Monopolistic Competition Model with Different Technological Innovation and Consumer Utility Levels

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Abstract

We consider a monopolistic competition model with the endogenous choice of technology. We study the impact of technological innovation on the equilibrium and socially optimal variables. We obtained the comparative statics of the equilibrium and socially optimal solutions with respect to the technological innovation parameter and utility level parameter.

1 Introduction

We study a monopolistic competition model with endogenous choice of technology in the closed economy case. We consider "technological innovation" parameter α that influences on costs. Moreover, we consider "consumer utility level" parameter β that influences on utility. The aim is to make comparative statistics of equilibrium and social optimal solutions with respect to parameters α and β.

Our key findings are:

- When parameter α increases,
  - consumption and investments in $R\&D$ both increase;
  - the behavior of the equilibrium and socially optimal variables does not depend on the properties of the costs as a function of investments in $R\&D$;
  - the behavior of the equilibrium variables depends on the elasticity of demand only;
  - the behavior of the socially optimal variables depends on the elasticity of utility only;
  - the equilibrium variables depend on the elasticity of demand and the socially optimal variables depend on the elasticity of utility in the identical way.
- When parameter β increases,
  - the behavior of the equilibrium individual investments in $R\&D$, individual consumption, and mass of firms depend on the behavior of the demand elasticity;

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– the behavior of the social optimal individual investments in $R\&D$, individual consumption, and mass of firms depend on the behavior of the utility elasticity;

– the behavior of the equilibrium total investments in $R\&D$ depends on the behavior of the elasticities of both demand and marginal costs;

– the behavior of the social optimal total investments in $R\&D$ depends on the behavior of the elasticities of both utility and marginal costs.

We discuss the generalization the results to another monopolistic competition models.

The paper concerns with [Antoshchenkova & Bykadorov, 2017]. Our research technique uses [Zhelobodko et al., 2012].

2 The Basic Model of Closed Economy

In this section we set the basic monopolistic competition model for closed economy (one country case). We will use the ArrowPratt measure of concavity defined for any function $g(z)$ as

$$r_g(z) = -\frac{g''(z)\,z}{g'(z)}.$$

Note that for sub-utility function $u(\cdot)$, ArrowPratt measure r_u means the "relative love for variety."

Denote by L the number of consumers and let $[0, N]$ be the endogenous interval of the firms.

2.1 Main Assumptions of Monopolistic Competition

Due to [Chamberlin, 1933] and [Dixit & Stiglitz, 1977], the main assumptions of Monopolistic Competition are:

- consumers are identical, each endowed with one unit of labor;
- labor is the only production factor; consumption, output, prices etc. are measured in labor;
- firms are identical, but produce "varieties" ("almost the same") of good;
- each firm produces one variety as a price-maker, but its demand is influenced by other varieties;
- each variety is produced by one firm that produces a single variety;
- each demand function results from additive utility function;
- number of firms is big enough to ignore firm's influence on the whole industry/economy;
- free entry drives all profits to zero;
- labor supply/demand in each country is balanced.

2.2 Consumer

Each consumer maximizes the total utility function under budget constraint by choosing an infinite-dimensional consumption vector $X = (x_i)_{i\in[0,N]}$ with coordinates $x_i : [0, N] \to \mathbb{R}_+$. Since consumers are identical, we omit the index of a consumer:

$$\begin{cases} \int_0^N u(x_i)\,di \to \max\limits_X \\ \int_0^N p_i x_i di \le w + \frac{\int_0^N \pi_i di}{L} = 1. \end{cases}$$

Here N is number (mass) of firms determined endogenously. Scalar x_i is consumption of variety i by each consumer. We assume that sub-utility function $u(\cdot)$ satisfies the conditions

$$u(0) = 0,\ u'(x_i) > 0,\ u''(x_i) < 0,$$

i.e., it is strictly increasing and strictly concave.

In the budget constraint, w is wage, p_i is the unit price of the variety i, π_i is the profit of firm i. Due to the free entry condition, $\pi_i = 0$ in the equilibrium. Since we consider the general equilibrium model, wage can be normalized to $w \equiv 1$.

The First Order Condition (FOC) for the consumer's problem entails the inverse demand for variety i:

$$p\left(x_i, \lambda\right) = \frac{u'\left(x_i\right)}{\lambda}, \tag{1}$$

where λ is the Lagrange multiplier associated with the budget constraint.

2.3 Producer

We assume that each variety is produced by one firm that produces a single variety. However, unlike the classical setting, each producer chooses the technology level. Namely, if he spends f units of labor as fixed costs, then the total costs of producing y units of output are $c(f)y + f$ units of labor. It is natural to suppose that the function $c(f)$ satisfies the condition $c'\left(f\right) < 0$.

Using (1) the profit maximization problem of the producer i with respect to x_i and f_i can be formulated as

$$\pi_i\left(x_i, f_i, \lambda\right) = \left(p\left(x_i, \lambda\right) - c\left(f_i\right)\right) L x_i - f_i = \left(\frac{u'\left(x_i\right)}{\lambda} - c\left(f_i\right)\right) L x_i - f_i \to \max_{x_i \geq 0, f_i \geq 0}.$$

2.4 Equilibrium

The producers are assumed identical, and hence the producer's problem acquires the same form for each producer. Accordingly, further analysis focuses on the symmetric equilibria $x_i = x, f_i = f$ for any i.

The FOC for the producer's problem are

$$\frac{u''(x)x + u'(x)}{\lambda} - c(f) = 0, \quad c'(f)Lx + 1 = 0. \tag{2}$$

while the Second Order Conditions (SOC) are

$$r_{u'}(x) < 2, \quad -\frac{\left(u'''(x) + 2u''(x)\right) c''(f)x}{\lambda} - \left(c'(f)\right)^2 > 0. \tag{3}$$

Like in the standard monopolistic competition framework, the firms enter into the market until their profit remains positive. Therefore, free entry implies the zero-profit condition

$$\frac{u'(x)}{\lambda} - c(f) = \frac{f}{Lx}. \tag{4}$$

The labor balance condition can be written as

$$\int_0^N \left(c\left(f_i\right) x_i L + f_i\right) di = N(c(f)xL + f) = L. \tag{5}$$

Summarizing, we define the ***symmetric equilibrium*** as a bundle $(x^*, p^*, \lambda^*, f^*, N^*)$ satisfying the following:

- the rational consumption condition (1);
- the rational production conditions (2) and (3);
- the free entry condition (4) and the labor balance condition (5).

Proposition 1. [Antoshchenkova & Bykadorov, 2017] *The equilibrium consumption/investment couple* (x^*, f^*) *is the solution of the system*

$$\frac{r_u(x)x}{1 - r_u(x)} = \frac{f}{Lc(f)}, \quad \left(1 - r_{\ln c}(f) + r_c(f)\right)\left(1 - r_u(x)\right) = 1,$$

under the conditions

$$r_u(x) < 1, \qquad (2 - r_{u'}(x))\, r_c(f) > 1.$$

The equilibrium mass of firms N^*, price p^* and markup are

$$N^* = \frac{L}{c(f^*)\, x^* L + f^*}, \qquad p^* = \frac{c(f^*)}{1 - r_u(x^*)}, \qquad \frac{p^* - c(f^*)}{p^*} = r_u(x^*) = \frac{N^* f^*}{L}.$$

2.5 Social Optimality

Let us consider another optimization problem:maximize the total (social) utility subject to the labor balance condition. Within the framework of our model where $x_i = x, f_i = f$ for any i, the problem is

$$\begin{cases} Nu(x) \to \max\limits_{N,x,f} \\ N(c(f)xL + f) = L, \end{cases}$$

i.e.,

$$\frac{Lu(x)}{c(f)\, xL + f} \to \max_{x,f}.$$

A bundle $(x^{opt}, f^{opt}, N^{opt})$ that solves this problem will be called ***socially optimal***.

In what follows we use the concept of elasticity. The elasticity of a one-variable function $g(x)$ is

$$\varepsilon_g(z) = \frac{g'(z)z}{g(z)}.$$

Note that $r_g(z) = -\varepsilon_{g'(z)}$ and $r_g(z) + \varepsilon_g(z) = r_{\ln g}(z)$.

Proposition 2. [Antoshchenkova & Bykadorov, 2017] *In the social optimality setting, the FOC is*

$$\begin{cases} r_{\ln u}(x) - r_u(x) = \dfrac{c(f)xL}{c(f)xL + f}, \\ c'(f)xL = -1, \end{cases}$$

while the SOC is

$$\varepsilon_c + r_u(x) r_c(f) \equiv r_{\ln c}(f) - (1 - r_u(x))\, r_c(f) > 0.$$

3 Generalization 1: the Case $c = c(f, \alpha)$

Let us study a more complicated case where the cost function depends on investments and also on the parameter α showing how technological innovation affects the costs. Let $c = c(f, \alpha)$ with

$$\frac{\partial c}{\partial f} < 0, \qquad \frac{\partial^2 c}{\partial f^2} > 0, \qquad \frac{\partial c}{\partial \alpha} < 0, \qquad \frac{\partial^2 c}{\partial f \partial \alpha} < 0.$$

The solution is the same as in the case $c = c(f)$, Proposition 1 and Proposition 2 remain valid under the notation

$$r_c := r_c(f, \alpha) := -\frac{\partial^2 c}{\partial f^2} \cdot \frac{f}{\frac{\partial c}{\partial f}} > 0, \qquad r_{\ln c} := r_{\ln c}(f, \alpha) := \frac{\partial^2 \ln c}{\partial f^2} \cdot \frac{f}{\frac{\partial \ln c}{\partial f}}.$$

We study the elasticities $E_{x/\alpha} = \frac{dx}{d\alpha} \cdot \frac{\alpha}{x}, E_{f/\alpha}, E_{N/\alpha}, E_{Nf/\alpha}, E_{p/\alpha}$ with respect to the parameter α. Note that

$$\varepsilon_u = \frac{du}{dx} \cdot \frac{x}{u} > 0, \quad \varepsilon_{c/\alpha} := \frac{\partial c}{\partial \alpha} \cdot \frac{\alpha}{c} < 0, \quad \varepsilon_{c/f} := \frac{\partial c}{\partial f} \cdot \frac{f}{c} < 0, \quad \varepsilon_{c'_f/\alpha} := \frac{\partial}{\partial \alpha}\left(\frac{\partial c}{\partial f}\right) \cdot \frac{\alpha}{\frac{\partial c}{\partial f}} = \frac{\partial^2 c}{\partial f \partial \alpha} \cdot \frac{\alpha}{\frac{\partial c}{\partial f}} > 0.$$

3.1 Comparative Statics w.r.t. α

Here we study the behavior of the equilibrium and socially optimal solutions when the technological innovation parameter α increase. More precisely, we study the signs of the derivatives w.r.t. α. We present the results in terms of the elasticities w.r.t. α. By total differentiation w.r.t. α the equations for the equilibrium (see Proposition 1) and social optimality (see Proposition 2)

Proposition 3. *The elasticities of the equilibrium variables x^*, f^*, N^*, N^*f^* and p^* w.r.t. α are*

$$E_{x^*/\alpha} = \frac{\varepsilon_{c'_f/\alpha} - (1 - r_u)\, r_c \varepsilon_{c/\alpha}}{(2 - r_{u'})\, r_c - 1} > 0, \quad E_{f^*/\alpha} = \frac{(2 - r_{u'})\, \varepsilon_{c'_f/\alpha} - (1 - r_u)\, \varepsilon_{c/\alpha}}{(2 - r_{u'})\, r_c - 1} > 0, \quad E_{N^*/\alpha} = \varepsilon_{r_u} \cdot E_{x^*/\alpha} - E_{f^*/\alpha},$$

$$E_{N^*f^*/\alpha} = E_{\frac{p^* - c(f^*)}{p^*}/\alpha} = \varepsilon_{r_u} \cdot E_{x^*/\alpha}, \qquad E_{p^*/\alpha} = \frac{-r_u \varepsilon_{c'_f/\alpha} + (1 - r_u)\left((2 - r_{u'})\, r_c - 1 + r_u r_c\right) \varepsilon_{c/\alpha}}{(2 - r_{u'})\, r_c - 1} < 0.$$

The elasticities of the socially optimal variables x^{opt}, f^{opt}, N^{opt} and $N^{opt}f^{opt}$ w.r.t. α are

$$E_{x^{opt}/\alpha} = \frac{\varepsilon_{c/f} \cdot \left(\varepsilon_u \cdot \varepsilon_{c/\alpha} \cdot r_{c/f} - \varepsilon_{c'_f/\alpha}\right)}{r_u \cdot r_{c/f} + \varepsilon_c} > 0, \qquad E_{f^{opt}/\alpha} = \frac{\varepsilon_{c'_f/\alpha} + E_{x^{opt}/\alpha}}{r_{c/f}} > 0,$$

$$E_{N^{opt}/\alpha} = -\varepsilon_u \cdot \left(\varepsilon_{c/\alpha} + E_{x^{opt}/\alpha}\right), \qquad E_{N^{opt}f^{opt}/\alpha} = \frac{\varepsilon_{\varepsilon_u} \cdot r_{c/f}}{r_u \cdot r_{c/f} + \varepsilon_{c/f}} \cdot \left(\varepsilon_u \cdot \varepsilon_{c/\alpha} - \frac{\varepsilon_{c'_f/\alpha}}{r_{c/f}}\right).$$

Let us compare the signs of the resulting elasticities of the equilibrium and socially optimal variables (Proposition 3). We summarize the results in Table 1 and Table 2. Note that the symbol "?" in the tables means that the sign of corresponding elasticity is not uniquely determined.

Table 1: Equilibrium: Comparative statics w.r.t. α

	$r'_u < 0$	$r'_u = 0$	$r'_u > 0$
$E_{x^*/\alpha}$	> 0	> 0	> 0
$E_{f^*/\alpha}$	> 0	> 0	> 0
$E_{N^*/\alpha}$	< 0	< 0	?
$E_{N^*f^*/\alpha}$	< 0	$= 0$	> 0

Table 2: Social Optimality: Comparative statics w.r.t. α

	$\varepsilon'_u > 0$	$\varepsilon'_u = 0$	$\varepsilon'_u < 0$
$E_{x^{opt}/\alpha}$	> 0	> 0	> 0
$E_{f^{opt}/\alpha}$	> 0	> 0	> 0
$E_{N^{opt}/\alpha}$	< 0	< 0	?
$E_{N^{opt}f^{opt}/\alpha}$	< 0	$= 0$	> 0

Therefore, *the equilibrium variables depend on the elasticity of demand in a similar way as the socially optimal variables depend on the elasticity of utility.*

4 Generalization 2: the Case $u = u(x, \beta)$

Now let us consider the situation when sub-utility function u depends not only on consumption x, but also on parameter β. We can interpret this parameter as the level of consumption utility (consumption quality). Thus, $u = u(x, \beta)$. Of course, it is natural to assume that $\dfrac{\partial u(x, \beta)}{\partial \beta} > 0$. But under comparative statics with respect to β, as we will see, the signs of equilibrium variables depends essentially on the partial elasticity w.r.t. β of the relative love for variety r_u,

$$\varepsilon_{r_u/\beta} := \frac{\partial}{\partial\beta}\left(r_u(x,\beta)\right)\cdot\frac{\beta}{r_u(x,\beta)} \equiv \frac{\partial}{\partial\beta}\left(\frac{\partial^2 u(x,\beta)}{\partial x^2}\cdot\frac{1}{\frac{\partial u(x,\beta)}{\partial x}}\right)\cdot\frac{\partial u(x,\beta)}{\partial x}\cdot\frac{1}{\frac{\partial^2 u(x,\beta)}{\partial^2 x}}\cdot\beta,$$

while the signs of socially optimal variables depends essentially on the partial elasticity w.r.t. β of the elasticity of sub-utility u,

$$\varepsilon_{\varepsilon_u/\beta} := \frac{\partial}{\partial\beta}\left(\varepsilon_u(x,\beta)\right)\cdot\frac{\beta}{\varepsilon_u(x,\beta)} \equiv \frac{\partial}{\partial\beta}\left(\frac{\partial u(x,\beta)}{\partial x}\cdot\frac{1}{u(x,\beta)}\right)\cdot u(x,\beta)\cdot\frac{1}{\frac{\partial u(x,\beta)}{\partial x}}\cdot\beta.$$

Proposition 4. *The elasticities of the equilibrium variables x^*, f^*, N^*, N^*f^* and p^* w.r.t. β are*

$$E_{x^*/\beta} = -\frac{r_c\cdot\varepsilon_{r_u/\beta}}{(2-r_{u'})r_c-1},\quad E_{f^*/\beta} = \frac{1}{r_c}\cdot E_{x^*/\beta},\quad E_{N^*/\beta} = -cx^*N^*\cdot E_{x^*/\beta},$$

$$E_{N^*f^*/\beta} = \frac{(1-r_u)\cdot\varepsilon_{\varepsilon_c}}{r_c}\cdot E_{x^*/\beta},\quad E_{p^*/\beta} = \varepsilon_{r_u/\beta}\cdot\varepsilon_{c/f}\cdot\frac{r_c(r_u-1)}{(2-r_{u'})r_c-1}.$$

The elasticities of the socially optimal variables x^{opt}, f^{opt}, N^{opt} and $N^{opt}f^{opt}$ w.r.t. β are

$$E_{x^{opt}/\beta} = \frac{r_c\cdot\varepsilon_{\varepsilon_u/\beta}}{\varepsilon_c+r_ur_c+r_c},\quad E_{f^{opt}/\beta} = \frac{1}{r_c}\cdot E_{x^{opt}/\beta},\quad E_{N^{opt}/\beta} = -\varepsilon_u\cdot E_{x^{opt}/\beta},\quad E_{N^{opt}f^{opt}/\beta} = \frac{\varepsilon_u\cdot\varepsilon_{\varepsilon_c}}{r_c}\cdot E_{x^{opt}/\beta}.$$

Let us summarize the results of Proposition 4 in Table 3 and Table 4.

Table 3: Equilibrium: Comparative statics w.r.t. β

	$\frac{\partial r_u}{\partial\beta} < 0$			$\frac{\partial r_u}{\partial\beta} > 0$		
	$\varepsilon_c' > 0$	$\varepsilon_c' = 0$	$\varepsilon_c' < 0$	$\varepsilon_c' > 0$	$\varepsilon_c' = 0$	$\varepsilon_c' < 0$
$E_{x^*/\beta}$	> 0	> 0	> 0	< 0	< 0	< 0
$E_{f^*/\beta}$	> 0	> 0	> 0	< 0	< 0	< 0
$E_{N^*/\beta}$	< 0	< 0	< 0	> 0	> 0	> 0
$E_{N^*f^*/\beta}$	< 0	$= 0$	> 0	> 0	$= 0$	< 0

Table 4: Social Optimality: Comparative statics w.r.t. β

	$\frac{\partial\varepsilon_u}{\partial\beta} > 0$			$\frac{\partial\varepsilon_u}{\partial\beta} < 0$		
	$\varepsilon_c' > 0$	$\varepsilon_c' = 0$	$\varepsilon_c' < 0$	$\varepsilon_c' > 0$	$\varepsilon_c' = 0$	$\varepsilon_c' < 0$
$E_{x^{opt}/\beta}$	> 0	> 0	> 0	< 0	< 0	< 0
$E_{f^{opt}/\beta}$	> 0	> 0	> 0	< 0	< 0	< 0
$E_{N^{opt}/\beta}$	< 0	< 0	< 0	> 0	> 0	> 0
$E_{N^{opt}f^{opt}/\beta}$	< 0	$= 0$	> 0	> 0	$= 0$	< 0

Therefore, *the equilibrium variables depend on the behavior of elasticity of demand w.r.t. β in a similar way as the socially optimal variables depend on the behavior of elasticity of utility w.r.t. β.*

5 Conclusions

We consider a monopolistic competition model with the endogenous choice of technology. We study the impact of technological innovation on the equilibrium and socially optimal variables, namely, consumption, costs, the mass of firms and prices (in the equilibrium case). We obtained the comparative statics of the equilibrium and socially optimal solutions with respect to the technological innovation parameter and utility level parameter.

The results can generalize to another monopolistic competition models: retailing [Bykadorov et al., 2014], market distortion [Bykadorov et al., 2016], international trade [Bykadorov et al., 2015], and to the marketing models: optimization of communication expenditure [Bykadorov et al., 2002] and the effectiveness of advertising [Bykadorov et al., 2009a], pricing [Bykadorov et al., 2009b].

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