Investigation of Speed Comparator Operating Features Above the Sea Surface

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Abstract

This article reflects the results of the investigation of an aircraft speed comparator operating features above the sea surface. The structure of the sea surface is considered, the method of modeling the water surface and the radio signal reflected from it is indicated. The results of full-scale tests and the results of mathematical simulation of the flight of an aircraft over the sea surface are given.

1 Introduction

In the course of full-scale tests, while testing the speed comparator of the aircraft, a flight was made over the marine agitated surface. In this case, a discrepancy in the longitudinal velocity (Vx) in the readings of the meter and the standard was revealed (Fig. 1). On the basis of work done [1], it was assumed that the error in the components measurement of the velocity vector during flight over the sea surface is due to the presence of the drift and orbital motion of the water surface, which arises from the presence of wind.

2 Structure of the Underlying Water Surface

A wind excited surface has a very complex relief, continuously changing in time. The surface of the excited sea consists of primary, largest and well-defined waves, and secondary, relatively small waves covering the primary waves. Some parts of the sea surface are covered with foam, bubbles, drops of scattered wave crests, in other areas these formations are absent [2].



Figure 1: The results of the investigation

Structure of surging

The sea waves are divided into the following types for convenience:

- wind waves waves in the area of acceleration. Waves arise under the influence of atmospheric influences
- nascent waves grow over time under the influence of wind force, until they become developed
- developed waves waves that occur when the wind operates at a great distance and for a fairly long time
- swell waves outside the acceleration region

- gravitational waves - a wavelength of more than 2.5 cm, damp for a long time, propagate in the direction specified by the wind

- capillary waves a wavelength of less than 2.5 cm, quickly damp, the wind depends relatively weakly Common elements for waves are (Figure 2) [3]:
- peak is the highest point of the wave crest;
- wave hollow the lowest point of the wave trough;
- height (h) exceeding the peak of the wave;

- length (L) - the horizontal distance between the vertices of two adjacent ridges on the wave profile drawn in the general direction of wave propagation;

- period (T) - time interval between the passage of two adjacent vertices of waves through a fixed vertical; In other words, it is the time interval during which the wave passes a distance equal to its length;

- slope (ϵ) is the ratio of the height of a given wave to its length. The steepness of the wave at different points of the wave profile is different. The average slope of a wave is determined by the ratio:



Figure 2: The basic elements of waves

The widest distribution and application was the representation of the agitated sea surface, as a probabilistic random process. While modeling sea waves, the statistics of wave elements are the most interesting.

The distribution of wave heights for the deep sea hB is well described by the Rayleigh distribution:

$$F(h_{\rm B}) = exp\left[-\frac{\pi}{4} \cdot \left(\frac{h_{\rm B}}{\overline{h_{\rm B}}}\right)^2\right]$$

where: $\overline{h_{B}}$ is the average wave height

The function of periods of waves distribution has the form:

$$F(T_{\rm B}) = exp\left[-\frac{\pi}{4.8} \cdot \left(\frac{T_{\rm B}}{\overline{T_{\rm B}}}\right)^4\right]$$

where: $\overline{T_{B}}$ – is the average wave height

To assess sea waves, a scale is used (Table 1)

Seawave,	Verbal	Wave rate		
ball	characteristic	height, m	length, m	period, s
0	absent	≈ 0	≈ 0	≈ 0
Ι	weak	до 0,25	5,0	2,0
II	moderate	0,25-0,75	5-15	2-3
III	moderate rough	0,75-1,25	15-25	3-4
IV	Rather rough	1,25-2,0	25-40	4-5
V	Very rough	2,0-3,5	4-75	5-7
VI	Very rough	3,5-6,0	75-125	7-9
VII	High	6,0-8,5	125-170	9-11
VIII	_	8,5-11,0	170-220	11-12
IX	confused	>11,0	>220	>12

Table 1: Geometric characteristics of water wave disturbance

The motion of reflecting elements of the sea surface is determined by different types of currents: tidal currents, drift current of the surface layer of water under the action of the local wind, trade-stable currents, for example, the Gulf Stream. In addition, in the case of wind waves in the water surface, orbital motion of the reflecting particles occurs, a moving foam forms on the sea surface, water splashes fly.

The speed of tidal currents, caused by the attraction of the mass of water by the Moon and the Sun, strongly depends on the place and relief of the surrounding land. In some narrow straits this speed can reach several kilometers per hour. In the open ocean, the tidal current does not exceed 0.2 km / h.

The drift current on the seas and oceans is caused by friction forces when the wind affects the elements of the water surface. The experimental value of the wind coefficient Vdr in the middle latitudes, according to the data given in [4, 5], reaches values of 0.02-0.04 m / s. The speed of the trade winds, stretching in the world's oceans for thousands and tens of thousands of kilometers, averages 0.25-0.5 m / s, reaching only values of up to 1 m / s in some sections.

The main effect on the offset of the frequency of the signal reflected from the sea surface is caused by the orbital motion of the reflecting particles of the water surface. As is well known, in the case of wave motion, the water particles in the deep sea move along orbits close to the circles with steady waves (Fig. 3). These circles lie in planes perpendicular to the crests of the waves. Velocity of motion The velocity of motion of a particle along the orbit Vorb at a wave period Tg and its height HB is

$$V_{orb} = \pi \cdot H_{B} / T_{g}$$

It should be noted that practically the particle velocity is not the same at different sites and reaches the maximum value at the crest of the wave.

The orbital motion of particles leads to frequency modulation of the reflected signal and to the spreading of the spectrum if the reflection from different parts of the wave is the same.

Figure 4 shows the experimental dependence of the velocity Vx of the total orbital and drift currents on the wind speed.



Figure 3: The profile of the sea wave and the trajectory of the reflecting particles in the case of wave motion: 1 - wave; 2 - a particle of water; 3 - orbit of the displacement of a water particle



Figure 4: Dependence of the velocity Vm of the orbital and drift currents in the direction of the wind on the wind speed.

Since the velocity of the orbital flow is much greater than the drift velocity, we can assume that VT is determined by the orbital current.

According to fig. 4 there is a minimum wind speed of about 3.6 km / h at which the orbital current appears.

The maximum speed of the orbital current is 11 km / h. In this case, the following formula is obtained, which describes the dependence of Vorb on the wind speed

$$V_{\rm orb} = (0.78 - 1) \cdot U^{1/3}$$

where V_{orb} , U - are expressed in km / h, and the coefficient depends on the wind speed, increasing with its grow.

Table 2: Values of orbital speed

Seawave,ball	1	2	3	4	5
Vorb , m/s	0,375	0,75	0,97	1,26	1,57

Table. 2 shows the values of VoB for different sea surface waves, which can be used to estimate the width of the signal spectrum reflected from the sea surface.

3 Modeling of the Water Surface

While modeling, the sea surface, its corresponding height (hv), length (LB) and the period (TB) of the wave are specified (Table 1). The basic direction of wave propagation (θ) is also selected.

According to [6], in the linear approximation, the surface can be represented as the sum of a number of simple waves (N) having different amplitudes, lengths, sea directions, and random phases. If the wave field does not experience abrupt changes in space, caused, for example, by a sharp change in depth, an agitated surface can be regarded as a quasi-stationary horizontal coordinate function with an ergodic property.



Figure 5: Water surface (h is the height and λ is the length of the crest L of the three-dimensional wave)

The surface of any three-dimensional object can be described by a faceted model represented in the form of an approximation by a polygonal mesh. The cell of such a grid is an elementary reflector (facet). The local elements of the scene are flat reflectors whose vertices lie on the surface of the object.

According to [7], the equation of a plane wave expressing the displacement of the oscillating point (ϵ) along the coordinate x as a function of time t, can be represented as:

$$\varepsilon(t) = a \cdot \cos\left(2\pi f t - \frac{2\pi}{\lambda}x\right),$$

where: f is the oscillation frequency;

 Λ is the wavelength.

On the basis of (1) for each facet k its current coordinates Xk (t), Zk (t) and Hk (t) are calculated in time. The displacement of a particle is determined by the sum of the displacements introduced by each of the N waves. So the current coordinate of the facet k along the H axis:

$$H_{k}(t) = \sum_{i=1}^{N} \frac{h_{\scriptscriptstyle B_{i}}}{2} \cdot \cos\left(\frac{2\pi}{T_{B_{i}}} \cdot t - \frac{2\pi}{L_{B_{i}}} \cdot xr_{i,k} + \phi_{\scriptscriptstyle B0_{i}}\right),$$

where: ϕ_{B0i} - random initial phase of wave i;

 $xr_{i,k} \\ is the coordinate of the facet <math display="inline">k$ along the axis of motion of the wave i:

$$xr_{i,k} = x_k \cdot \cos(\theta_i) + z_k \cdot \sin(\theta_i),$$

In addition to the oscillation of particles along the vertical axis H, movements along the X and Z axes also take place due to the orbital motion, which also represent oscillatory processes:

$$\begin{aligned} \mathrm{Xo}_{i,k}(t) &= \frac{\mathrm{h}_{^{\mathrm{B}i}}}{2} \cdot \sin\left(\frac{2\pi}{\mathrm{T}_{\mathrm{B}i}} \cdot t - \frac{2\pi}{\mathrm{L}_{\mathrm{B}i}} \cdot \mathrm{xr}_{i,k} + \varphi_{^{\mathrm{B}0}i}\right), \\ \mathrm{X}_{k}(t) &= \sum_{i=1}^{N} \mathrm{Xo}_{i,k}(t) \cdot \cos(\theta_{i}), \\ \mathrm{Z}_{k}(t) &= \sum_{i=1}^{N} \mathrm{Xo}_{i,k}(t) \cdot \sin(\theta_{i}). \end{aligned}$$

Fig. 6 shows the vibrational displacements of particles based on the orbital motion and without taking into account.



Figure 6: Wave motion of particles



Figure 7: Water surface (a - 3 balls, Hv = 1 m, $Vorb \approx 0.9 m / s$, b - 5 balls, Hv = 2.75 m, $Vorb \approx 1.4 m / s$)



Figure 8: Randomly distributed facets

4 Modeling of the Reflected Signal

In accordance with the geometry of the problem, the transmitting system emits a probing signal, which is a rectangular pulse signal of unit amplitude with a duration of ΔT . As the sounding signal directs at various times, different surface areas will be illuminated.

The resulting diffraction field of scattering of the radar scene is generally determined by coherent summation of the local scattering fields of individual elements belonging to different surface elements.

When modeling the radar signal reflected from the surface under consideration, the position of each facet and its slope on the surface were taken into account. For this, the plane of the water surface at each site is approximated by the five nearest points.

The equation of the plane has the form:

$$\mathbf{A} \cdot \mathbf{h} + \mathbf{B} \cdot \mathbf{x} + \mathbf{C} \cdot \mathbf{z} + \mathbf{D} = \mathbf{0},$$

Assuming that $A \neq 0$, the equation can be written in the form

$$\mathbf{h} = -\frac{\mathbf{B}}{\mathbf{A}} \cdot \mathbf{x} - \frac{\mathbf{C}}{\mathbf{A}} \cdot \mathbf{z} - \frac{\mathbf{D}}{\mathbf{A}}.$$

Let us introduce the following notation $b = -\frac{B}{A}c = -\frac{C}{A}d = -\frac{D}{A}$ then we can have

 $\mathbf{h} = \mathbf{b} \cdot \mathbf{x} + \mathbf{c} \cdot \mathbf{z} + \mathbf{d}.$

According to the method of least squares, the values of the parameters b, c and d are chosen so that the value Q = $\sum_{k=1}^{5} (b \cdot x_k + c \cdot z_k + d - h_k)^2$ takes the smallest value. In accordance with the theory of extrema, the parameters b, c and d are found from the condition that the following

partial derivatives

$$\begin{cases} \frac{\delta Q}{\delta b} = \sum_{k=1}^{5} 2 \cdot (b \cdot x_k + c \cdot z_k + d - h_k) \cdot x_k = 0, \\ \frac{\delta Q}{\delta c} = \sum_{k=1}^{5} 2 \cdot (b \cdot x_k + c \cdot z_k + d - h_k) \cdot z_k = 0, \\ \frac{\delta Q}{\delta d} = \sum_{k=1}^{5} 2 \cdot (b \cdot x_k + c \cdot z_k + d - h_k) = 0. \end{cases}$$

Transform the system of equations to the form

$$\begin{cases} b \cdot \sum_{k=1}^{5} x_{k}^{2} + c \cdot \sum_{k=1}^{5} x_{k} \cdot z_{k} + d \cdot \sum_{k=1}^{5} x_{k} - \sum_{k=1}^{5} h_{k} \cdot x_{k} = 0, \\ b \cdot \sum_{k=1}^{5} x_{k} \cdot z_{k} + c \cdot \sum_{k=1}^{5} z_{k}^{2} + d \cdot \sum_{k=1}^{5} z_{k} - \sum_{k=1}^{5} h_{k} \cdot z_{k} = 0, \\ b \cdot \sum_{k=1}^{5} x_{k} + c \cdot \sum_{k=1}^{5} z_{k} + d \cdot 5 - \sum_{k=1}^{5} h_{k} = 0 \end{cases}$$

And introduce the notation

$$\begin{split} S_x &= \sum_{k=1}^5 x_k, S_z = \sum_{k=1}^5 z_k, S_h = \sum_{k=1}^5 h_k, \\ S_{xz} &= \sum_{k=1}^5 x_k \cdot z_k, S_{hx} = \sum_{k=1}^5 h_k \cdot x_k, S_{hz} = \sum_{k=1}^5 h_k \cdot z_k, \\ S_{x2} &= \sum_{k=1}^5 x_k^2, S_{z2} = \sum_{k=1}^5 z_k^2. \end{split}$$

Taking into account the notation introduced, the system of equations takes the form

$$\begin{cases} b \cdot S_{x2} + c \cdot S_{xz} + d \cdot S_x - S_{hx} = 0\\ b \cdot S_{xz} + c \cdot S_{z2} + d \cdot S_z - S_{hz} = 0\\ b \cdot S_x + c \cdot S_z + d \cdot 5 - S_h = 0 \end{cases}$$

We solve it by the Cramer method and obtain

$$b = \frac{S_z^2 S_{hx} + S_{xz}(5S_{hz} - S_h S_z) - 5S_{z2}S_{hx} + S_h S_x S_{z2} - S_x S_z S_{hz}}{S_{z2}S_x^2 - 2S_x S_z S_{xz} + S_{x2}S_z^2 + 5S_{xz}^2 - 5S_{x2}S_{z2}},$$

$$c = \frac{S_x^2 S_{hz} + S_{xz}(5S_{hx} - S_h S_x) - 5S_{x2}S_{hz} + S_h S_z S_{x2} - S_x S_z S_{hx}}{S_{z2}S_x^2 - 2S_x S_z S_{xz} + S_{x2}S_z^2 + 5S_{xz}^2 - 5S_{x2}S_{z2}},$$

$$d = \frac{S_h S_{xz}^2 + S_x (S_{z2}S_{hx} - S_{hz}S_{x2}) - S_h S_{x2}S_{z2} + S_z S_{x2}S_{hz} - S_z S_{hx}S_{xz}}{S_{z2}S_x^2 - 2S_x S_z S_{xz} + S_{x2}S_z^2 + 5S_{xz}^2 - 5S_{x2}S_{hz}},$$

Thus, the coefficients b, c and d of the equation of the plane are found, we write it in the form

$$\mathbf{h} - \mathbf{b} \cdot \mathbf{x} - \mathbf{c} \cdot \mathbf{z} - \mathbf{d} = \mathbf{0}.$$

The vector of the normal to the plane

$$\vec{n} = \{1, -b, -c\}$$

Knowing the coefficients of the vector connecting the investigated facet and the aircraft

$$\vec{\mathbf{v}} = \{-\mathbf{y}\mathbf{t}, -\mathbf{x}\mathbf{t}, -\mathbf{z}\mathbf{t}\}$$

It is possible to determine the angle between these two vectors [8]:



Figure 9: The normal vector of the plane of the elementary reflector and the vector in the direction of the LA

5 Results

Modeling of the aircraft flight at the altitude of 100 m above the water surface with various wave parameters was carried out. The longitudinal component of the ground speed Vx was set at 200 m / s, transverse Vz = 0 m / s.

The results of the simulation are the values of the velocity components measured by the correlation meter during the flight of the aircraft, shown in Figures 10 and 11.



Figure 10: Values of Vx when a wave moves against the direction of flight of an aircraft



Figure 11: Values of Vx when the wave moves in the direction of flight of the aircraft

The data presented, obtained by calculation, confirm some influence of the orbital current on the readings of the correlation speed meter.

From the simulation results in Fig. 10, 11 it follows that, when flying over the sea surface, the measured values of the longitudinal velocity component differ from the reference velocity by an amount commensurate with the orbital velocity of the sea surface particles motion. In this case, the direction of wave relative to the direction of flight determines the sign of the introduced measurement error.

It was established that the orbital motion of the elements of the water surface introduces a systematic error (on the orthodrome) in the measurement of the velocity components. The value of the error depends on the parameters of the wave and its direction.

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