f - x_1(x_1 - 2L\omega(\Lambda + \omega)x_2)f + x_1^2 + x_2^2 - (1 + \rho_1)K_1 + \omega^2)f +$$

$$(-\beta_1 + 2Lx_2)g + (\Lambda - \omega)x_1 - L(x_1^2 + x_2^2)] +$$

$$\varepsilon^2\{[\omega^2 - (1 + \rho_1)^2 K_1]f - (1 + \rho_1)\beta_1 g + (1 + \rho_1)\omega x_1 - (1 + \rho_1)(x_1^2 + x_2^2)\} + \varepsilon^3(1 + \rho_1)^2(\Lambda - \omega)x_1.$$

The functions f, g are computed in the usual way [5]. Restricting ourselves linearly in x_1, x_2 terms to the third order and nonlinear - up to the second order in ε inclusive, we write the equations of motion with respect to the integral manifold in the form

$$q\dot{\omega} = \frac{\varepsilon^2}{K_1}[-(\Lambda - \omega)(3\Lambda + \omega)x_2 x_1 + (\Lambda + \omega)Lx_2(x_1^2 + x_2^2)]\,,$$

$$\dot{x_1} = -\Lambda x_2 + \frac{\varepsilon^2}{K_1}[(\Lambda - \omega)x_1^2 x_2 - 2L(\Lambda - \omega)(2\Lambda + \omega)x_2 x_1 +$$

$$2L^2(\Lambda + \omega)x_2(x_1^2 + x_2^2) - Lx_1\, x_2 x_1\,(x_1^2 + x_2^2)],$$

$$\dot{x_2} = \Lambda x_1 + \varepsilon^2[-\frac{1}{K_1}(\Lambda + \omega)(\Lambda - \omega)^2(1 - \frac{\varepsilon L^2}{K_1})x_1 - \frac{\varepsilon}{K_1^2}\,(\Lambda(\Lambda + \omega)(\Lambda - \omega)^2 x_2\beta_1) + \frac{1}{K_1}2L(\Lambda - \omega)((\Lambda + \omega)x_1^2 - \Lambda x_2^2)$$

$$- 2L(\Lambda + \omega)x_1\,(x_1^2 + x_2^2) + L(x_1^2 - \omega^2)(x_1^2 + x_2^2)].$$

After linearizing the equations on an integral manifold for variables x_1, x_2 we obtain the linear with respect to x_1, x_2 subsystem

$$\dot{x_1} = -\Lambda x_2,$$

$$\dot{x_2} = \Lambda x_1 + \varepsilon^2[-\frac{1}{K_1}(\Lambda + \omega)(\Lambda - \omega)^2(1 - \frac{\varepsilon L^2}{K_1})x_1 - \frac{\varepsilon}{K_1^2}\,(\Lambda(\Lambda + \omega)(\Lambda - \omega)^2 x_2\beta_1)\,].$$

The condition of asymptotic stability with respect to variables x_1, x_2 is

$$-\Lambda(\Lambda + \omega)(\Lambda - \omega)^2 < 0.$$

For the integral manifold of slow motions, the following principle is valid: the variety of stationary states of the initial system is stable (unstable, asymptotically stable with respect to some of the variables, is stabilizable) if and only if it is stable (unstable, asymptotically stable with respect to a part of the variables, stabilizable) the variety of stationary states of a system describing the motion on an integral manifold. It is clear that a violation of the resulting inequality entails a loss of stability. This is confirmed by the results of numerical experiments. In the figures below, one can see oscillations with increasing amplitude for the variables x_1, x_2 and ω.

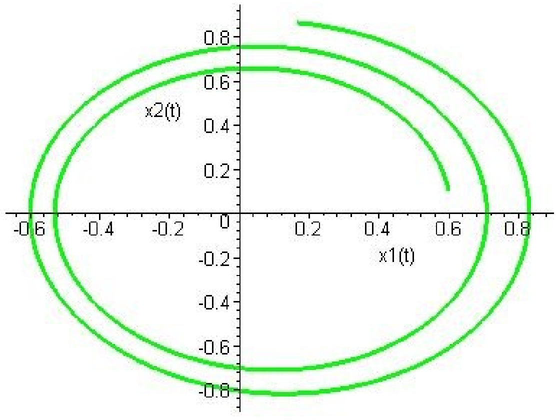

Fig. 1. Projection of the trajectory on the plane of variables x_1, x_2 (the movement is made counter-clockwise).

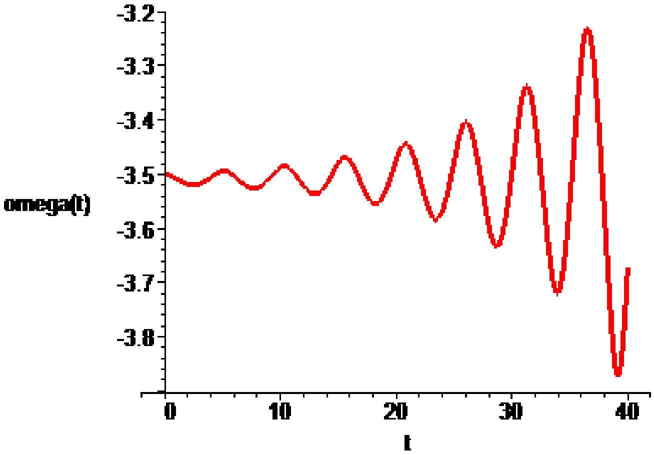

Fig 2. Solution graph for variable ω.

5. Conclusion

In the present work, the mathematical model of a satellite stabilized by rotation has been studied by the methods of the geometric theory of singular perturbations. A reduction of the system was carried out, as a result of which, instead of the original system of five differential equations, its projection onto a three-dimensional slow integral manifold was investigated. It should be noted that, due to the validity of the reduction principle for a slow integral manifold, the reduction is carried out correctly, and the reduced system of three differential equations preserves the basic qualitative properties of the original model. An inequality is obtained, in violation of which the satellite loses the required orientation in space.

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