Reasoning about exceptions in ontologies: a skeptical preferential approach (Extended Abstract)

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1 Introduction

Reasoning about exceptions in ontologies is nowadays one of the challenges the description logics community is facing, a challenge which is at the very roots of the development of non-monotonic reasoning in the 80s. Many non-monotonic extensions of Description Logics (DLs) have been developed incorporating non-monotonic features from most of the non-monotonic formalisms in the literature [1, 10, 12, 19, 5, 4, 7, 24, 11, 3, 20, 6, 18, 15, 16], or defining new constructions and semantics such as in [2].

The abstract describes a preferential approach for dealing with exceptions in description logics [14], where a typicality operator is used to select the typical (or most preferred) instances of a concept. This approach, as well as the preferential approach in [5], has been developed along the lines of the preferential semantics introduced by Kraus, Lehmann and Magidor [21, 22].

We focus on the rational closure for DLs [7, 9, 6, 16] and, in particular, on the construction developed in [16], which is semantically characterized by minimal preferential models. While the rational closure provides a simple and efficient approach for reasoning with exceptions, exploiting polynomial reductions to standard DLs [13], the rational closure does not allow an independent handling of the inheritance of different defeasible properties of concepts so that, if a subclass of C is exceptional for a given aspect, it is exceptional tout court and does not inherit any of the typical properties of C.

To cope with this problem Lehmann [23] introduced the notion of the lexicographic closure, which was extended to DLs by Casini and Straccia [8], while in [17] Gliozzi proposed a semantic approach in which models are equipped with several preference relations. The lexicographic closure allows for stronger inferences with respect to rational closure, computing the defeasible consequences in the lexicographic closure may require to compute several alternative *bases* [23](namely, consistent sets of defeasible inclusions which are maximal with respect to some specificity ordering).

In this extendedabstract we propose an alternative notion of closure, the *skeptical closure*, which can be regarded as a skeptical variant of the lexicographic closure. It is a refinement of rational closure which allows for stronger inferences, but it is weaker than the lexicographic closure and its computation does not require to generate all the alternative maximally consistent bases. The construction is based on the idea of building a single base, i.e. a single maximal consistent set of defeasible inclusions, starting with the defeasible inclusions with highest rank and progressively adding less specific inclusions, if consistent, but excluding the defeasible inclusions which produce a conflict at a certain stage without considering alternative consistent bases.

2 The rational closure

We briefly recall the logic $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ which is at the basis of a rational closure construction proposed in [16] for $\mathcal{ALC}$. The idea underlying $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ is that of extending the standard $\mathcal{ALC}$ with concepts of the form $\mathbf{T}(C)$, whose intuitive meaning is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C, to distinguish between the properties that hold for all instances of concept C ($C \sqsubseteq D$), and those that only hold for the typical such instances ($\mathbf{T}(C) \sqsubseteq D$). The $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ language is defined as follows: $C_R := A \mid \top \mid \bot \mid \neg C_R \mid C_R \sqcap C_R \mid C_R \sqcup C_R \mid \forall R.C_R \mid \exists R.C_R$, and $C_L := C_R \mid \mathbf{T}(C_R)$, where A is a concept name and R a role name. A KB is a pair (TBox, ABox). TBox contains a finite set of concept inclusions $C_L \sqsubseteq C_R$. ABox contains a finite set of assertions of the form $C_R(a)$ and aRb, for a, b individual names.
The semantics of $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ is defined in terms of rational models: ordinary models of $\mathcal{ALC}$ are equipped with a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements: $x < y$ means that x is more typical than y. The instances of $\mathbf{T}(C)$ are the instances of concept C that are minimal with respect to $<$. We refer to [16] for a detailed description of the semantics and we denote by $\models_{\mathcal{ALC}+\mathbf{T}_{\mathsf{R}}}$ entailment in $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$.

In [16] the rational closure construction has been defined for $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$, extending to DLs the notion of rational closure introduced by Lehmann and Magidor [22]. The definition is based on the notion of exceptionality. Roughly speaking $\mathbf{T}(C) \sqsubseteq D$ holds in the rational closure of K if C is less exceptional than $C \sqcap \neg D$. We shortly recall this construction of the rational closure of a TBox and we refer to [16] for full details.

Definition 1 (Exceptionality of concepts and inclusions). *Let E be a TBox and C a concept. C is* exceptional *for E if and only if $E \models_{\mathcal{ALC}+\mathbf{T}_R} \mathbf{T}(\top) \sqsubseteq \neg C$. An inclusion $\mathbf{T}(C) \sqsubseteq D$ is exceptional for E if C is exceptional for E. The set of inclusions in TBox which are exceptional for E will be denoted by $\mathcal{E}(E)$.*

Given a TBox, it is possible to define a sequence of non increasing subsets of TBox ordered according to the exceptionality of the elements $E_0 \supseteq E_1 \supseteq E_2 \ldots$ by letting $E_0 =$ TBox and, for $i > 0$, $E_i = \mathcal{E}(E_{i-1}) \cup \{C \sqsubseteq D \in$ TBox s.t. $\mathbf{T}$ does not occurr in $C\}$. Observe that, being KB finite, there is an $n \geq 0$ such that, for all $m > n$, $E_m = E_n$ or $E_m = \emptyset$. A concept C has *rank* i (denoted $rank(C) = i$) for TBox, iff i is the least natural number for which C is not exceptional for E_i. If C is exceptional for all E_i then $rank(C) = \infty$ (C has no rank). Rational closure builds on this notion of exceptionality:

Definition 2 (Rational closure of TBox). *Let KB = (TBox, ABox) be a DL knowledge base. The rational closure of TBox is defined as: $\overline{TBox} = \{\mathbf{T}(C) \sqsubseteq D \mid$ either $rank(C) < rank(C \sqcap \neg D)$ or $rank(C) = \infty\} \cup \{C \sqsubseteq D \mid KB \models_{\mathcal{ALC}+\mathbf{T}_R} C \sqsubseteq D\}$, where C and D are $\mathcal{ALC}$ concepts.*

Exploiting the fact that entailment in $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ can be polynomially encoded into entailment in $\mathcal{ALC}$, it is easy to see that deciding if an inclusion $\mathbf{T}(C) \sqsubseteq D$ belongs to the rational closure of TBox is a problem in EXPTIME [16].

Example 1. Let $K = \{\mathbf{T}(Student) \sqsubseteq \neg Pay\_Taxes, \quad \mathbf{T}(WStudent) \sqsubseteq Pay\_Taxes, \mathbf{T}(Student) \sqsubseteq Young, \quad WStudent \sqsubseteq Student\}$ be a knowledge base stating that typical students do not pay taxes, but typical working students (which are students) do pay taxes and that typical students are young. It is possible to see that $E_0 = \{\mathbf{T}(Student) \sqsubseteq \neg Pay\_Taxes, \mathbf{T}(Student) \sqsubseteq Young, WStudent \sqsubseteq Student\}$, $E_1 = \{\mathbf{T}(WStudent) \sqsubseteq Pay\_Taxes, WStudent \sqsubseteq Student\}$ and that the defeasible inclusions $\mathbf{T}(Student \sqcap Italian) \sqsubseteq \neg Pay\_Taxes$ and $\mathbf{T}(WStudent \sqcap Italian) \sqsubseteq Pay\_Taxes$ both belong, as expected, to the rational closure of K, as being Italian is irrelevant with respect to being or not a typical student. However, we cannot conclude that $\mathbf{T}(Student) \sqsubseteq Young$: concept $WStudent$ is exceptional w.r.t. $Student$ concerning the property of paying taxes and, hence, it does not inherit any defeasible property of $Student$.

In the example above the rational closure is too weak to infer that typical working students, as typical student, are young. The lexicographic closure [23] strengthens the rational closure and allows to conclude that typical working students are young. The property of typical students of being toung is inherited by working students, as it is consistent with all the other (strict or defeasible) properties of working students.

3 From the lexicographic to the skeptical closure

Given a concept B, one wants to identify the defeasible properties of the B-elements. Assume that the rational closure of the knowledge base K has already been constructed and that k is the rank of concept B in the rational closure. The typical B elements are clearly compatible with all the defeasible inclusions in E_k, but they might satisfy further defeasible inclusions with lower ranks, i.e. those included in $E_0, E_1, \ldots, E_{k-1}$. In general, there may be alternative maximal sets of defeasible inclusions compatible with B, among which one would prefer those that maximize the number of defeasible inclusions with higher rank. This is indeed what is done by the lexicographic closure [23], which considers alternative maximally preferred sets of defaults called "bases", which, roughly speaking, maximize the number of defaults of higher ranks with respect to those lower ranks (degree of seriousness), and where situations which violate a number of defaults with a certain rank are considered to be less plausible than situations which violates a lower number of defaults with the same rank. In general, there may be exponentially many alternative sets of defeasible inclusions (bases) which are maximal and consistent for a given concept, and the lexicographic closure has to consider all of them to check if a defeasible inclusion is accepted. Instead, in the following, we aim at defining a construction which skeptically builds a single set of defeasible inclusions compatible with B.

Let S^B be the set of typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ in K which are *individually compatible with B (with respect to E_k)*, that is

$S^B = \{\mathbf{T}(C) \sqsubseteq D \in \text{TBox} \mid \; E_k \cup \{\mathbf{T}(C) \sqsubseteq D\} \not\models_{\mathcal{ALC}+\mathbf{T}_\mathsf{R}} \mathbf{T}(\top) \sqsubseteq \neg B\}$.

Clearly, although each defeasible inclusion in S^B is compatible with B, it might be the case that overall set S^B *is not compatible with* B, i.e., $E_k \cup S^B \models_{\mathcal{ALC}+\mathbf{T}_\mathsf{R}} \mathbf{T}(\top) \sqsubseteq \neg B$.

When compatible with B, S^B is the unique maximal basis with respect to the *seriousness ordering* [23]. Let $\delta(E_i)$ denote the set of defeasible inclusions in E_i. When S^B is not compatible with B, we cannot use the defeasible inclusions in S^B to derive conclusions about typical B elements. In this case, we can either use just the defeasible inclusions in E_k, as in the rational closure, or we can additionally use all the defeasible inclusions in $S^B_{k-1} \in \delta(E_{k-1})$, with rank $k-1$, provided they are compatible with B

and E_k and, possibly, we can add all the defeasible inclusions in $S^B_{k-2} \in \delta(E_{k-2})$ (with rank $k-2$) provided they are compatible with B, E_k and S^B_{k-1}, and so on for lower ranks. This leads to the construction below. For each rank j of the rational closure construction, let S^B_j be a set of the defeasible inclusions in E_j as follows: $S^B_j = \{\mathbf{T}(C) \sqsubseteq D \in \delta(E_j) \mid \; E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_{j+1} \cup \{\mathbf{T}(C) \sqsubseteq D\} \not\models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq \neg B\}$ Informally, S^B_j is the set of the defeasible inclusions with rank j, which are not (individually) overridden by defeasible inclusions with higher ranks (from $j+1$ to k).

Definition 3. *Let B be a concept such that $rank(B) = k$. We define the* skeptical closure $S^{sk,B}$ of B *as follows:* $S^{sk,B} = E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_h$, *where h is the least integer j such that* $0 \leq j \leq k-1$ *and* $E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_j \not\models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq \neg B$, *if such a j exists;* $S^{sk,B} = E_k$, *otherwise.*

Intuitively, $S^{sk,B}$ contains, for each rank j, all the defeasible inclusions having rank j which are compatible with B and with the more specific defeasible inclusions (with rank $> j$). As S^B_{h-1} is not included in the skeptical closure, $E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S_h \cup S^B_{h-1} \models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq \neg B$ i.e., the set S^B_{h-1} contains conflicting defeasible inclusions which are not overridden by more specific ones. The inclusions in S^B_{h-1} (and, similarly, all the defeasible inclusions with rank lower than $h-1$) are not added to the skeptical closure of B. Let us now define when a defeasible inclusion is derivable from the skeptical closure of a TBox.

Definition 4. *Let $\mathbf{T}(B) \sqsubseteq D$ be a query and let $k = rank(B)$ be the rank of concept B in the rational closure.* $\mathbf{T}(B) \sqsubseteq D$ is derivable from the skeptical closure of TBox *if* $S^{sk,B} \models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq (\neg B \sqcup D)$.

The identification of the defeasible inclusions in $S^{sk,B}$ requires a number of entailment checks which is linear in the number of defeasible inclusions in TBox. In Example 1 the inclusion $\mathbf{T}(\mathit{WStudent}) \sqsubseteq \mathit{Young}$ is derivable from the skeptical closure of TBox, as $\mathit{WStudent}$ has rank 1 and inclusion $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$ in E_0 is compatible with $\mathit{WStudent}$. No other inclusions in $\delta(E_0)$ are compatible with E_1. Instead, the inclusion $\mathbf{T}(\mathit{WStudent}) \sqsubseteq \mathit{Young}$ is not derivable from the skeptical closure of the KB $K' = \{\mathbf{T}(\mathit{Student}) \sqsubseteq \neg \mathit{Pay\_Taxes}$, $\mathbf{T}(\mathit{Worker}) \sqsubseteq \mathit{Pay\_Taxes}$, $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$, $\mathit{WStudent} \sqsubseteq \mathit{Student} \sqcap \mathit{Worker}\}$. as S_0^{WStudent} is not compatible with $\mathit{WStudent}$ (w.r.t. E_1), due to the conflicting defaults concerning tax payment for Worker and $\mathit{Student}$ (both with rank 0). Hence, the defeasible property that typical students are young is not inherited by typical working students.

Notice that, the property that typical working students are young is accepted in the lexicographic closure of K', as there are two bases (the one including $\mathbf{T}(\mathit{Student}) \sqsubseteq \neg \mathit{Pay\_Taxes}$ and the other $\mathbf{T}(\mathit{Worker}) \sqsubseteq \mathit{Pay\_Taxes}$), both containing $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$. The skeptical closure is indeed weaker than the lexicographic closure. Also, while in the logic $\mathcal{DL}^N$ [2], given the knowledge base K', the concept $\mathit{WStudent}$ has an inconsistent prototype, in the skeptical closure one cannot conclude that $\mathbf{T}(\mathit{WStudent}) \sqsubseteq \bot$ and, using the terminology in [2], the conflict is "silently removed". In this respect, the skeptical closure appears to be weaker than $\mathcal{DL}^N$, although it shares with $\mathcal{DL}^N$ (and with lexicographic closure) a notion of overriding. Detailed comparisons and the study of the semantics underlying the skeptical closure will be subject of future work.

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